RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4726:** Further Pure Mathematics 2

Mark Scheme for June 2012

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

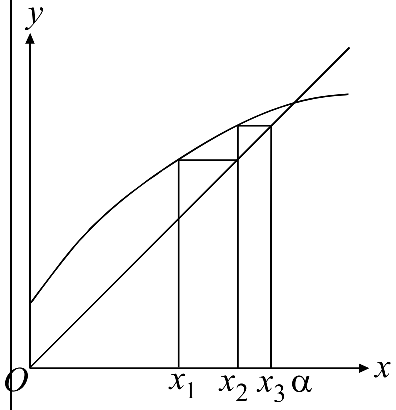
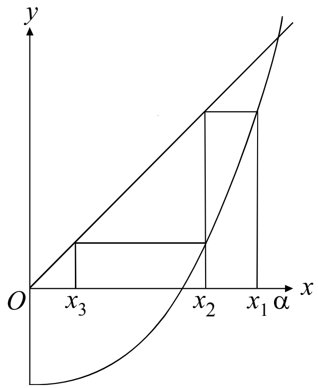
h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

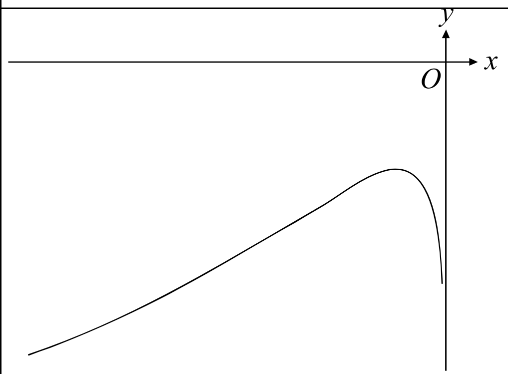
Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			$\operatorname{sech} 2x = \dfrac{2}{e^{2x}+e^{-2x}}$	B1	For sech2x expression **oe**	
			$u = e^{2x} \Rightarrow du = 2e^{2x}dx$ **or** $x = \dfrac{1}{2}\ln u \Rightarrow dx = \dfrac{1}{2u}du$	M1	For differentiating substitution correctly and substituting into *their* integral	
			$\Rightarrow I = \int \operatorname{sech} 2x\,dx = \int \dfrac{2}{e^{2x}+e^{-2x}}dx$			
			$= \int \dfrac{2}{\left(e^{2x}+e^{-2x}\right)}.\dfrac{du}{2e^{2x}}$	A1	For correct integral	
			$= \int \dfrac{1}{u^2+1}du$			
			$= \tan^{-1} u\ (+c) = \tan^{-1}\left(e^{2x}\right) + c$	M1 A1 **[5]**	For integration to $\tan^{-1}(\ )$ For correct expression (c required)	

Question			Answer	Marks	Guidance	
2	(i)		$r = 0 \Rightarrow \cos\theta = 0, \sin 2\theta = 0$	M1	For $r = 0$ **(soi)** and attempt to solve for θ	
			$\Rightarrow \theta = 0, \frac{1}{2}\pi$	A1	For both values and no others (ignore values outside range)	
				[2]		
2	(ii)		$\frac{dr}{d\theta} = -\sin\theta \sin 2\theta + 2\cos 2\theta \cos\theta$	M1	For attempt to find $\frac{dr}{d\theta}$ using product rule	
			$= 0$	A1	For correct $\frac{dr}{d\theta}$ set $= 0$ **soi**	
			Alternatively: $r = 2\cos^2\theta \sin\theta \Rightarrow \frac{dr}{d\theta} = 2\cos^3\theta - 4\cos\theta \sin^2\theta$			
			$\Rightarrow 2\sin^2\theta \cos\theta = 2(1 - 2\sin^2\theta)\cos\theta$			
			$\Rightarrow \sin\theta = \frac{1}{\sqrt{3}} \left(\cos\theta = \frac{\sqrt{2}}{\sqrt{3}},\ \tan\theta = \frac{1}{\sqrt{2}} \right)$	A1	For correct value of $\sin\theta$ (**OR** $\cos\theta$ *OR* $\tan\theta$) or decimal equivalent; $\sin\theta = 0.546$ or $\cos\theta = 0.816$ or $\tan\theta = 0.707$	
			$\Rightarrow r = \frac{4}{3\sqrt{3}} = \frac{4}{9}\sqrt{3}$	A1	For correct r or anything that rounds to 0.77	
				[4]		
2	(iii)		$x = r\cos\theta,\ y = r\sin\theta$	M1	For substituting $x = r\cos\theta$ **OR** $y = r\sin\theta$	
			$\Rightarrow r = \frac{x}{r}.2\frac{y}{r}\frac{x}{r}$	M1	For $r^2 = x^2 + y^2$ soi	
			$\Rightarrow \left(x^2 + y^2\right)^2 = 2x^2 y$	A1	For a correct cartesian equation **Any equivalent form** without fractions	
				[3]		

