OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

Mechanics 3

4730

Candidates answer on the answer booklet.

OCR supplied materials:

- 8 page answer booklet (sent with general stationery)
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Monday 20 June 2011
Morning

Duration: 1 hour 30 minutes

* 4 7 3 0 *

INSTRUCTIONS TO CANDIDATES

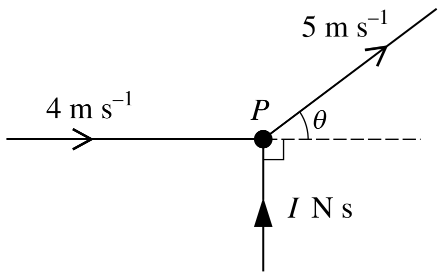
- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

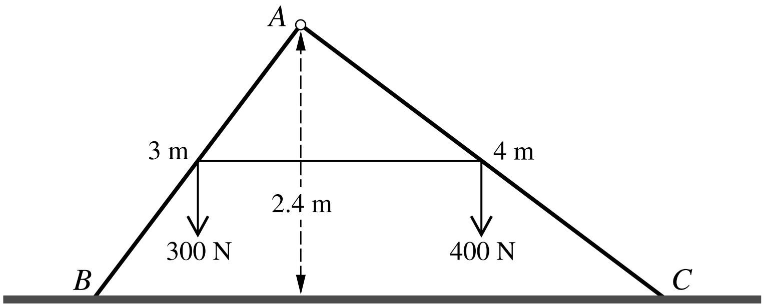
Turn over

1



A particle P of mass 0.3 kg is moving in a straight line with speed 4 m s^{-1} when it is deflected through an angle θ by an impulse of magnitude I N s. The impulse acts at right angles to the initial direction of motion of P (see diagram). The speed of P immediately after the impulse acts is 5 m s^{-1}. Show that $\cos\theta = 0.8$ and find the value of I. **[4]**

2



Two uniform rods AB and AC, of lengths 3 m and 4 m respectively, have weights 300 N and 400 N respectively. The rods are freely jointed at A. The mid-points of the rods are joined by a light inextensible string. The rods are in equilibrium in a vertical plane with the string taut and B and C in contact with a smooth horizontal surface. The point A is 2.4 m above the surface (see diagram).

(i) Show that the force exerted by the surface on AB is 374 N and find the force exerted by the surface on AC. **[4]**

(ii) Find the tension in the string. **[3]**

(iii) Find the horizontal and vertical components of the force exerted on AB at A and state their directions. **[3]**

3 A particle P of mass 0.25 kg is projected horizontally with speed 5 m s^{-1} from a fixed point O on a smooth horizontal surface and moves in a straight line on the surface. The only horizontal force acting on P has magnitude $0.2v^2$ N, where v m s^{-1} is the velocity of P at time t s after it is projected from O. This force is directed towards O.

(i) Find an expression for v in terms of t. **[5]**

The particle P passes through a point X with speed 0.2 m s^{-1}.

(ii) Find the average speed of P for its motion between O and X. **[5]**

4 One end of a light inextensible string of length 2 m is attached to a fixed point O. A particle P of mass 0.2 kg is attached to the other end of the string. P is held at rest with the string taut so that OP makes an angle of 0.15 radians with the downward vertical. P is released and t seconds afterwards OP makes an angle of θ radians with the downward vertical.

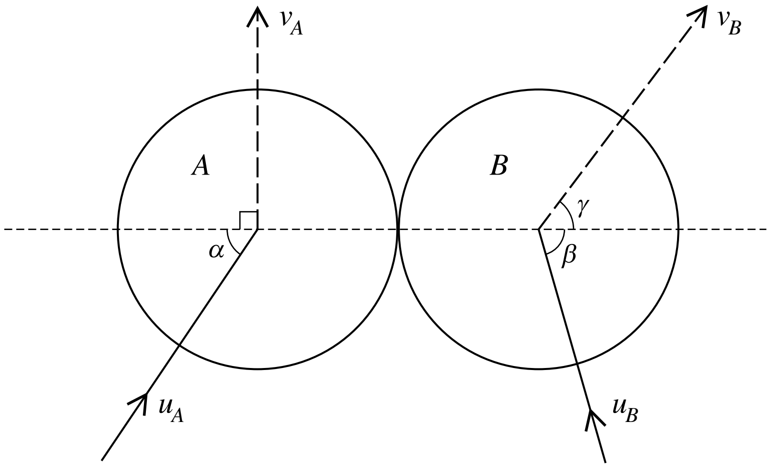
(i) Show that $\dfrac{d^2\theta}{dt^2} = -4.9\sin\theta$ and give a reason why the motion is approximately simple harmonic. **[3]**

