RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4727:** Further Pure Mathematics 3

Mark Scheme for June 2012

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

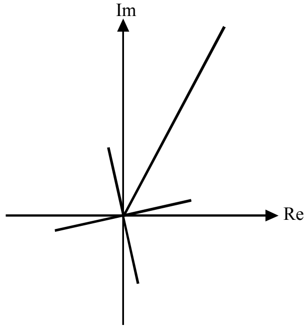
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			METHOD 1			
			$\mathbf{b} = [1, -3, 4] \times [3, 1, 2] = [-10, 10, 10]$	M1	For attempt to find vector product of directions	
			$= k[-1, 1, 1]$	M1	Correct calculation of vector product	Allow 1 error
				A1	For correct **b** .	
			$\Rightarrow\ \mathbf{r} = [1, 4, 2] + t[-1, 1, 1]$	B1 FT	For correct equation. FT from **b**	
				[4]		
			METHOD 2			
			$[x, y, z]\,.\,[1, -3, 4] = 0 \Rightarrow x - 3y + 4z = 0$		For an equation from l_2 perpendicular to normal of plane	
			$[x, y, z]\,.\,[3, 1, 2] = 0 \Rightarrow 3x + y + 2z = 0$	M1	and an equation from l_2 perpendicular to l_1	
			Solving $\Rightarrow [x, y, z] = \mathbf{b} = k[-1, 1, 1]$	M1 A1		
			$\Rightarrow\ \mathbf{r} = [1, 4, 2] + t[-1, 1, 1]$	B1FT	For correct equation. FT. from **b**	Must show "**r** ="
2	(i)		$z^4 = 4\left(\frac{1}{2} + \mathrm{i}\frac{\sqrt{3}}{2}\right) = 4\,\mathrm{cis}\,\frac{1}{3}\pi$	B1	For $\arg(z^4) = \frac{1}{3}\pi$ soi	
				M1	For dividing $\arg(z^4)$ by 4	
			$z = \sqrt{2}\,\mathrm{cis}\left(k\frac{\pi}{12}\right),\quad k = 1, 7, 13, 19$	A1	For any 2 correct values of k	
				A1	For all 4 values of k *and no extras.* Ignore values outside range	For second A1, must be in correct form.
				B1	For modulus of all stated roots = $\sqrt{2}$	Don't accept 1.41.. or $\sqrt[4]{4}$
					SR For $\arg(z^4) = \frac{1}{6}\pi$ award B0 M1 A1 FT for all $\mathrm{cis}\left(k\frac{\pi}{24}\right), k = 1, 13, 25, 37$, A0 B0/B1	
				[5]		

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th colspan="2">Guidance</th></tr>
<tr><td>2</td><td>(ii)</td><td></td><td></td><td>B1

B1

B1

[3]</td><td>For roots forming a square, centre O, on equal-scale axes.

For z^4 and only one root in first quadrant with arguments in ratio approximately 3:1

For $\left|z^4\right|:\left|z\right| \approx 4:\sqrt{2}$ (allow (2,4):1)</td><td>Must be roots distinct from z^4

Penalise once use of points not lines

For all four roots</td></tr>
<tr><td>3</td><td></td><td></td><td>Integrating factor = $e^{\int \cot x \, dx} = e^{\ln \sin x} = \sin x$

$\Rightarrow \frac{d}{dx}(y \sin x) = 2x \sin x$

$\Rightarrow y \sin x = -2x \cos x + \int 2 \cos x \, dx$

$\Rightarrow y \sin x = -2x \cos x + 2 \sin x \ (+c)$
$\left(\frac{1}{6}\pi, 2\right) \Rightarrow c = \frac{1}{6}\pi\sqrt{3}$

$\Rightarrow y = -2x \cot x + 2 + \frac{1}{6}\pi\sqrt{3} \operatorname{cosec} x$</td><td>M1
A1
M1

M1*

A1
A1
M1dep *
A1 FT
A1

[9]</td><td>For IF = $e^{\pm \ln \sin x}$ OR $e^{\pm \ln \cos x}$
For simplified IF
For $\frac{d}{dx}(y.\text{their IF}) = 2x.\text{their IF}$
For attempt to integrate RHS using parts for $\int x \begin{Bmatrix} \sin x \\ \cos x \end{Bmatrix} dx$
For correct RHS 1st stage
oe
For substituting $\left(\frac{1}{6}\pi, 2\right)$ into their GS (with c)
For correctly finding c (FT from GS)
For correct solution AEF of standard notation $y = f(x)$</td><td>

