RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE **A2 7890 - 2**

Advanced Subsidiary GCE **AS 3890 - 2**

Mark Schemes for the Units

January 2010

3890-2/7890-2/MS/10J

Oxford Cambridge and RSA Examinations

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MARK SCHEMES FOR THE UNITS

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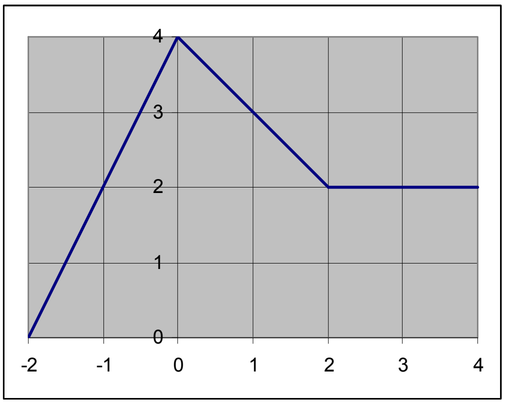
4721 Core Mathematics 1

1		$\left[(x-6)^2-36\right]+1$	B1		$(x-6)^2$
		$=(x-6)^2-35$	M1		$q = 1 - (\text{their } p)^2$
			A1	3	$q=-35$
				$\boxed{3}$	
2	(i)		B1		For $x < 0$, straight line joining (–2, 0) and (0, 4)
			B1	2	For $x > 0$, line joining (0,4) to (2, 2) and horizontal line joining (2,2) and (4,2)
	(ii)	Translation	B1		
		1 unit right parallel to x axis	B1	2	Allow: 1 unit right, 1 along the x axis, 1 in x **direction**, allow vector notation e.g. $\begin{pmatrix}1\\0\end{pmatrix}$, 1 unit horizontally
				$\boxed{4}$	
3		$\frac{dy}{dx}=3x^2-8x$	M1		Attempt to differentiate (one of $3x^2$, $-8x$)
			A1		Correct derivative
		When $x=2$, $\frac{dy}{dx}=-4$	M1 A1		Substitutes $x = 2$ into their $\frac{dy}{dx}$
		$\therefore$ Gradient of normal to curve $= \frac{1}{4}$	B1 ft		Must be numerical $= -1 \div$ their m
		$y+1=\frac{1}{4}(x-2)$	M1		Correct equation of straight line through (2, –1), any non-zero numerical gradient
		$x-4y-6=0$	A1	7	Correct equation in required form
				$\boxed{7}$	

4	(i)	$m = 4$	B1	1	May be embedded
	(ii)	$6p^2 = 24$ $p^2 = 4$ $p = 2$ or $p = -2$	M1 A1 A1	 3	$(\pm)6p^2 = 24$ or $36p^4 = 576$
	(iii)	$5^{2n+4} = 25$ $\therefore 2n + 4 = 2$ $n = -1$	M1 M1 A1	 3 $\boxed{7}$	Addition of indices as powers of 5 Equate powers of 5 or 25
5		$k = \sqrt{x}$ $k^2 - 8k + 13 = 0$	M1*		Use a substitution to obtain a quadratic (may be implied by squaring or rooting later) or factorise into 2 brackets each containing $\sqrt{x}$
		$k - 4 = \pm\sqrt{3}$ or $k = \dfrac{8 \pm \sqrt{(-8)^2 - 4\times1\times13}}{2}$	M1 dep A1		Correct method to solve resulting quadratic
		$k = 4 \pm \sqrt{3}$	A1		$k = 4 \pm \sqrt{3}$ or $k = \dfrac{8 \pm \sqrt{12}}{2}$ or $k = 4 \pm \dfrac{\sqrt{12}}{2}$
		$\therefore x = (4+\sqrt{3})^2$ or $x = (4-\sqrt{3})^2$	M1 M1		Recognise the need to square to obtain x Correct method for squaring $a + \sqrt{b}$ (3 or 4 term expansion)
		$x = 19 \pm 8\sqrt{3}$ or $19 \pm 4\sqrt{12}$	A1	7 $\boxed{7}$	
6	(i)	$\dfrac{dy}{dx} = 2x$ When $x = 1$, $\dfrac{dy}{dx} = 2$	B1* B1 dep	 2	
	(ii)	$\dfrac{a^2 + 5 - 6}{a - 1} = 2.3$	M1 A1		uses $\dfrac{y_2 - y_1}{x_2 - x_1}$ correct expression
		$a^2 - 2.3a + 1.3 = 0$ $(a - 1.3)(a - 1) = 0$	M1		correct method to solve a quadratic or correct factorisation and cancelling to get $a + 1 = 2.3$
		$a = 1.3$	A1	4	1.3 only