Using the simple harmonic approximation,

(ii) obtain an expression for θ in terms of t and hence find the values of t at the first and second occasions when $\theta = -0.1$, **[5]**

(iii) find the angular speed of OP and the linear speed of P when $t = 0.5$. **[3]**

5



Two uniform smooth identical spheres A and B are moving towards each other on a horizontal surface when they collide. Immediately before the collision A and B are moving with speeds u_A m s^{-1} and u_B m s^{-1} respectively, at acute angles α and β, respectively, to the line of centres. Immediately after the collision A and B are moving with speeds v_A m s^{-1} and v_B m s^{-1} respectively, at right angles and at acute angle γ, respectively, to the line of centres (see diagram).

(i) Given that $\sin\beta = 0.96$ and $\dfrac{v_B}{u_B} = 1.2$, find the value of $\sin\gamma$. **[2]**

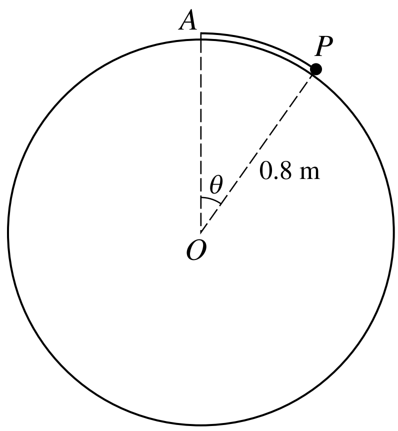
(ii) Given also that, before the collision, the component of A's velocity parallel to the line of centres is 2 m s^{-1}, find the values of u_B and v_B. **[5]**

(iii) Find the coefficient of restitution between the spheres. **[3]**

(iv) Given that the kinetic energy of A immediately before the collision is $6.5m$ J, where m kg is the mass of A, find the value of v_A. **[2]**

[Questions 6 and 7 are printed overleaf.]

6



A particle P of weight 6 N is attached to the highest point A of a fixed smooth sphere by a light elastic string. The sphere has centre O and radius 0.8 m. The string has natural length $\frac{1}{10}\pi$ m and modulus of elasticity 9 N. P is released from rest at a point X on the sphere where OX makes an angle of $\frac{1}{4}\pi$ radians with the upwards vertical. P remains in contact with the sphere as it moves upwards to A. At time t seconds after the release, OP makes an angle of θ radians with the upwards vertical (see diagram). When $\theta = \frac{1}{6}\pi$, P passes through the point Y.

(i) Show that as P moves from X to Y its gravitational potential energy increases by $2.4(\sqrt{3} - \sqrt{2})$ J and the elastic potential energy in the string decreases by 0.4π J. **[5]**

(ii) Verify that the transverse acceleration of P is zero when $\theta = \frac{1}{6}\pi$, and hence find the maximum speed of P. **[6]**

7 One end of a light inextensible string of length 0.8 m is attached to a fixed point O. A particle P of mass 0.3 kg is attached to the other end of the string. P is projected horizontally from the point 0.8 m vertically below O with speed 5.6 m s^{-1}. P starts to move in a vertical circle with centre O. The speed of P is $v\text{ m s}^{-1}$ when the string makes an angle θ with the downward vertical.

(i) While the string remains taut, show that $v^2 = 15.68(1 + \cos\theta)$, and find the tension in the string in terms of θ. **[7]**

(ii) For the instant when the string becomes slack, find the value of θ and the value of v. **[3]**

(iii) Find, in either order, the speed of P when it is at its greatest height after the string becomes slack, and the greatest height reached by P above its point of projection. **[4]**