(Must use u = (2)x)

c = 0.907</td></tr>
</table>

Question			Answer	Marks	Guidance	
4	(i)		$\begin{array}{c\|cccc} H & e & r & r^2 & r^3 \\ \hline e & e & r & r^2 & r^3 \\ r & r & r^2 & r^3 & e \\ r^2 & r^2 & r^3 & e & r \\ r^3 & r^3 & e & r & r^2 \end{array}$ $\begin{array}{c\|cccc} K & e & p & q & pq \\ \hline e & e & p & q & pq \\ p & p & e & pq & q \\ q & q & pq & e & p \\ pq & pq & q & p & e \end{array}$	B2 B2 **[4]**	For correct table for H For correct table for K **SR** In both tables allow B1 for 1 or 2 errors	
4	(ii)		Identity = b	B1 **[1]**	For correct identity	
4	(iii)		G is isomorphic to H $\begin{array}{c\|c\|c} G & H & H \\ \hline a & r^2 & r^2 \\ b & e & e \\ c & r & r^3 \\ d & r^3 & r \end{array}$	B1 B1 B1 B1 **[4]**	For H identified as isomorphic to G (may be implied by table) For $a \leftrightarrow r^2$ at least once For $c, d \leftrightarrow r, r^3$ either way For $c, d \leftrightarrow r, r^3$ both ways **and** b corresponds to e explicit. Award fourth **B1** only for completely correct answer. If none of last 3 marks gained, then **SC1** for order of all elements of G and H	
5	(i)		METHOD 1 $\sin^3\theta\cos^2\theta = \left(\dfrac{e^{i\theta}-e^{-i\theta}}{2i}\right)^3\left(\dfrac{e^{i\theta}+e^{-i\theta}}{2}\right)^2$	B1	z may be used for $e^{i\theta}$ throughout For $\left(\dfrac{e^{i\theta}-e^{-i\theta}}{2i}\right)$ *OR* $\left(\dfrac{e^{i\theta}+e^{-i\theta}}{2}\right)$ soi	
			$= -\dfrac{1}{32i}\left(z^3-3z+3z^{-1}-z^{-3}\right)\left(z^2+2+z^{-2}\right)$	M1	For expanding brackets (binomial theorem or otherwise)	
				M1 B1	For full expansion with 12 terms. For $-\dfrac{1}{32i}$	two brackets expanded soi by alternate method
			$= -\dfrac{1}{32i}\left(\left(z^5-z^{-5}\right)-\left(z^3-z^{-3}\right)-2\left(z-z^{-1}\right)\right)$	M1	For grouping terms	Can be seen at any stage
			$= -\dfrac{1}{16}\left(\dfrac{z^5-z^{-5}}{2i}-\dfrac{z^3-z^{-3}}{2i}-2\dfrac{z-z^{-1}}{2i}\right)$		This step, oe, is needed for the final mark	oe includes replacing z^5-z^{-5} with 2isin5θ etc
			$= -\frac{1}{16}(\sin 5\theta - \sin 3\theta - 2\sin\theta)$	A1 **[6]**	For simplification to **AG** WWW	

Question			Answer	Marks	Guidance	
			METHOD 2 $\sin^3\theta\cos^2\theta = \sin^3\theta - \sin^5\theta$ $2i\sin\theta = z - \frac{1}{z}$ $-8i\sin^3\theta = z^3 - 3z + \frac{3}{z} - \frac{1}{z^3}$ $= (z^3 - \frac{1}{z^3}) - (3z - \frac{3}{z})$ $= 2i\sin 3\theta - 6i\sin\theta$ $32i\sin^5\theta = z^5 - 5z^3 + 10z - \frac{10}{z} + \frac{5}{z^3} - \frac{1}{z^5}$ $= (z^5 - \frac{1}{z^5}) - (5z^3 - \frac{5}{z^3}) + (10z - \frac{10}{z})$ $= 2i\sin 5\theta - 10i\sin 3\theta + 20i\sin\theta$ $\sin^3\theta\cos^2\theta$ $= -\frac{1}{32i}(4(2i\,s3\theta - 6i\,s\theta) + (2i\,s5\theta - 10i\,s3\theta + 20i\,s\theta))$ $= -\frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 4\sin 3\theta + 10\sin\theta - 12\sin\theta)$ $= -\frac{1}{16}(\sin 5\theta - \sin 3\theta - 2\sin\theta)$	B1 M1 M1 B1 B1 A1	 For RHS * For grouping terms For RHS of this line and line * above For $-\frac{1}{32i}$ For **ag** www	
5	(ii)		$\sin^3\theta\cos^2\theta = 0 \Rightarrow \sin\theta = 0$ *OR* $\cos\theta = 0$ $\Rightarrow \theta = r\pi$ *OR* $\theta = (2r+1)\frac{1}{2}\pi$ $\Rightarrow \theta = \frac{n\pi}{2}$	M1 A1 A1 **[3]**	For either equation Accept also Sinθ = +/-1 For either solution, **AEF** including a list of the first few For both of above solutions leading to general solution in form of **AG** where $k = 2$	Can be implied by the A mark plus at least $\sin^3\theta = 0$ or similar. At least 2 in list (and no wrong solution)

Question			Answer	Marks	Guidance	
6	(i)		METHOD 1			
			$m^2+4m=0 \Rightarrow m=0,-4$	M1	For attempt to solve correct auxiliary equation	
			CF = $A+Be^{-4x}$	A1	For correct CF	
			PI $y=pe^{2x} \Rightarrow 4p+8p=12$	B1	For PI of correct form seen	Beware poor use of pxe^{2x}
				M1	For differentiating PI and substituting	Scores maximum
			$\Rightarrow p=1$	A1	For correct p	of **M1 A1 B0 M1**
			GS $y=A+Be^{-4x}+e^{2x}$	B1 FT	For using GS = CF + PI with 2 arbitrary constants in GS and none in PI	**A0 B0**
				[6]		
			METHOD 2			
			Integrating $\Rightarrow \dfrac{dy}{dx}+4y=6e^{2x}+c$	M1 B1	For attempt to integrate equation For $+c$ included	
			IF $e^{4x} \Rightarrow \dfrac{d}{dx}\left(ye^{4x}\right)=6e^{6x}+ce^{4x}$	B1√ M1	For correct IF. f.t. from their DE For multiplying through by their IF and attempting to integrate	
			$\Rightarrow ye^{4x}=e^{6x}+\frac{1}{4}ce^{4x}+B$	A1	For correct integration both sides, including $+B$	
			$\Rightarrow y=e^{2x}+A+Be^{-4x}$	A1	For correct solution	Must include "y ="
6	(ii)		$\dfrac{dy}{dx}=-4Be^{-4x}+2e^{2x}$	M1	For differentiating "their GS" with 2 arbitrary constants and substituting values to obtain an equation	If "their CF" is $(A+Bx)e^{-4x}$
			$\left(0, \dfrac{dy}{dx}=6\right) \Rightarrow -4B+2=6 \Rightarrow B=-1$	A1	For correct B	can score max of **M1 A0 B1 A0**
			$(y \approx e^{2x} \Rightarrow) A=0$	B1	For correct A and consistent with" their GS"	
			$\Rightarrow y=-e^{-4x}+e^{2x}$	A1	For correct equation **www**	
				[4]		
7	(i)		$\mathbf{m}=\mathbf{v}+\frac{1}{2}(\mathbf{w}-\mathbf{v}) \Rightarrow$	M1	For using vector triangle, or equivalent, for M	$\overrightarrow{UM}=\overrightarrow{UV}+\overrightarrow{VM}$ $=(\mathbf{v}-\mathbf{u})+\dfrac{1}{2}(\mathbf{w}-\mathbf{v})$
			$\overrightarrow{UM}=\mathbf{v}+\frac{1}{2}(\mathbf{w}-\mathbf{v})-\mathbf{u}=\frac{1}{2}(\mathbf{v}+\mathbf{w}-2\mathbf{u})$	A1	For correct expression **AG**	
					SR Allow use of ratio theorem	Minimum $-\mathbf{u}+\dfrac{1}{2}(\mathbf{v}+\mathbf{w})$
				[2]		

Question			Answer	Marks	Guidance	
7	(ii)		METHOD 1 (first 3 marks)			
			$\overrightarrow{UM}$ is $\mathbf{r}=\mathbf{u}+\frac{1}{2}t(\mathbf{v}+\mathbf{w}-2\mathbf{u})$	M1*	For equation of *UM*	
				M1*	For attempt to find a suitable value of t	
			$t=\frac{2}{3} \Rightarrow \mathbf{u}+\frac{1}{3}(\mathbf{v}+\mathbf{w}-\mathbf{2u})=\frac{1}{3}(\mathbf{u}+\mathbf{v}+\mathbf{w})$	A1	For $t=\frac{2}{3}$ and *G* obtained **AG**	
			METHOD 2 (first 3 marks)			
			$\overrightarrow{UG}=\frac{1}{3}(\mathbf{u}+\mathbf{v}+\mathbf{w})-\mathbf{u}=\frac{1}{3}(\mathbf{v}+\mathbf{w}-\mathbf{2u})$	M1* M1*	For finding directions of *UG* or *MG* For comparing with *UM*	
			OR			
			$\overrightarrow{MG}=\frac{1}{3}(\mathbf{u}+\mathbf{v}+\mathbf{w})-\frac{1}{2}(\mathbf{v}+\mathbf{w})=-\frac{1}{6}(\mathbf{v}+\mathbf{w}-2\mathbf{u})$			
			$\Rightarrow$ *U*, *G*, *M* collinear	A1	For showing *G* lies on *UM* **AG**	
			By symmetry of $\overrightarrow{OG}$ in **u**, **v**, **w**	B1	For use of symmetry, or by repeating method for UM twice more.	
			G also lies on *VN*, *WP* $\Rightarrow$ *UM*, *VN*, *WP* intersect at *G*	B1dep *	For complete reasoning to **AG**	
				[5]		
7	(iii)		Line is $\mathbf{r}=\frac{1}{3}(\mathbf{u}+\mathbf{v}+\mathbf{w})+t(\mathbf{u}-\mathbf{v})\times(\mathbf{u}-\mathbf{w})$ (etc)	B1	For $r=\frac{1}{3}(\mathbf{u}+\mathbf{v}+\mathbf{w})+t\times$"any vector"	
				B1	For a correct **n**, using any 2 of $\pm(\mathbf{u}-\mathbf{v})$, $\pm(\mathbf{v}-\mathbf{w})$, $\pm(\mathbf{w}-\mathbf{u})$	Allow $\overrightarrow{UV}\times\overrightarrow{VW}$ or similar
				[2]		