		Alternative method: Equation of straight line through (1,6) with $m = 2.3$ found then $a^2 + 5 = 2.3a +$ "c" seen **M1** with $c = 3.7$ **A1** then as main scheme			
	(iii)	A value between 2 and 2.3	**B1**	1 $\boxed{7}$	2 < value < 2.3 (strict inequality signs)
7	**(i)**	(a) Fig 3 (b) Fig 1 (c) Fig 4	**B1** **B1** **B1**	3	
	(ii)	$-(x-3)^2$ $y = -(x-3)^2$	**M1** **A1**	2 $\boxed{5}$	Quadratic expression with correct x^2 term and correct y-intercept and/or roots for **their unmatched diagram** (e.g. negative quadratic with y-intercept of –9 or root of 3 for Fig 2) Completely correct equation for **Fig 2**
8	**(i)**	Centre (–3, 2) $(x+3)^2 - 9 + (y-2)^2 - 4 - 4 = 0$ $r^2 = 17$ $r = \sqrt{17}$	**B1** **M1** **A1**	3	 Correct method to find r^2 Correct radius
	(ii)	$x^2 + (3x+4)^2 + 6x - 4(3x+4) - 4 = 0$ $10x^2 + 18x - 4 = 0$ $(5x-1)(x+2) = 0$ $x = \frac{1}{5}$ or $x = -2$ $y = \frac{23}{5}$ or $y = -2$	**M1*** **A1** **A1** **M1 dep** **A1** **A1**	6 $\boxed{9}$	substitute for x/y or attempt to get an equation in 1 variable only correct unsimplified expression obtain correct 3 term quadratic correct method to solve their quadratic **SR** If A0 A0, one correct pair of values, spotted or from correct factorisation **www** **B1**
9	**(i)**	$f'(x) = -x^{-2} - \frac{1}{2}x^{-\frac{1}{2}}$	**M1** **A1** **A1**	3	Attempt to differentiate $-x^{-2}$ or $-\frac{1}{2}kx^{-\frac{1}{2}}$ **www** Fully correct expression

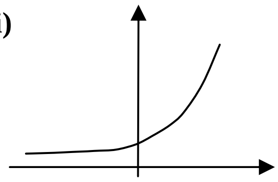
	(ii)	$f''(x) = 2x^{-3} + \frac{1}{4}x^{-\frac{3}{2}}$	M1		Attempt to differentiate their $f'(x)$
			A1 ft		One correctly differentiated term
			A1		Fully correct expression **www** in either part of the question
		$f''(4) = \frac{2}{4^3} + \frac{1}{4}.\frac{1}{8}$	M1		Substitution of $x = 4$ into their $f''(x)$
		$= \frac{1}{16}$	A1	5 $\boxed{8}$	oe single fraction **www** in either part of the question
10		$(-30)^2 - 4 \times k \times 25k = 0$	M1		Attempts $b^2 - 4ac$ involving k
		$900 - 100k^2 = 0$	M1		States their discriminant = 0
		$k = 3$	B1		
		or $k = -3$	B1	4 $\boxed{4}$	
11	(i)	$P = 2 + x + 3x + 2 + 5x + 5x$ $= 14x + 4$	M1		Adds lengths of all 4 edges with attempt to use Pythagoras to find the missing length
			A1	2	May be left unsimplified
	(ii)	Area of rectangle = $3x(2 + x) = 6x + 3x^2$ Area of triangle $= \frac{1}{2}(3x)(4x) = 6x^2$	M1		Correct method – splitting or formula for area of trapezium
		Total area = $9x^2 + 6x$	A1	2	Convincing working leading to given expression **AG**
	(iii)	$14x + 4 \geq 39$	B1 ft		ft on their expression for P from **(i)** unless restarted in **(iii)**. (Allow >)
		$\frac{5}{2}$	B1		o.e. (e.g. $\frac{35}{14}$) soi by subsequent working
		$9x^2 + 6x < 99$	B1		
		$3x^2 + 2x - 33 < 0$			Allow ≤
		$(3x + 11)(x - 3) < 0$	M1		
		$\left(-\frac{11}{3} <\right) x < 3$			Correct method to find critical values
			B1		$x < 3$ identified
			M1		root from linear < x < upper root from quadratic
		$\therefore \frac{5}{2} \leq x < 3$	A1	7 $\boxed{11}$	Fully correct including inequality signs or exact equivalent in words **cwo**
		Total		**72**	