Question			Answer	Marks	Guidance	
3	(i)		$\tanh 2x \equiv \dfrac{\sinh 2x}{\cosh 2x} \equiv \dfrac{2\sinh x\cosh x}{\cosh^2 x+\sinh^2 x}$	M1	For $\dfrac{\sinh 2x}{\cosh 2x}$ and use double angle formulae	
			$\equiv \dfrac{2\tanh x}{1+\tanh^2 x}$	A1	For division by $\cosh^2 x$ seen	N.B. Tanh($A+B$) not in formula book
				[2]		
3	(ii)		$\dfrac{10t}{(t^2+1)}=(1+6t)$	M1	For using (i) to obtain equation in *t*.	
			$\Rightarrow 6t^3+t^2-4t+1=0$	A1	Correct cubic equation	
			$\Rightarrow (t+1)(3t-1)(2t-1)=0$	M1	Attempt to solve cubic (calculator OK)	
			$\Rightarrow t=(-1), \frac{1}{3}, \frac{1}{2}$	A1	Solution. Ignore any extra values at this stage	
			$x=\frac{1}{2}\ln\dfrac{1+t}{1-t} \Rightarrow x=\frac{1}{2}\ln 2, \frac{1}{2}\ln 3$	M1 A1	For using ln form for $\tanh^{-1}$ Correct 2 values (only) **oe**	
				[6]		
			Alternative: M1 $e^{4x}-5e^{2x}+6=0$ A1 $\Rightarrow (e^{2x}-2)(e^{2x}-3)=0$ M1 $\Rightarrow e^{2x}=2,\ 3$ A1 $\Rightarrow 2x=\ln 2,\ \ln 3$ M1 $\Rightarrow x=\dfrac{1}{2}\ln 2,\ \dfrac{1}{2}\ln 3$ A1		Use exponentials to obtain a quadratic in e^{2x} Correct Solve quadratic Soln Take logs	

Question			Answer	Marks	Guidance	
4	(i)		 $x_2 = 1.3869...$ $x_3 = 1.3938$	B1 B1 B1 **[3]**	For correct value (4 d.p. or better) For correct value. For sketch showing staircase towards α. (Vertical lines do not need to be labelled)	
4	(ii)			B1 B1 **[2]**	For sketch like $y = \frac{1}{2}\left(x^4 - 1\right)$ and $y = x$ (curve or continuation of curve cuts - y axis.) For sketch showing staircase away from α.("Away" means labelling or arrows required.) Labelling means x_1, x_2,... in right place or numeric values.	
4	(iii)		$x_{n+1} = x_n - \frac{x_n^4 - 2x_n - 1}{4x_n^3 - 2}$ $1.35 \to 1.398268$ $\to 1.395348 \to 1.395337$ $\Rightarrow 1.3953$	M1 A1 A1 A1 **[4]**	For deriving the iterative formula For correct formula For 1st value For correct 4dp α with 2 iterates equal to 4 dp. (i.e. last two iterates agree to 4dp) **www**	

Question			Answer	Marks	Guidance	
5	(i)		$f'(x)=\frac{1}{\sqrt{1+x^2}}+\frac{1}{\sqrt{1+\frac{1}{x^2}}}.\frac{-1}{x^2}$	M1 B1	For attempt to differentiate using chain rule. First term correct	
			$=\frac{1}{\sqrt{1+x^2}}\left(1-\frac{1}{x}\right)$			
			$=0\Rightarrow x=1$	M1 A1	For attempt to solve their f '(x) = 0 For correct value of x (ignore $x=-1$)**www**	
			$f(1)=2\sinh^{-1}1=2\ln\left(1+\sqrt{2}\right)$	A1	For correct value obtained **www AG**	
				[5]		
5	(ii)			B1	For correct shape in 3rd quadrant only(condone inclusion of the 1st quadrant part given)	
			$\left\{f(x)\geqslant 2\ln\left(1+\sqrt{2}\right),\ f(x)\leqslant -2\ln\left(1+\sqrt{2}\right)\right\}$	B1 B1	For one part of range For other part of range **SC** B1 Both ranges correct but < and > used	
				[3]		

