OXFORD CAMBRIDGE AND RSA EXAMINATIONS

AS GCE

4725

MATHEMATICS

Further Pure Mathematics 1

QUESTION PAPER

FRIDAY 20 JANUARY 2012: Afternoon

DURATION: 1 hour 30 minutes

SUITABLE FOR VISUALLY IMPAIRED CANDIDATES

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4725
List of Formulae (MF1)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- **The Question Paper will be found in the centre of the Printed Answer Book.**
- **Candidates answer on the Printed Answer Book or any suitable paper provided by the Centre. The Printed Answer Book may be enlarged by the Centre.**
- **Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).**
- **Use black ink. HB pencil may be used for graphs and diagrams only.**
- **Answer ALL the questions.**
- **Read each question carefully. Make sure you know what you have to do before starting your answer.**
- **You are permitted to use a scientific or graphical calculator in this paper.**
- **Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.**

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- **The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.**
- **YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.**
- **The total number of marks for this paper is 72.**
- **The Printed Answer Book consists of 12 pages.**

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- **Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.**

1 The complex number $a + 5\mathrm{i}$, where a is positive, is denoted by z. Given that $|z| = 13$, find the value of a and hence find $\arg z$. [4]

2 The matrices A and B are given by $\mathbf{A} = \begin{pmatrix} 3 & 4 \\ 2 & -3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 4 & 6 \\ 3 & -5 \end{pmatrix}$, and I is the 2×2 identity matrix.

Given that $p\mathbf{A} + q\mathbf{B} = \mathbf{I}$, find the values of the constants p and q. [5]

3 Use an algebraic method to find the square roots of $3 + (6\sqrt{2})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are exact real numbers. [6]

4 Find $\sum_{r=1}^{n} r(r^2 - 3)$, expressing your answer in a fully factorised form. [6]

5 (a) Find the matrix that represents a reflection in the line $y = -x$. [2]

(b) The matrix C is given by $\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$.

(i) Describe fully the geometrical transformation represented by C. [2]

(ii) State the value of the determinant of C and describe briefly how this value relates to the transformation represented by C. [2]

6 Sketch, on a single Argand diagram, the loci given by $|z - \sqrt{3} - \mathrm{i}| = 2$ and $\arg z = \frac{1}{6}\pi$. [6]

7 The matrix M is given by $\mathbf{M} = \begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix}$.

(i) Show that $\mathbf{M}^4 = \begin{pmatrix} 81 & 0 \\ 80 & 1 \end{pmatrix}$. [3]

(ii) Hence suggest a suitable form for the matrix $\mathbf{M}^n$, where n is a positive integer. [2]

(iii) Use induction to prove that your answer to part (ii) is correct. [4]

8 (i) Show that $\frac{r}{r+1} - \frac{r-1}{r} \equiv \frac{1}{r(r+1)}$. [2]

(ii) Hence find an expression, in terms of n, for

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \ldots + \frac{1}{n(n+1)}.$$ [4]

(iii) Hence find $\sum_{r=n+1}^{\infty} \frac{1}{r(r+1)}$. [2]

9 The matrix X is given by $\mathrm{X} = \begin{pmatrix} a & 2 & 9 \\ 2 & a & 3 \\ 1 & 0 & -1 \end{pmatrix}$.

(i) Find the determinant of X in terms of a. [3]

(ii) Hence find the values of a for which X is singular. [3]

(iii) Given that X is non-singular, find X^{-1} in terms of a. [4]

10 The cubic equation $3x^3 - 9x^2 + 6x + 2 = 0$ has roots α, β and γ.

(i) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. [3]

The cubic equation $x^3 + ax^2 + bx + c = 0$ has roots α^2, β^2 and γ^2.

(ii) Show that $c = -\frac{4}{9}$ and find the values of a and b. [9]

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THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE