OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4737:** Decision Mathematics 2

Mark Scheme for June 2011

Oxford Cambridge and RSA Examinations

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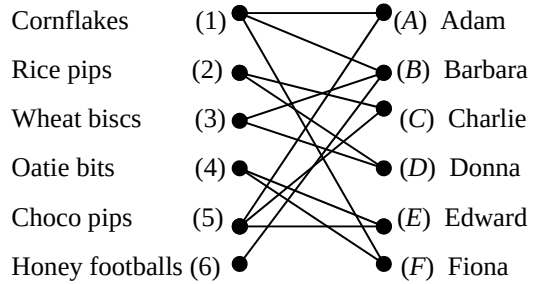
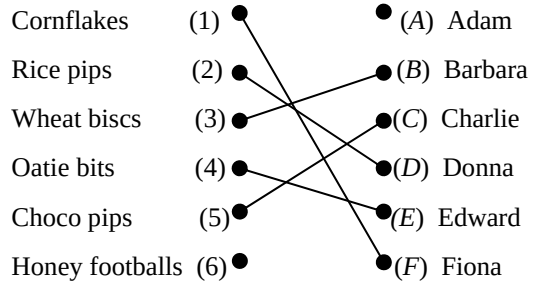
1	(i)		B1	**[1]**	Bipartite graph correct	Condone any extra labels (working)
	(ii)		B1	**[1]**	Incomplete matching correct	Condone any extra labels (working) but not extra arcs (working), apart from an *X* if used Alternating path shown (even if dashed) $\Rightarrow$ B0
	(iii)	$6 = B - 3 = D - 2 = C - 5 = A$ Cornflakes = Fiona Rice pips = Charlie Wheat biscs = Donna Oatie bits = Edward Choco pips = Adam Honey footballs = Barbara	B1 B1	**[2]**	This alternating path written down, not just read off from labels on graph This matching written down in words (or numbers and letters)	Written in any unambiguous form (condone reversed) Written in any unambiguous form A = Choco pips (5) B = Honey footballs (6) C = Rice pips (2) D = Wheat biscs (3) E = Oatie bits (4) F = Cornflakes (1)
	(iv)	$A = 1 - F = 4 - E = 5$ Cornflakes = Adam Rice pips = Charlie Wheat biscs = Donna Oatie bits = Fiona Choco pips = Edward Honey footballs = Barbara	B1 B1	**[2]**	This alternating path written down, not just read off from labels on graph This matching written down in words (or numbers and letters)	Written in any unambiguous form Not a longer path Written in any unambiguous form A = Cornflakes (1) B = Honey footballs (6) C = Rice pips (2) D = Wheat biscs (3) E = Choco pips (5) F = Oatie bits (4)

2 (i)

Dummy columns are needed to make a square table — B1

Square
Because there are more cards than people

So that (number of) people equals (number of) cards; one person for each card (or vv)

Dummies should not have positive scores because they should not be more suitable than any of the family members — B1

Smallest value in table is 0
Less than or equal to all values in table
So they do not affect the row reductions

Not 'dummies are not suitable for anyone' (dummies are people not cards)
Not vague statements

Modify resulting table by subtracting all scores from 8 (or any fixed constant ≥ 8) — B1

Saying how to convert to a minimisation problem

Must be written down, not just shown in working

[3]

(ii)

Add dummy columns and convert to a minimisation problem

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
P	6	4	6	2	8	4	8	8
Q	5	3	5	3	5	4	8	8
R	2	1	2	2	2	0	8	8
S	4	2	4	4	8	4	8	8
T	7	8	7	4	8	3	8	8
U	5	4	4	4	5	5	8	8
V	1	3	1	2	1	3	8	8
W	4	2	3	4	3	3	8	8

M1

This matrix seen or implied

Condone up to three individual arithmetic slips
Dummy columns missing $\Rightarrow$ M0

Reduce rows

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
P	4	2	4	0	6	2	6	6
Q	2	0	2	0	2	1	5	5
R	2	1	2	2	2	0	8	8
S	2	0	2	2	6	2	6	6
T	4	5	4	1	5	0	5	5
U	1	0	0	0	1	1	4	4
V	0	2	0	1	0	2	7	7
W	2	0	1	2	1	1	6	6