4722 Core Mathematics 2

1	**(i)**	$2(1 - \cos^2 x) = 5\cos x - 1$ $2\cos^2 x + 5\cos x - 3 = 0$ **A.G.**	M1 A1	 **2**	Use $\sin^2 x = 1 - \cos^2 x$ Show given equation correctly
	(ii)	$(2\cos x - 1)(\cos x + 3) = 0$ $\cos x = ½$ $x = 60^\circ$ $x = 300^\circ$	M1 M1 A1 A1√	 **4**	Recognise equation as quadratic in $\cos x$ and attempt recognisable method to solve Attempt to find x from root(s) of quadratic Obtain 60° or $^{\pi}/_3$ or 1.05 rad Obtain 300° only (or 360° – their x) and no extra in range **SR** answer only is B1 B1
				$\boxed{6}$	
2	**(i)**	$\int (6x - 4)\,dx = 3x^2 - 4x + c$ $y = 3x^2 - 4x + c \Rightarrow 5 = 12 - 8 + c$ $\Rightarrow c = 1$ Hence $y = 3x^2 - 4x + 1$	M1* A1 M1dep* A1	 **4**	Attempt integration (inc. in power for at least one term) Obtain $3x^2 - 4x$ (or unsimplified equiv), with or without $+ c$ Use (2, 5) to find c Obtain $y = 3x^2 - 4x + 1$
	(ii)	$3p^2 - 4p + 1 = 5$ $3p^2 - 4p - 4 = 0$ $(p - 2)(3p + 2) = 0$ $p = {}^{-2}/_3$	M1* M1dep* A1	 **3**	Equate their y (from integration attempt) to 5 Attempt to solve three term quadratic Obtain $p = {}^{-2}/_3$ (allow any variable) from correct working; condone $p = 2$ still present, but A0 if extra incorrect solution
				$\boxed{7}$	
3	**(i)**	$(2 - x)^7 = 128 - 448x + 672x^2 - 560x^3$	M1 A1 A1 A1	 **4**	Attempt (at least) two relevant terms – product of binomial coeff, 2 and x (or expansion attempt that considers all 7 brackets) Obtain $128 - 448x$ Obtain $672x^2$ Obtain $-560x^3$
	(ii)	$-560 \times ({}^{1}/_{4})^3 = {}^{-35}/_4$	M1 A1	 **2**	Attempt to use coeff of x^3 from **(i)**, with clear intention to cube $^{1}/_{4}$ Obtain ${}^{-35}/_4$ (w^6), (allow ${}^{35}/_4$ from $+560x^3$ in **(i)**)
				$\boxed{6}$	

4	(i)	$\int_3^5 \log_{10}(2+x)\,dx \approx \frac{1}{2}\times\frac{1}{2}\times(\log 5 + 2\log 5.5 +$	M1		Attempt y-coords for at least 4 of the correct 5 x-coords only
		$2\log 6 + 2\log 6.5 + \log 7)$	M1		Use correct trapezium rule, any h, to find area between x = 3 and x = 5
			M1		Correct h (soi) for their y-values
		$\approx$ 1.55	A1	**4**	Obtain 1.55
	(ii)	$\int_3^5 \log_{10}(2+x)^{\frac{1}{2}}\,dx = \frac{1}{2}\int_3^5 \log_{10}(2+x)\,dx$	B1√		Divide by 2, or equiv, at any stage to obtain 0.78 or 0.77,
		$\approx$ ½ × 1.55			following their answer to **(i)**
		$\approx$ 0.78	B1	**2**	Explicitly use log √a = ½ log a on a single term
				6	
5		$\int_1^3 \{(11-9x^{-2})-(x^2+1)\}dx = \left[9x^{-1}-\frac{1}{3}x^3+10x\right]_1^3$	M1		Attempt subtraction (correct order) at any point
		= (3 – 9 + 30) – (9 – $^1/_3$ +10)	M1		Attempt integration – inc. in power for at least one term
		= 24 – 18$^2/_3$	A1		Obtain ± (– $^1/_3x^3$ + 10x) or 11x and $^1/_3x^3 + x$
		= 5$^1/_3$	M1		Obtain remaining term of form kx^{-1}
		OR	A1		Obtain ± 9x^{-1} or any unsimplified equiv
		$\left[11x+9x^{-1}\right]_1^3 - \left[\frac{1}{3}x^3+x\right]_1^3$	M1		Use limits x = 1, 3 – correct order & subtraction
		= [(33 + 3) – (11 + 9)] – [(9 + 3) – ($^1/_3$ + 1)]	A1	**7**	Obtain 5$^1/_3$, or exact equiv
		= 16 – 10$^2/_3$			
		= 5$^1/_3$			
				7	
6	(i)	f(–3) = 0 ⇒ –54 + 9a – 3b + 15 = 0	M1		Attempt f(–3) and equate to 0, or equiv method
		3a – b = 13	A1		Obtain 3a – b = 13, or unsimplified equiv
		f(2) = 35 ⇒ 16 + 4a + 2b + 15 = 35	M1		Attempt f(2) and equate to 35, or equiv method
		2a + b = 2	A1		Obtain 2a + b = 2, or unsimplified equiv
		Hence a = 3, b = –4	M1		Attempt to solve simultaneous eqns
			A1	**6**	Obtain a = 3, b = –4
	(ii)	f(x) = (x + 3)(2x^2 – 3x + 5)	M1		Attempt complete division by (x + 3), or equiv
			A1		Obtain 2x^2 – 3x + c or 2x^2 + bx + 5, from correct f(x)
		ie quotient is (2x^2 – 3x + 5)	A1	**3**	Obtain 2x^2 – 3x + 5 (state or imply as quotient)
				9	

