OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS 4724

Core Mathematics 4

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Friday 14 January 2011
Afternoon

Duration: 1 hour 30 minutes

* 4 7 2 4 *

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Turn over

1 **(i)** Expand $(1-x)^{\frac{1}{2}}$ in ascending powers of x as far as the term in x^2. **[3]**

(ii) Hence expand $\left(1-2y+4y^2\right)^{\frac{1}{2}}$ in ascending powers of y as far as the term in y^2. **[3]**

2 **(i)** Express $\dfrac{7-2x}{(x-2)^2}$ in the form $\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}$, where A and B are constants. **[3]**

(ii) Hence find the exact value of $\displaystyle\int_4^5 \frac{7-2x}{(x-2)^2}\,\mathrm{d}x$. **[4]**

3 **(i)** Show that the derivative of $\sec x$ can be written as $\sec x \tan x$. **[4]**

(ii) Find $\displaystyle\int \frac{\tan x}{\sqrt{1+\cos 2x}}\,\mathrm{d}x$. **[4]**

4 A curve has parametric equations

$$x = 2 + t^2, \qquad y = 4t.$$

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. **[2]**

(ii) Find the equation of the normal at the point where $t = 4$, giving your answer in the form $y = mx + c$. **[3]**

(iii) Find a cartesian equation of the curve. **[2]**

5 In this question, I denotes the definite integral $\displaystyle\int_2^5 \frac{5-x}{2+\sqrt{x-1}}\,\mathrm{d}x$. The value of I is to be found using two different methods.

(i) Show that the substitution $u = \sqrt{x-1}$ transforms I to $\displaystyle\int_1^2 (4u-2u^2)\,\mathrm{d}u$ and hence find the exact value of I. **[5]**

(ii) **(a)** Simplify $\left(2+\sqrt{x-1}\right)\left(2-\sqrt{x-1}\right)$. **[1]**

(b) By first multiplying the numerator and denominator of $\dfrac{5-x}{2+\sqrt{x-1}}$ by $2-\sqrt{x-1}$, find the exact value of I. **[3]**

6 The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix} + s\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$. The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} + t\begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$.

(i) Find the acute angle between l_1 and l_2. **[4]**

(ii) Show that l_1 and l_2 are skew. **[4]**

(iii) One of the numbers in the equation of line l_1 is changed so that the equation becomes $\mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ a \end{pmatrix} + s\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$. Given that l_1 and l_2 now intersect, find a. **[2]**

7 Show that $\int_0^{\pi} (x^2 + 5x + 7)\sin x\,\mathrm{d}x = \pi^2 + 5\pi + 10$. **[7]**

8 The points P and Q lie on the curve with equation

$$2x^2 - 5xy + y^2 + 9 = 0.$$

The tangents to the curve at P and Q are parallel, each having gradient $\frac{3}{8}$.

(i) Show that the x- and y-coordinates of P and Q are such that $x = 2y$. **[5]**

(ii) Hence find the coordinates of P and Q. **[3]**

9 Paraffin is stored in a tank with a horizontal base. At time t minutes, the depth of paraffin in the tank is x cm. When $t = 0$, $x = 72$. There is a tap in the side of the tank through which the paraffin can flow. When the tap is opened, the flow of the paraffin is modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -4(x-8)^{\frac{1}{3}}.$$

(i) How long does it take for the level of paraffin to fall from a depth of 72 cm to a depth of 35 cm? **[7]**

(ii) The tank is filled again to its original depth of 72 cm of paraffin and the tap is then opened. The paraffin flows out until it stops. How long does this take? **[3]**

There are no questions printed on this page.