M1

Substantially correct attempt at reducing (their) rows (or columns)

Condone up to three individual arithmetic slips, must be convincingly reducing rows (or columns)

Not dependent on first method mark

	Reduce columns A1		[3]	Reducing columns for a correct row reduced matrix to achieve this (given) reduced cost matrix	Correct method, with no errors leading to the given matrix Could get away with reducing rows correctly and then saying that reducing columns gives the matrix in the question

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
P	4	2	4	0	6	2	2	2
Q	2	0	2	0	2	1	1	1
R	2	1	2	2	2	0	4	4
S	2	0	2	2	6	2	2	2
T	4	5	4	1	5	0	1	1
U	1	0	0	0	1	1	0	0
V	0	2	0	1	0	2	3	3
W	2	0	1	2	1	1	2	2

(iii)	Cross out 0's using 5 lines M1			This is not follow through from (ii) Cross out 0's using these 5 lines	 May be seen in answer space for (ii), allow this if it happens, but check here first

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
P	4	2	4	0	6	2	2	2
Q	2	0	2	0	2	1	1	1
R	2	1	2	2	2	0	4	4
S	2	0	2	2	6	2	2	2
T	4	5	4	1	5	0	1	1
U	1	0	0	0	1	1	0	0
V	0	2	0	1	0	2	3	3
W	2	0	1	2	1	1	2	2

Augment M1		Substantially correct attempt at augmenting (from 5 lines crossed out, or more)	

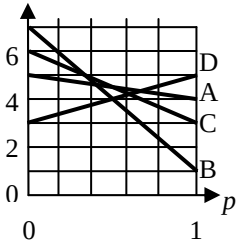
	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
P	3	2	3	0	5	2	1	1
Q	1	0	1	0	1	1	0	0
R	1	1	1	2	1	0	3	3
S	1	0	1	2	5	2	1	1
T	3	5	3	1	4	0	0	0
U	1	1	0	1	1	2	0	0
V	0	3	0	2	0	3	3	3
W	1	0	0	2	0	1	1	1

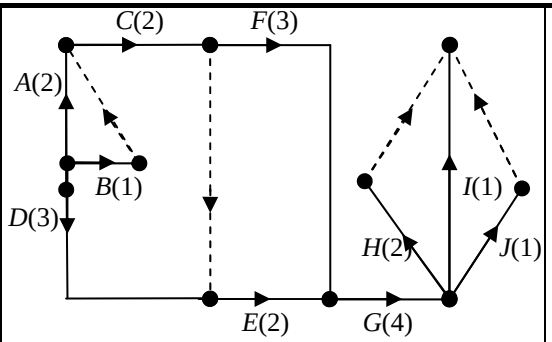
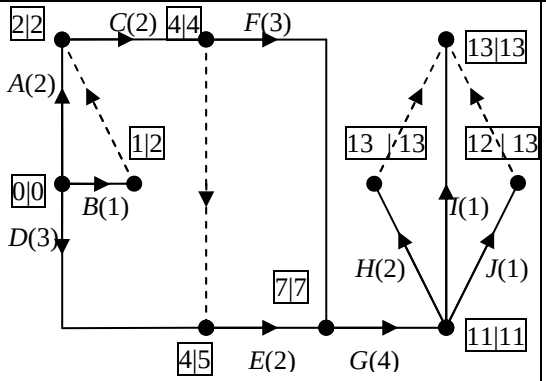
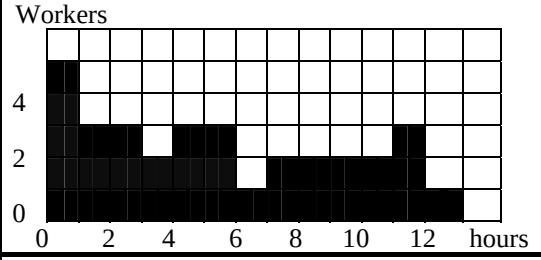
A1	This final matrix (cao)	All entries correct	