7	(i)	$13^2 = 10^2 + 14^2 - 2 \times 10 \times 14 \times \cos\theta$	M1		Attempt to use correct cosine rule in ΔABC
		$\cos\theta = 0.4536$			
		$\theta = 1.10$ **A.G.**	A1	2	Obtain 1.10 radians (allow 1.1 radians) **SR** B1 only for verification of 1.10, unless complete method
	(ii)	arc $EF = 4 \times 1.10 = 4.4$	B1		State or imply $EF = 4.4$cm (allow 4×1.10)
		perimeter $= 4.4 + 10 + 13 + 6$	M1		Attempt perimeter of region - sum of arc and three sides with attempt to subtract 4 from at least one relevant side
		$= 33.4$ cm	A1	3	Obtain 33.4 cm
	(iii)	area $AEF = \frac{1}{2} \times 4^2 \times 1.1$	M1		Attempt use of $(\frac{1}{2})\, r^2\theta$, with $r = 4$ and $\theta = 1.10$
		$= 8.8$	A1		Obtain 8.8
		area $ABC = \frac{1}{2} \times 10 \times 14 \times \sin 1.1$	M1		Attempt use of $(\frac{1}{2})ab\sin\theta$, sides consistent with angle used
		$= 62.4$	A1		Obtain 62.4 or better (allow 62.38 or 62.39)
		hence total area $= 53.6$ cm^2	A1	5	Obtain total area as 53.6 cm^2
				10	
8	(i)	$u_5 = 8 + 4 \times 3$	M1		Attempt $a + (n-1)d$ or equiv inc list of terms
		$= 20$ **A.G.**	A1	2	Obtain 20
	(ii)	$u_n = 3n + 5$ ie $p = 3$, $q = 5$	B1		Obtain correct expression, poss unsimplified, eg $8 + 3(n-1)$
			B1	2	Obtain correct $3n + 5$, or $p = 3$, $q = 5$ stated
	(iii)	arithmetic progression	B1	1	Any mention of arithmetic
	(iv)	$\frac{2N}{2}(16 + (2N-1)3) - \frac{N}{2}(16 + (N-1)3) = 1256$	M1		Attempt S_N, using any correct formula (inc $\sum (3n+5)$)
			M1		Attempt S_{2N}, using any correct formula, with $2N$ consistent (inc $\sum (3n+5)$)
		$26N + 12N^2 - 13N - 3N^2 = 2512$			
		$9N^2 + 13N - 2512 = 0$	M1*		Attempt subtraction (correct order) and equate to 1256
		$(9N + 157)(N - 16) = 0$	M1dep*		Attempt to solve quadratic in N
		$N = 16$	A1	5	Obtain $N = 16$ only, from correct working
					OR: alternative method is to use $^n/_2\,(a + l) = 1256$
			M1		Attempt given difference as single summation with N terms
			M1		Attempt $a = u_{N+1}$
			M1		Attempt $l = u_{2N}$
			M1		Equate to 1256 and attempt to solve quadratic
			A1		Obtain $N = 16$ only, from correct working
				10	

9	(i)		M1		Reasonable graph in both quadrants
			A1		Correct graph in both quadrants
			B1	**3**	State or imply (0, 6)
	(ii)	$9^x = 150$	M1		Introduce logarithms throughout, or equiv with $\log_9$
		$x \log 9 = \log 150$	M1		Use $\log a^b = b \log a$ and attempt correct method to find x
		$x = 2.28$	A1	**3**	Obtain $x = 2.28$
	(iii)	$6 \times 5^x = 9^x$	M1		Form eqn in x and take logs throughout (any base)
		$\log_3 (6 \times 5^x) = \log_3 9^x$	M1		Use $\log a^b = b \log a$ correctly on $\log 5^x$ or $\log 9^x$ or legitimate combination of these two
		$\log_3 6 + x \log_3 5 = x \log_3 9$	M1		Use $\log ab = \log a + \log b$ correctly on $\log (6 \times 5^x)$ or $\log 6$
		$\log_3 3 + \log_3 2 + x \log_3 5 = 2x$	M1		Use $\log_3 9 = 2$ or equiv (need base 3 throughout that line)
		$x (2 - \log_3 5) = 1 + \log_3 2$			
		$x = \frac{1+\log_3 2}{2-\log_3 5}$ **A.G.**	A1	**5**	Obtain $x = \frac{1+\log_3 2}{2-\log_3 5}$ convincingly (inc base 3 throughout)
				$\boxed{11}$	

4723 Core Mathematics 3

1		Obtain integral of form $k(2x-7)^{-1}$	M1		any constant k
		Obtain correct $-5(2x-7)^{-1}$	A1		or equiv
		Include … + c	B1	**3**	at least once; following any integral
				$\boxed{3}$	
2	**(i)**	Use $\sin 2\theta = 2\sin\theta\cos\theta$	B1		
		Attempt value of $\sin\theta$ from $k\sin\theta\cos\theta = 5\cos\theta$	M1		any constant k; or equiv
		Obtain $\frac{5}{12}$	A1	**3**	or exact equiv; ignore subsequent work
	(ii)	Use $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ or $\operatorname{cosec}^2\theta = 1+\cot^2\theta$	B1		or equiv
		Attempt to produce equation involving $\cos\theta$ only	M1		using $\sin^2\theta = \pm 1 \pm \cos^2\theta$ or equiv
		Obtain $3\cos^2\theta + 8\cos\theta - 3 = 0$	A1		or equiv
		Attempt solution of 3-term quadratic equation	M1		using formula or factorisation or equiv
		Obtain $\frac{1}{3}$ as only final value of $\cos\theta$	A1	**5**	or exact equiv; ignore subsequent work
				$\boxed{8}$	
3	**(i)**	Obtain or clearly imply $60\ln x$	B1		
		Obtain ($60\ln 20 - 60\ln 10$ and hence) $60\ln 2$	B1	**2**	with no error seen
	(ii)	Attempt calculation of form $k(y_0 + 4y_1 + y_2)$	M1		any constant k; using y-value attempts
		Identify k as $\frac{5}{3}$	A1		
		Obtain $\frac{5}{3}(6+4\times4+3)$ and hence $\frac{125}{3}$ or 41.7	A1	**3**	or equiv
	(iii)	Equate answers to parts **(i)** and **(ii)**	M1		provided $\ln 2$ involved
		Obtain $60\ln 2 = \frac{125}{3}$ and hence $\frac{25}{36}$	A1	**2**	AG; necessary detail required including clear use of an exact value from **(ii)**
				$\boxed{7}$	
4	**(i)**	Attempt correct process for composition	M1		numerical or algebraic
		Obtain (7 and hence) 0	A1	**2**	
	(ii)	Attempt to find x-intercept	M1		
		Obtain $x \le 7$	A1	**2**	or equiv; condone use of $<$
	(iii)	Attempt correct process for finding inverse	M1		
		Obtain $\pm(2-y)^3 - 1$ or $\pm(2-x)^3 - 1$	A1		
		Obtain correct $(2-x)^3 - 1$	A1	**3**	or equiv in terms of x
	(iv)	Refer to reflection in $y = x$	B1	**1**	or clear equiv
				$\boxed{8}$	

5	(i)	Obtain derivative of form $kx(x^2+1)^7$	M1		any constant k
		Obtain $16x(x^2+1)^7$	A1		or equiv
		Equate first derivative to 0 and confirm $x=0$ or substitute $x=0$ and verify first derivative zero	M1		AG; allow for deriv of form $kx(x^2+1)^7$
		Refer, in some way, to $x^2+1=0$ having no root	A1	**4**	or equiv
	(ii)	Attempt use of product rule	*M1		obtaining … + … form
		Obtain $16(x^2+1)^7$ + …	A1√		follow their $kx(x^2+1)^7$
		Obtain … + $224x^2(x^2+1)^6$	A1√		follow their $kx(x^2+1)^7$; or unsimplified equiv
		Substitute 0 in attempt at second derivative	M1		dep *M
		Obtain 16	A1	**5**	from second derivative which is correct at some point
				$\boxed{9}$	

6		Integrate e^{3x} to obtain $\frac{1}{3}e^{3x}$ or $e^{-\frac{1}{2}x}$ to obtain $-2e^{-\frac{1}{2}x}$	B1		or both
		Obtain indefinite integral of form $m_1e^{3x}+m_2e^{-\frac{1}{2}x}$	M1		any constants m_1 and m_2
		Obtain correct $\frac{1}{3}ke^{3x}-2(k-2)e^{-\frac{1}{2}x}$	A1		or equiv
		Obtain $e^{3\ln 4}=64$ or $e^{-\frac{1}{2}\ln 4}=\frac{1}{2}$	B1		or both
		Apply limits and equate to 185	M1		including substitution of lower limit
		Obtain $\frac{64}{3}k-(k-2)-\frac{1}{3}k+2(k-2)=185$	A1		or equiv
		Obtain $\frac{17}{2}$	A1	**7**	or equiv
				$\boxed{7}$	

7	(a)	Either: State or imply either $\frac{dA}{dr}=2\pi r$ or $\frac{dA}{dt}=250$	B1		or both
		Attempt manipulation of derivatives to find $\frac{dr}{dt}$	M1		using multiplication / division
		Obtain correct $\frac{250}{2\pi r}$	A1		or equiv
		Obtain 1.6	A1	**4**	or equiv; allow greater accuracy
		Or: Attempt to express r in terms of t	M1		using $A=250t$
		Obtain $r=\sqrt{\frac{250t}{\pi}}$	A1		or equiv
		Differentiate $kt^{\frac{1}{2}}$ to produce $\frac{1}{2}kt^{-\frac{1}{2}}$	M1		any constant k
		Substitute $t=7.6$ to obtain 1.6	A1	**(4)**	allow greater accuracy

