

Friday 1 June 2012 – Morning

AS GCE MATHEMATICS

4725 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4725
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

- 1 The complex numbers z and w are given by $z = 6 - i$ and $w = 5 + 4i$. Giving your answers in the form $x + iy$ and showing clearly how you obtain them, find
- (i) $z + 3w$, [2]
- (ii) $\frac{z}{w}$. [3]
- 2 The matrices \mathbf{A} and \mathbf{B} are given by $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$. Find
- (i) \mathbf{AB} , [2]
- (ii) $\mathbf{B}^{-1}\mathbf{A}^{-1}$. [3]
- 3 One root of the quadratic equation $x^2 + ax + b = 0$, where a and b are real, is the complex number $4 - 3i$. Find the values of a and b . [4]
- 4 Find $\sum_{r=1}^n (3r^2 - 3r + 2)$, expressing your answer in a fully factorised form. [7]
- 5 Prove by induction that, for $n \geq 1$, $\sum_{r=1}^n 4 \times 3^r = 6(3^n - 1)$. [5]
- 6 The quadratic equation $2x^2 + x + 5 = 0$ has roots α and β .
- (i) Use the substitution $x = \frac{1}{u+1}$ to obtain a quadratic equation in u with integer coefficients. [3]
- (ii) Hence, or otherwise, find the value of $\left(\frac{1}{\alpha} - 1\right)\left(\frac{1}{\beta} - 1\right)$. [3]
- 7 The loci C_1 and C_2 are given by $|z - 3 - 4i| = 4$ and $|z| = |z - 8i|$ respectively.
- (i) Sketch, on a single Argand diagram, the loci C_1 and C_2 . [6]
- (ii) Hence find the complex numbers represented by the points of intersection of C_1 and C_2 . [2]
- (iii) Indicate, by shading, the region of the Argand diagram for which
- $$|z - 3 - 4i| \leq 4 \text{ and } |z| \geq |z - 8i|. \quad [2]$$
- 8 (i) Show that $\frac{1}{r} - \frac{1}{r+2} \equiv \frac{2}{r(r+2)}$. [1]
- (ii) Hence find an expression, in terms of n , for $\sum_{r=1}^n \frac{2}{r(r+2)}$. [6]
- (iii) Given that $\sum_{r=N+1}^{\infty} \frac{2}{r(r+2)} = \frac{11}{30}$, find the value of N . [4]

- 9 (i) The matrix \mathbf{X} is given by $\mathbf{X} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. Describe fully the geometrical transformation represented by \mathbf{X} . [2]

- (ii) The matrix \mathbf{Z} is given by $\mathbf{Z} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2}(2 + \sqrt{3}) \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2}(1 - 2\sqrt{3}) \end{pmatrix}$. The transformation represented by \mathbf{Z} is

equivalent to the transformation represented by \mathbf{X} , followed by another transformation represented by the matrix \mathbf{Y} . Find \mathbf{Y} . [5]

- (iii) Describe fully the geometrical transformation represented by \mathbf{Y} . [2]

- 10 The matrix \mathbf{D} is given by $\mathbf{D} = \begin{pmatrix} a & 2 & -1 \\ 2 & a & 1 \\ 1 & 1 & a \end{pmatrix}$.

- (i) Find the determinant of \mathbf{D} in terms of a . [3]

- (ii) Three simultaneous equations are shown below.

$$ax + 2y - z = 0$$

$$2x + ay + z = a$$

$$x + y + az = a$$

For each of the following values of a , determine whether or not there is a unique solution. If the solution is not unique, determine whether the equations are consistent or inconsistent.

(a) $a = 3$

(b) $a = 2$

(c) $a = 0$

[7]

THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE



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