Question			Answer	Marks	Guidance	
7	(iv)		METHOD 1 $\mathbf{n} = [1, 0, -1] \times [0, 1, -1]$ (etc) $= k[1, 1, 1]$	M1*	For attempt to find $\mathbf{n}$	May see use of $\dfrac{\lvert p.n - d \rvert}{\lvert n \rvert}$
			UVW is $\mathbf{r}.\mathbf{n} = [1, 0, 0].[1, 1, 1] = 1$	M1dep*	For substituting a point	
			$\Rightarrow d = \dfrac{1}{\sqrt{3}}$	A1	For correct d	
				[3]		
			METHOD 2 UVW is $x + y + z = 1$ (from given $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$)	M2	For attempt to find cartesian equation	
			$\Rightarrow d = \dfrac{1}{\sqrt{3}}$	A1	For correct d	
			METHOD 3 $\overrightarrow{OG} = \frac{1}{3}(\mathbf{u} + \mathbf{v} + \mathbf{w})$	M1*	For stating or implying $\lvert\overrightarrow{OG}\rvert$ is d	
			$\Rightarrow OG = \sqrt{\frac{1}{9} + \frac{1}{9} + \frac{1}{9}}$	M1dep*	For finding magnitude	
			$\Rightarrow d = \dfrac{1}{\sqrt{3}}$	A1	For correct d	

Question			Answer	Marks	Guidance	
8	(i)		For R, $\cos^2\theta + \sin^2\theta = 1$ $\Rightarrow$ad-bc =1 ($\Rightarrow R \subset M$)	B1	For showing $R \subset M$	
			$R(\theta)R(\phi) = R(\theta+\phi)$ and hence closed, since $\cos\theta\cos\phi - \sin\theta\sin\phi = \cos(\theta+\phi)$ and	M1	For multiplying 2 distinct elements	
			$\pm(\cos\theta\sin\phi + \sin\theta\cos\phi) = \pm\sin(\theta+\phi)$	A1	For obtaining $R(\theta)R(\phi) \in R$	Must demonstrate use of compound angles or explain rotations.
			Identity $\theta = 0 \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \in R$	B1	For identity element related to θ = 0	
			Inverse $R(-\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$	B1	For inverse element …	
			$= \begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{pmatrix}$	B1	...converted to form of elements of R	
				[6]		
			SR For use of $\left(a, b \in R \Rightarrow ab^{-1} \in R\right) \Leftrightarrow R$ is a subgroup of M			
			For R, $\cos^2\theta + \sin^2\theta = 1$ $\Rightarrow$ $R \subset M$	B1	For showing $R \subset M$	
			$R(\theta)R(\phi)^{-1} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{pmatrix}$	B1 B1 M1 A1	For considering $R(\theta)R(\phi)^{-1}$ For correct inverse For multiplying elements For correct product	
			$= \begin{pmatrix} \cos(\theta-\phi) & -\sin(\theta-\phi) \\ \sin(\theta-\phi) & \cos(\theta-\phi) \end{pmatrix} \in R$			
			Set is non-empty	B1	Can be implied by identity element related to θ = 0	

Question			Answer	Marks	Guidance	
8	(ii)		For $\theta = \frac{1}{3}k\pi$ elements are	B1	For $\theta = \frac{1}{3}\pi$ soi	Allow degrees instead of radians.
			$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} \frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}$, $\begin{pmatrix} -\frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & -\frac{1}{2} \end{pmatrix}$,	M1	For using "their θ" in $\begin{pmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{pmatrix}$ for at least 2 values of k, or lists all 6 values of θ	
			$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, $\begin{pmatrix} -\frac{1}{2} & \frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & -\frac{1}{2} \end{pmatrix}$, $\begin{pmatrix} \frac{1}{2} & \frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}$	A1 A1 A1 **[5]**	For identity and one other element other than (-I) For 2 more elements For all 6 elements correct	

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