	(b)	State $\frac{dm}{dt} = -150k e^{-kt}$	B1		
		Equate to $(\pm)3$ and attempt value for t	M1		using valid process; condone sign confusion
		Obtain $-\frac{1}{k}\ln(\frac{1}{50k})$ or $\frac{1}{k}\ln(50k)$ or $\frac{\ln 50 + \ln k}{k}$	A1	**3**	or equiv but with correct treatment of signs
				$\boxed{7}$	
8	**(i)**	State scale factor is $\sqrt{2}$	B1		allow 1.4
		State translation is in negative x-direction …	B1		or clear equiv
		… by $\frac{3}{2}$ units	B1	**3**	
	(ii)	Draw (more or less) correct sketch of $y = \sqrt{2x+3}$	B1		'starting' at point on negative x-axis
		Draw (more or less) correct sketch of $y = \frac{N}{x^3}$	B1		showing both branches
		Indicate one point of intersection	B1	**3**	with both sketches correct
		[SC: if neither sketch complete or correct but diagram correct for both in first quadrant B1]			
	(iii) (a)	Substitute 1.9037 into $x = N^{\frac{1}{3}}(2x+3)^{-\frac{1}{6}}$	M1		or into equation $\sqrt{2x+3} = \frac{N}{x^3}$; or equiv
		Obtain 18 or value rounding to 18	A1	**2**	with no error seen
	(b)	State or imply $2.6282 = N^{\frac{1}{3}}(2\times 2.6022+3)^{-\frac{1}{6}}$	B1		
		Attempt solution for N	M1		using correct process
		Obtain 52	A1	**3**	concluding with integer value
				$\boxed{11}$	
9	**(i)**	Identify $\tan 55°$ as $\tan(45°+10°)$	B1		or equiv
		Use correct angle sum formula for $\tan(A+B)$	M1		or equiv
		Obtain $\frac{1+p}{1-p}$	A1	**3**	with $\tan 45°$ replaced by 1
	(ii)	<u>Either</u>: Attempt use of identity for $\tan 2A$	*M1		linking $10°$ and $5°$
		Obtain $p = \frac{2t}{1-t^2}$	A1		
		Attempt solution for t of quadratic equation	M1		dep *M
		Obtain $\frac{-1+\sqrt{1+p^2}}{p}$	A1	**4**	or equiv; and no second expression
		<u>Or (1)</u>: Attempt expansion of $\tan(60°-55°)$	*M1		
		Obtain $\frac{\sqrt{3}-\frac{1+p}{1-p}}{1+\sqrt{3}\,\frac{1+p}{1-p}}$	A1√		follow their answer from **(i)**
		Attempt simplification to remove denominators	M1		dep *M
		Obtain $\frac{\sqrt{3}(1-p)-(1+p)}{1-p+\sqrt{3}(1+p)}$	A1	**(4)**	or equiv

Or (2):	State or imply $\tan 15° = 2-\sqrt{3}$	B1	
	Attempt expansion of $\tan(15°-10°)$	M1	with exact attempt for $\tan 15°$
	Obtain $\dfrac{2-\sqrt{3}-p}{1+p(2-\sqrt{3})}$	A2 **(4)**	
Or (3):	State or imply $\tan 15° = \frac{\sqrt{3}-1}{\sqrt{3}+1}$	B1	or exact equiv
	Attempt expansion of $\tan(15°-10°)$	M1	with exact attempt for $\tan 15°$
	Obtain $\dfrac{\sqrt{3}-1-p\sqrt{3}-p}{\sqrt{3}+1+p\sqrt{3}-p}$	A2 **(4)**	or equiv
Or (4):	Attempt expansion of $\tan(10°-5°)$	*M1	
	Obtain $t = \dfrac{p-t}{1+pt}$	A1	
	Attempt solution for t of quadratic equation	M1	dep *M
	Obtain $\dfrac{-2+\sqrt{4+4p^2}}{2p}$	A1 **(4)**	or equiv; and no second expression
(iii)	Attempt expansion of both sides	M1	
	Obtain $3\sin\theta\cos 10° + 3\cos\theta\sin 10° = 7\cos\theta\cos 10° + 7\sin\theta\sin 10°$	A1	or equiv
	Attempt division throughout by $\cos\theta\cos 10°$	M1	or by $\cos\theta$ (or $\cos 10°$) only
	Obtain $3t + 3p = 7 + 7pt$	A1	or equiv
	Obtain $\dfrac{3p-7}{7p-3}$	A1 **5**	or equiv
		$\boxed{12}$	