'View from air' is sent to Adam 'Scenic view' is sent to Barbara 'University' is sent to Charlie 'Painted barges' is sent to Donna 'Windmills' is sent to Edward 'Reichsmuseum' is sent to Fiona 'Quaint houses' and 'Tulips' are not used	M1 A1 B1	A valid matching from the 0's in their 8×8 reduced cost matrix This matching, in words or symbols *Q* and *T* not used	Matching the six cards using their entries with reduced cost = 0 from an 8x8 matrix cao cao

[6]

3	(i)	If it is rainy D > A > C > B but if it is sunny B > C > A > D so for each pair the better choice for rainy weather is the worse choice for sunny weather	M1 A1	[2]	A substantially correct explanation for at least one pair in words or using inequalities A correct explanation for all pairs: A&B, A&C, A&D, B&C, B&D, C&D or equivalent (eg why each cannot always be best)	'B is best for sunny but worst for rainy'; 'no magazine gives a higher income for both sunny and rainy weather over <u>any</u> <u>other</u> magazine' Explaining how what they have done shows that no magazine can be rejected by dominance
	(ii)	<table><tr><th></th><th>R</th><th>S</th><th>row min</th></tr><tr><td>A</td><td>4000</td><td>5000</td><td>4000</td></tr><tr><td>B</td><td>1000</td><td>7000</td><td>1000</td></tr><tr><td>C</td><td>3000</td><td>6000</td><td>3000</td></tr><tr><td>D</td><td>5000</td><td>3000</td><td>3000</td></tr><tr><td>col max</td><td>5000</td><td>7000</td><td></td></tr></table> Play-safe is 'Activity holidays' Play-safe for weather is 'rainy'= 5000 $4000 \neq 5000$ so unstable	M1 A1 M1 A1	[4]	Calculating row minima (condone 1 error) (or maximin) *A* (cao, without wrong working) Calculating column minimax or substantially correct reasoned argument Hence showing that game is unstable (may be reasoned in words)	4000, 1000, 3000, 3000 (or 4, 1, 3, 3) (May also see *R*) If Basil chooses A(plays safe) and it is rainy he would have been better with D (or if it is sunny he would have been better with B or with C) He would want to change from his play-safe strategy, so game is unstable
	(iii)	A: (0.4)(4000)+(0.6)(5000) = £4600 B: (0.4)(1000)+(0.6)(7000) = £4600 C: (0.4)(3000)+(0.6)(6000) = £4800 D: (0.4)(5000)+(0.6)(3000) = £3800	M1 A1	[2]	Correct method for any one magazine All four correct	Any one calculation seen correctly or any one final value correct 4600 4600 4800 3800 (need not have £)
	(iv)	British beaches Dining experiences	B1	[1]	$p = 0$ (sunny) $\Rightarrow$ B $p = 1$ (rainy) $\Rightarrow$ D	cao

	(v)	Income (£1000)  Graph shows negative of E(win) for 'weather', assuming that it is a zero-sum game. Upper boundary of this graph corresponds to lower boundary of inverted graph. $0 \leq p < \frac{1}{3}$ choose B $p = \frac{1}{3}$ choose B or C $\frac{1}{3} < p < \frac{1}{2}$ choose C $p = \frac{1}{2}$ choose C or A $\frac{1}{2} < p < \frac{2}{3}$ choose A $p = \frac{2}{3}$ choose A or D $\frac{2}{3} < p \leq 1$ choose D	B1 M1 A1	**[3]**	Correct graph (labels and scales may be implied, but must be consistent) This explanation is not required Identifying any two of the p values 1/3, 1/2 , 2/3 0.3 to 0.4 for 1/3 and 0.6 to 0.7 for 2/3 Complete and correct answer presented in any form. Accept either choice (or no choice) on boundaries at $p = 0, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 1$	Using the grid given in the answer booklet in any reasonable way Need not label A, B, C and D Follow through their graph if possible, (unless it significantly simplifies the solution) Follow through their graph if possible (unless it simplifies the solution)

4	**(i)**		M1 M1dep A1	**[3]**	Durations not necessary Correct structure, even without directions shown Activities (letters) must be labelled Exactly four directed dummies used correctly Completely correct, with exactly four dummies used and all arcs directed	Activity on node scores 0 Precedences must be correct, other than possibly due to any unmarked directions Dummies may appear before or after *B*, *H*, *J* Dummy attached to *B* could alternatively be attached to *A* (before or after) With single start and finish
	(ii)		M1 M1 A1ft		Follow through their activity network if possible Substantially correct attempt at forward pass Substantially correct attempt at backward pass Both passes wholly correct	 No more than two independent errors or omissions No more than two independent errors or omissions Follow through if possible
		Minimum project completion time = 13 hours Critical activities: *A*, *C*, *F*, *G*, *H*	B1 B1	**[5]**	13 cao *A C F G H* cao (in any order) and no extras	Condone wrong units
	(iii)		M1 A1	**[2]**	Axes scaled appropriately and a plausible histogram with no holes or overhangs Axes also labelled and histogram completely correct, cao	At least as far as 10 on time axis (horizontal) and first hour at height 5 workers Values and appropriate label on each axis, shape correct
	(iv)	Delay *D* by 1 hour, so that it starts after *B* has finished and does not delay any other activity.	B1	**[1]**	Delay *D* by 1 hour (or 2)	Start *D* after 1 hour (or 2) BOD 'start *D* at 2' (or 3)
	(v)	Both do *A* then *B* (or *B* then *A*) immediately followed by *C*; then one does *D* while other does *F*; then both do *E* followed by *G*; finally one does *H* and other does *I* and *J* 16 hours	M1 A1 B1	**[3]**	A reasonable attempt to describe a valid schedule A correct schedule described unambiguously May be given as a timetable 16 cao	A schedule that never needs more than two workers and that begins with *A*, *B* or *B*, *A* immediately followed by *C* Accept, for example, *A B C D E G H F I J*

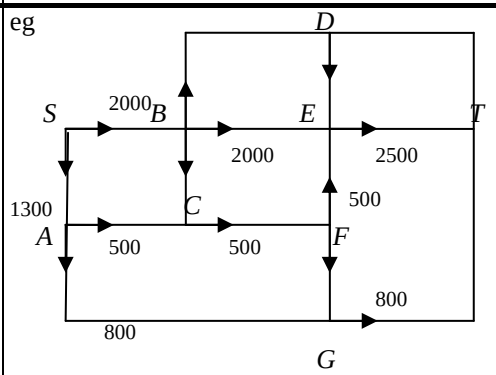
For reference:

Activity	Duration (hours)	Immediate predecessors	Number of workers
A: Choose summerhouse	2	-	2
B: Buy slabs for base	1	-	2
C: Take goods home	2	*A, B*	2
D: Level ground	3	-	1
E: Lay slabs	2	*C, D*	2
F: Treat wood	3	*C*	1
G: Make floor, walls and roof	4	*E, F*	2
H: Fit windows and door	2	*G*	1
I: Fit patio rail	1	*G*	1
J: Fit shelving	1	*G*	1

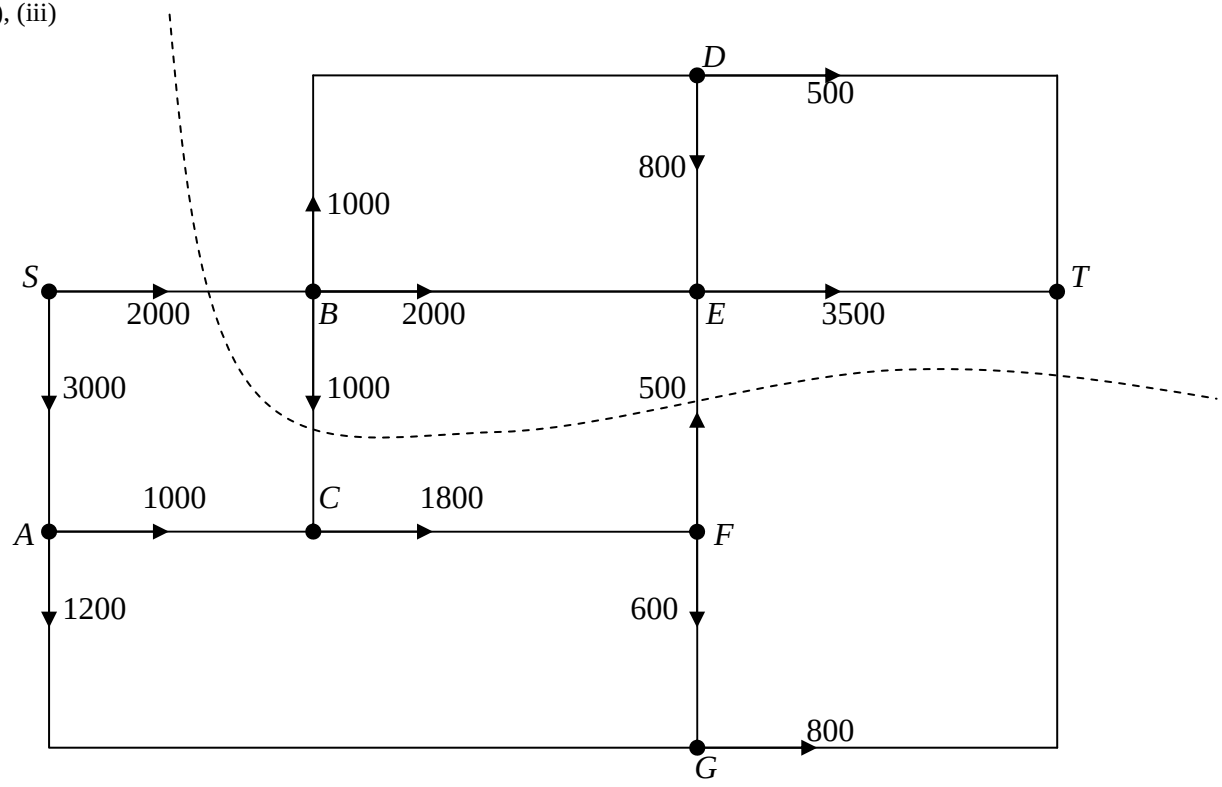
Part (v)

Activity	Start time
A: Choose summerhouse	0
B: Buy slabs for base	2
C: Take goods home	3
D: Level ground	5
E: Lay slabs	8
F: Treat wood	5
G: Make floor, walls and roof	10
H: Fit windows and door	14
I: Fit patio rail	14
J: Fit shelving	15

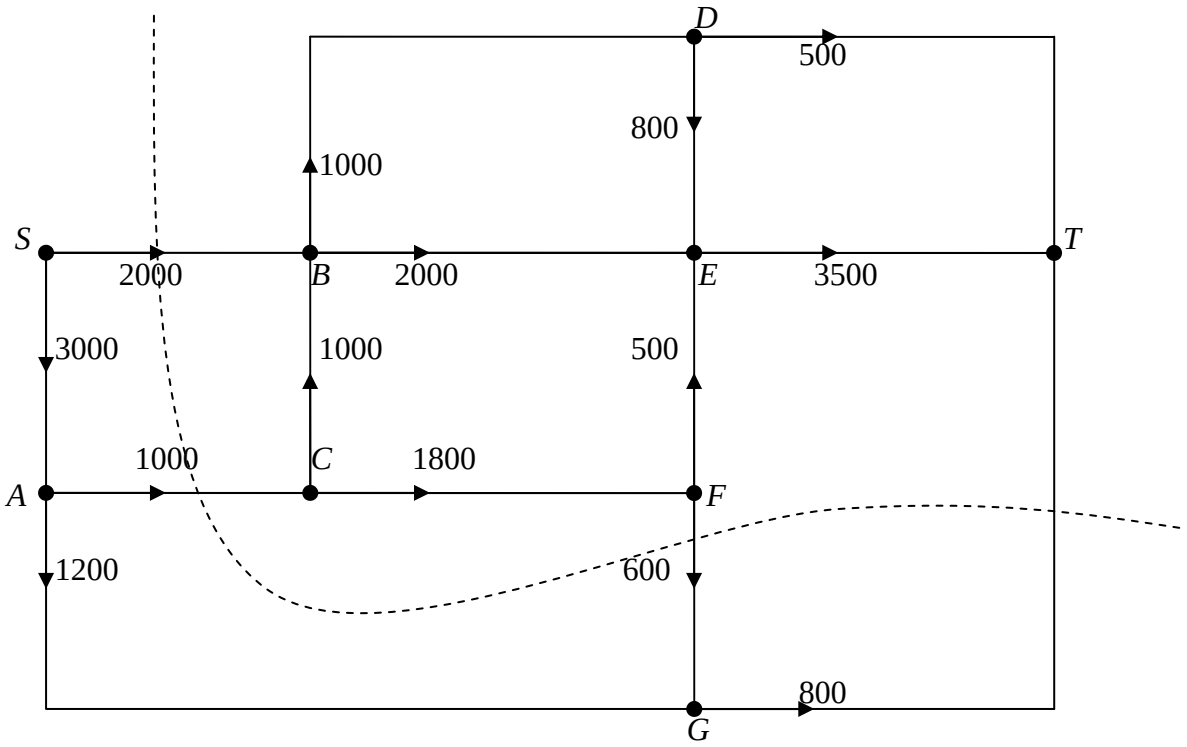
I and J may be interchanged

5	**(i)**	2000 + 0 + 1800 + 0 + 800 = 4600	M1 A1	**[2]**	2000+1800+800, no backflow 4600 cao	For method: condone <u>one</u> error or omission, if working is seen
	(ii)	*SA* ... because at most 2200 can leave *A* *ET* ... because at most 3300 can enter *E* *AC* and *BC* ... because at most 1800 can leave *C*	B1 B1 B1	**[3]**	Valid explanation of why $SA < 3000$ Valid explanation of why $ET < 3500$ Valid explanation of why $AC+BC < 2000$	$AC+AG = 1000+1200$, calc not req^d (could have eg $AG \leq 800$ so $AC+AG \leq 1800$) $DE+BE+FE = 800+2000+500$, calc not req^d (eg $BE+BD \leq 2000$ so into $E \leq 2500$) $AC+BC = 1000+1000$ but $CF = 1800$
	(iii)	eg  Cut {*S*, *A*, *C*, *F*, *G*}, {*B*, *D*, *E*, *T*}	 M1 A1 B1	**[3]**	Assume blanks are zero flow A flow of 3300 from *S* to *T* No arc capacities are exceeded This cut, represented in any way	May list flows 3300 out of *S*, 3300 into *T* and flow in = flow out at *A*, *B*, *C*, *D*, *E*, *F*, *G* Not labelling procedure Or shown on diagram If multiple answers given mark written work rather than diagram
	(iv)	Arrows in original direction of flow show excess capacities equal to arc capacities Arrows opposing original direction of flow show potential backflows equal to 0	B1 B1	**[2]**	Not reversed Values must be visible	Working will all be on one diagram, try to interpret the intention of the candidate
	(v)	Arrows *SB*, *BE*, *ET* decreased by 2000 Arrows *TE*, *EB*, *BS* increased by 2000	B1 B1	**[2]**	Decreasing their excess capacities *SBET* by 2000 Increasing their potential backflows *SBET* by 2000	Values must be visible. Follow through their labelling if possible
	(vi)	eg *SACBDT* flow 500 *SACBDET* flow 500 *SAGT* flow 800 New excess capacities and potential backflows Maximum flow = 3800	M1 A1 B1 B1	**[4]**	Listing any valid flow augmentation from *S* to *T* (route and flow), apart from *SBET* Valid routes that saturate *AC* and *GT* Values updated on diagram appropriately 3800 stated (cao)	 Valid routes with $AC = 1000$ and $GT = 800$ Flow must be 3800 3800 must be written in this part
	(vii)	Showing a valid flow of 3800 Cut {*S*, *A*, *G*}, {*B*, *C*, *D*, *E*, *F*, *T*} = 3800 Max flow = min cut, we have a flow of 3800 and a cut of 3800 so this is the max flow	B1 M1 A1	**[3]**	Follow through their potential backflows if possible, or a fresh start Identifying this cut in any way (or in words) Explaining how this shows that 3800 is max flow	May need to check back to (vi) (follow through may not lead to 3800) Cut may be marked on diagram Need a flow of 3800 seen or described correctly