Question			Answer	Marks	Guidance	
6	(i)		$I_n = \left[-x^n \cos x\right]_0^{\pi} + n\int_0^{\pi} x^{n-1} \cos x \, dx$	M1 A1	For attempt to integrate by parts For correct result before limits	
			$= \pi^n + n\left\{\left[x^{n-1} \sin x\right]_0^{\pi} - (n-1)\int_0^{\pi} x^{n-2} \sin x \, dx\right\}$	M1 A1	For attempt at second integration by parts For correct result before limits	
			$\Rightarrow I_n = \pi^n - n(n-1)I_{n-2}$	A1 **[5]**	For correct result **www AG**	
6	(ii)		$I_1 = \left[-x \cos x\right]_0^{\pi} + \int_0^{\pi} \cos x \, dx$	M1	For integrating by parts for I_1	
			$\Rightarrow I_1 = \pi + \left[\sin x\right]_0^{\pi} = \pi$	A1	For correct I_1 SC B1 $I_1 = \pi$ with no working	
			$I_3 = \pi^3 - 6I_1$, $I_5 = \pi^5 - 20I_3$	M1	For substituting $n = 3$ or 5 in reduction formula	
			$\Rightarrow I_5 = \pi^5 - 20\pi^3 + 120\pi$	A1 **[4]**	For correct result	

Question			Answer	Marks	Guidance	
7	(i)		$a=2,\ b=n$ $c=1,\ \ d=n-1$	B1 B1 B1 **[3]**	for any 2 correct for the third correct for all four correct. Allow values inserted in series. **SC** treat $a=\frac{1}{2}$ etc as MR –1 once	
7	(ii)		$\int_1^n \frac{1}{x}\,dx = \ln n$ $1+\frac{1}{2}+\ldots+\frac{1}{n}<1+\ln n$ $\Rightarrow\ f(n)<1$ (upper bound) $\Rightarrow\ f(n)>\frac{1}{n}$ (lower bound)	B1 M1 A1 A1 **[4]**	For integral evaluated **soi** (Definite integral between 1 and n) For adding 1 *OR* $\frac{1}{n}$ to series For correct upper bound For correct lower bound	
7	(iii)		$f(n+1)-f(n)=\frac{1}{n+1}-\ln(n+1)+\ln n$ $=\frac{1}{n+1}-\ln\left(\frac{n+1}{n}\right)\approx\frac{1}{n+1}-\left(\frac{1}{n}-\frac{1}{2n^2}\right)$ $\approx\frac{1}{n+1}-\frac{2n-1}{2n^2}$ $\approx-\frac{n-1}{2n^2(n+1)}$	B1 M1 M1 A1 A1 **[5]**	For correct expression For combining ln terms For attempt to expand $\ln\left(1+\frac{1}{n}\right)$ Correct expansion of $\ln\left(1+\frac{1}{n}\right)$ For correct expression **AG**	Any expansion of $\ln(1+n)$ oe is 0

Alternative answer to 7(iii)

Question			Answer	Marks	Guidance	
7	(iii)		$f(n+1)-f(n)=\frac{1}{n+1}-\ln(n+1)+\ln n$ $=\frac{1}{n+1}-\ln\left(\frac{n+1}{n}\right)$ $=\frac{1}{n+1}+\ln\left(\frac{n}{n+1}\right)$ $=\frac{1}{n+1}+\ln\left(1-\frac{1}{(n+1)}\right)$ $=\frac{1}{n+1}+\left(-\frac{1}{(n+1)}-\frac{1}{2(n+1)^2}\right)$ $=-\frac{1}{2(n+1)^2}$	B1 M1 M1 A1	For correct expression For combining ln terms and attempt to expand For attempt to expand $\ln\left(1-\frac{1}{(n+1)}\right)$ Correct expansion of $\ln\left(1-\frac{1}{(n+1)}\right)$ **Max 4**	

Question			Answer	Marks	Guidance	
8	(i)		$\mathrm{q}(x) = x+2$	B1	For correct $\mathrm{q}(x)$ **soi oe**	
			$y = \dfrac{A}{x+2} + \frac{1}{2}x + 1$	M1	For expressing y in this form. Allow $cx+d$ for A	
			$\left(-1, \frac{17}{2}\right) \Rightarrow A = 8$	A1	For correct A	
			$y = \dfrac{\frac{1}{2}x^2 + 2x + 10}{x+2} \Rightarrow \mathrm{p}(x) = \frac{1}{2}x^2 + 2x + 10$	A1 **[4]**	For correct $\mathrm{p}(x)$ Allow equal multiples of $\mathrm{p}(x)$ and $\mathrm{q}(x)$	
			Alternative: $\mathrm{q}(x) = x+2$ B1		For correct $\mathrm{q}(x)$ **soi oe**	
			$y = \dfrac{ax^2+bx+c}{\mathrm{q}(x)} = ax + (b-2a) + \dfrac{c-2b+4a}{x+2}$ M1		For division by *their* $\mathrm{q}(x)$	1st line of division and 1st term in quotient should be seen for correct method
			$y = \frac{1}{2}x + 1 \Rightarrow a = \frac{1}{2}, b = 2$ A1		For correct a and b **oe**	
			$\left(-1, \frac{17}{2}\right) \Rightarrow c - 2b + 4a = 8 \Rightarrow c = 10$ A1		For correct c **oe**	
8	(ii)		$\frac{1}{2}x^2 + (2-y)x + 10 - 2y = 0$	M1	For attempt to rearrange as quadratic in x	
			$b^2 - 4ac \geqslant 0 \Rightarrow (2-y)^2 \geqslant 2(10-2y)$	M1	For use of $b^2 - 4ac$ ($\le$ or $\ge$ or $=$ or $<$ or $>$)	
			$\Rightarrow y^2 \ge 16 \Rightarrow \{y \le -4, \; y \ge 4\}$	A1 A1	For critical values ± 4 For correct range. (Must be $\le$ and $\ge$) **www**	
			(pto for alternative)	**[4]**		
8	(iii)		$\left(\frac{1}{2}x+1\right)^2 = \dfrac{\frac{1}{2}x^2 + 2x + 10}{x+2}$ **OR** $y^2 = \dfrac{4}{y} + y$	B1ft	For a correct equation derived from intersection of C_2 with $y = \frac{1}{2}x + 1$ FT from (i)	
			$\Rightarrow x^3 + 4x^2 + 4x - 32 = 0$ **OR** $y^3 - y^2 - 4 = 0$	M1 A1	For obtaining a cubic Correct cubic	
			$\Rightarrow (2, 2)$	A1 **[4]**	Coordinates correct **www**	

Alternative to 8(ii)

Question			Answer	Marks	Guidance	
8	(ii)		$y=\dfrac{\frac{1}{2}x^2+2x+10}{x+2}$ $\Rightarrow \dfrac{dy}{dx}=\dfrac{(x+2)(x+2)-\left(\frac{1}{2}x^2+2x+10\right)}{(x+2)^2}$	M1	Diffn using quotient rule	
			$=0$ when $(x+2)(x+2)=\left(\frac{1}{2}x^2+2x+10\right)$ $\Rightarrow \frac{1}{2}x^2+2x-6=0 \Rightarrow x^2+4x-12=0$	M1	Attempt to find soln using $\dfrac{dy}{dx}=0$	
			$\Rightarrow (x+6)(x-2)=0$ $\Rightarrow x=2,\ y=4$ $\quad x=-6,\ y=-4$	A1		
			$\{y\le -4,\ \ y\ge 4\}$	A1	For correct range. (Must be ≤ and ≥) **www**	
			Alternatively: $y=\frac{1}{2}x+1+\dfrac{8}{x+2}$ $\Rightarrow \dfrac{dy}{dx}=\dfrac{1}{2}-\dfrac{8}{(x+2)^2}$ M1 $=0$ when $\dfrac{1}{2}-\dfrac{8}{(x+2)^2}\Rightarrow (x+2)^2=16$ M1 $\Rightarrow x+2=\pm 4\Rightarrow x=2$ or -6 $\Rightarrow y=4$ or -4 A1 $\{y\le -4,\ \ y\ge 4\}$ A1		Diffn using chain rule Attempt to find soln using $\dfrac{dy}{dx}=0$ For correct range. (Must be ≤ and ≥) **www**	

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