4724 Core Mathematics 4

1		Long division method			
		Correct leading term x^2 in quotient	B1		
		Evidence of correct div process	M1		Sufficient to convince
		(Quotient $=$) x^2+6x-4	A1		
		(Remainder $=$) $11x+9$	A1		
		Identity method			
		$x^4+11x^3+28x^2+3x+1=Q\left(x^2+5x+2\right)+R$	M1		
		$Q=ax^2+bx+c$ or x^2+bx+c; $R=dx+e$ & ≥ 3 ops	M1		N.B. $a=1 \Rightarrow$ 1 of the 3 ops
		$a=1, b=6, c=-4, d=11, e=9$ (for all 5)	A2		S.R. B1 for 3 of these
			$\boxed{4}$		
2	(i)	Find at least 2 of $\left(\overrightarrow{AB} \text{ or } \overrightarrow{BA}\right), \left(\overrightarrow{BC} \text{ or } \overrightarrow{CB}\right), \left(\overrightarrow{AC} \text{ or } \overrightarrow{CA}\right)$	M1		irrespect of label; any notation
		Use correct method to find scal prod of any 2 vectors	M1		or use corr meth for modulus
		Use $\overrightarrow{AB}.\overrightarrow{BC}=0$ or $\dfrac{\overrightarrow{AB}.\overrightarrow{BC}}{\lvert AB\rvert\lvert BC\rvert}=0$	M1		or use $\lvert\overrightarrow{AB}\rvert^2+\lvert\overrightarrow{BC}\rvert^2=\lvert\overrightarrow{AC}\rvert^2$
		Obtain $p=1$ (dep 3 @ M1)	A1	4	
	(ii)	Use equal ratios of appropriate vectors	M1		or scalar product method
		Obtain $p=-8$	A1	2	
			$\boxed{6}$		
3		Use $\cos 2x = a\cos^2 x+b \,/\, \pm\cos^2 x-\sin^2 x \,/\, 1-2\sin^2 x$	*M1		
		Obtain $\lambda+\mu\sec^2 x$	dep*M1		using 'reasonable' Pythag attempt
		$\int \lambda+\mu\sec^2 x\,\mathrm{d}x=\lambda x+\mu\tan x$	A1		(λ or μ may be 0 here/prev line)
		Obtain correct result $2x-\tan x$	A1		no follow-through
		$\frac{1}{6}\pi-\sqrt{3}+1$ ISW	A1		exact answer required
			$\boxed{5}$		
4		Attempt to connect du and dt or find $\frac{\mathrm{d}u}{\mathrm{d}t}$ or $\frac{\mathrm{d}t}{\mathrm{d}u}$	M1		not du = dt but no accuracy
		$\mathrm{d}u=\frac{1}{t}\mathrm{d}t$ or $\frac{\mathrm{d}u}{\mathrm{d}t}=\frac{1}{t}$ or $\mathrm{d}t=\mathrm{e}^{u-2}\mathrm{d}u$ or $\frac{\mathrm{d}t}{\mathrm{d}u}=\mathrm{e}^{u-2}$	A1		
		Indef int $\rightarrow \int \frac{1}{u^2}(\mathrm{d}u)$	A1		no t or dt in evidence
		$=-\frac{1}{\mathrm{u}}$	A1		
		Attempt to change limits if working with $\mathrm{f}(u)$	M1		or re-subst & use 1 and e
		$\frac{1}{6}$ ISW	A1		ln e must be changed to 1, ln 1 to 0
			$\boxed{6}$		

5	**(i)** $(1+x)^{\frac{1}{3}} = 1 + \frac{1}{3}x + ...$	B1		
	$\ldots\ -\frac{1}{9}x^2$	B1	**2**	$-\frac{2}{18}x^2$ acceptable
	(ii) (a) $(8+16x)^{\frac{1}{3}} = 8^{\frac{1}{3}}(1+2x)^{\frac{1}{3}}$	B1		not $16^{\frac{1}{3}}\left(\frac{1}{2}+x\right)^{\frac{1}{3}}$
	$(1+2x)^{\frac{1}{3}}$ = their **(i)** expansion with $2x$ replacing x	M1		not dep on prev B1
	$= 1+\frac{2}{3}x-\frac{4}{9}x^2 + \ldots$	√A1		$-\frac{8}{18}x^2$ acceptable
	Required expansion = 2 (expansion just found)	√B1	**4**	accept equiv fractions
	N.B. If not based on part **(i)**, award M1 for $8^{1/3}+\frac{1}{3}.8^{-2/3}(16x)+\frac{\frac{1}{3}.-\frac{2}{3}}{1.2}8^{-5/3}(16x)^2$, allowing $16x^2$ for $(16x)^2$, with 3 @ A1 for $2...+\frac{4}{3}x...-\frac{8}{9}x^2$, accepting equivalent fractions & ISW			
	(ii) (b) $-\frac{1}{2} < x < \frac{1}{2}$ or $\lvert x\rvert < \frac{1}{2}$	B1	**1**	no equality
		$\boxed{7}$		
6	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	M1		quoted/implied
	$\frac{dx}{dt} = 9 - \frac{9}{9t}$ ISW	B1		
	$\frac{dy}{dt} = 3t^2 - \frac{3t^2}{t^3}$ ISW	B1		
	Stating/implying $\frac{3t^2-\frac{3}{t}}{9-\frac{1}{t}} = 3 \Rightarrow t^2 = 9$ or $t^3 - 9t = 0$	A1		WWW, totally correct at this stage
	$t = 3$ as final ans with clear log indication of invalidity of -3; ignore (non) mention of $t = 0$	A2		**S.R.** A1 if $t = \pm 3$ or $t = -3$ or ($t = 3$ **&** wrong/no indication)
		$\boxed{6}$		
7	Treat $\frac{d}{dx}(x^2y)$ as a product	M1		
	$\frac{d}{dx}(y^3) = 3y^2\frac{dy}{dx}$	B1		
	$3x^2 + 2x^2\frac{dy}{dx} + 4xy = 3y^2\frac{dy}{dx}$	A1		Ignore $\frac{dy}{dx} =$ if not used
	Subst (2, 1) and solve for $\frac{dy}{dx}$ or vice-versa	M1		
	$\frac{dy}{dx} = -4$ WWW	A1		
	grad normal = $-\frac{1}{\text{their }\frac{dy}{dx}}$	√A1		stated or used
	Find eqn of line, through (2, 1), with either gradient	M1		using their $\frac{dy}{dx}$ or $-\frac{1}{\text{their }\frac{dy}{dx}}$
	$x - 4y + 2 = 0$	A1		AEF with integral coefficients
		$\boxed{8}$		