For reference:
Parts (i), (ii), (iii)



For reference:
Parts (iv), (v), (vi), (vii)



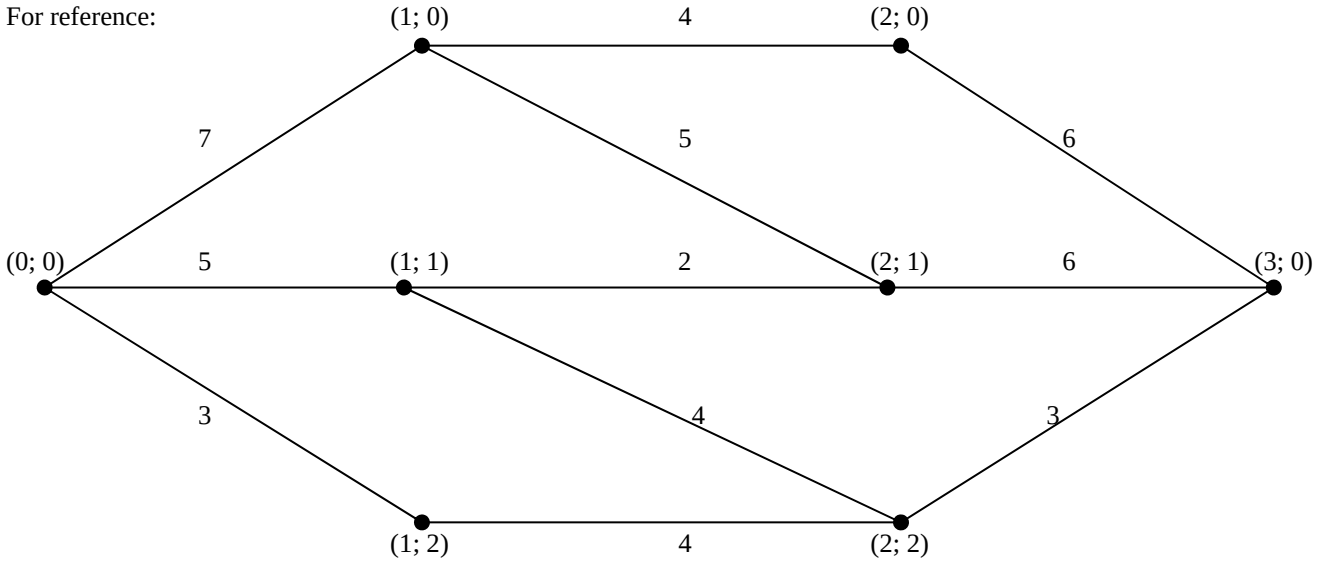
6

Stage	State	Action	Working	Suboptimal maximin
2	0	0	6	6
	1	0	6	6
	2	0	3	3
1	0	0	min(4,6)=4	5
		1	min(5,6)=5	
	1	1	min(2,6)=2	3
		2	min(4,3)=3	
	2	2	min(4,3)=3	3
0	0	0	min(7,5)=5	5
		1	min(5,3)=3	
		2	min(3,3)=3	

Route: (0; 0) – (1; 0) – (2; 1) – (3; 0)

B1		Structure of table correct	Stage, state, action and $\geq$ 1 other column
B1		Actions labelled correctly	Condone working forwards for B mark
M1		Stages labelled correctly, working backwards from stage 2	Not (stage; state), must be working backwards
A1	**[4]**	States labelled correctly	Not (stage; state)
M1		Working values correct for stage 1 (4, 5, 2, 3, 3)	If working forwards penalise the A marks and use diagram to deduce values
A1		Suboptimal maximin values correct for stage 1	Must be solving maximin problem
		Follow through if possible from stage 1	Follow through if possible from their previous stage
M1		Working values correct for stage 0 (5, 3, 3)	
A1		Suboptimal maximin value correct for stage 0	Must be solving a maximin problem
B1		cao	May be given in reverse Accept with (0; 0) and/or (3; 0) missing Accept 0-0-1-0 but only with all four states and in this order, in this case
	[5]		

For reference:



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