8 (i)	$-\sin x\,e^{\cos x}$	B1	**1**	
(ii)	$\int \sin x\,e^{\cos x}\,dx = -e^{\cos x}$	B1		anywhere in part **(ii)**
	Parts with split $u = \cos x, dv = \sin x\,e^{\cos x}$	M1		result $f(x) +/- \int g(x)\,dx$
	Indef Integ, 1st stage $-\cos x\,e^{\cos x} - \int \sin x\,e^{\cos x}\,dx$	A1		accept $\ldots - \int -e^{\cos x}.-\sin x\,dx$
	Second stage = $-\cos x\,e^{\cos x} + e^{\cos x}$	*A1		
	Final answer = 1	dep*A2	**6**	
		$\boxed{7}$		
9 (i)	P is $\begin{pmatrix}3\\1\\1\end{pmatrix}+\begin{pmatrix}1\\-1\\2\end{pmatrix}=\begin{pmatrix}4\\0\\3\end{pmatrix}$	B1		
	direction vector of ℓ is $\begin{pmatrix}1\\-1\\2\end{pmatrix}$ and of $\overrightarrow{OP}$ is their P	√B1		
	Use $\cos\theta = \dfrac{\mathbf{a}.\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ for $\begin{pmatrix}1\\-1\\2\end{pmatrix}$ and their OP	M1		
	$\theta = 35.3$ or better (0.615… rad)	A1	**4**	
(ii)	Use $\begin{pmatrix}1\\-1\\2\end{pmatrix}.\begin{pmatrix}3+t\\1-t\\1+2t\end{pmatrix}=0$	M1		
	$1(3+t)-1(1-t)+2(1+2t)=0$	A1		
	$t = -\dfrac{2}{3}$	A1		
	Subst. into $\begin{pmatrix}3+t\\1-t\\1+2t\end{pmatrix}$ to produce $\begin{pmatrix}7/3\\5/3\\-1/3\end{pmatrix}$ ISW	A1	**4**	
(iii)	Use $\sqrt{x^2+y^2+z^2}$ where $\begin{pmatrix}x\\y\\z\end{pmatrix}$ is part **(ii)** answer	M1		
	Obtain $\sqrt{\dfrac{75}{9}}$ AEF, 2.89 or better (2.8867513….)	A1	**2**	
		$\boxed{10}$		

10 (i)	$\frac{\frac{1}{3}}{3-x}$ ……. $-\frac{\frac{1}{3}}{6-x}$	B1+1 **2**	
(ii)	**(a)** Separate variables $\int \frac{1}{(3-x)(6-x)}\,dx = \int k\,dt$	M1	or invert both sides
	Style: For the M1, dx & dt must appear on correct sides or there must be $\int$ sign on both sides		
	Change $\frac{1}{(3-x)(6-x)}$ into partial fractions from **(i)**	√B1	
	$\int \frac{A}{3-x}\,dx = \left(-A \text{ or } -\frac{1}{A}\right)\ln(3-x)$	B1	or $\int \frac{B}{6-x}\,dx = \left(-B \text{ or } -\frac{1}{B}\right)\ln(6-x)$
	$-\frac{1}{3}\ln(3-x)+\frac{1}{3}\ln(6-x)=kt\ (+c)$	√A1	f.t. from wrong multiples in **(i)**
	Subst (x = 0, t = 0) & (x = 1, t = 1) into eqn with 'c'	M1	and solve for 'k'
	Use $\ln a + \ln b = \ln ab$ or $\ln a - \ln b = \ln \frac{a}{b}$	M1	
	Obtain $k = \frac{1}{3}\ln\frac{5}{4}$ with sufficient working & WWW	A1 **7**	**AG**
	(b) Substitute $k = \frac{1}{3}\ln\frac{5}{4}$, $t = 2$ & their value of 'c'	*M1	
	Reduce to an eqn of form $\frac{6-x}{3-x} = \lambda$	dep*M1	where λ is a const
	Obtain $x = \frac{27}{17}$ or 1.6 or better (1.5882353…)	A2 **4**	S.R. A1√ for $x = \frac{3\lambda-6}{\lambda-1}$
		$\boxed{13}$	

4725 Further Pure Mathematics 1

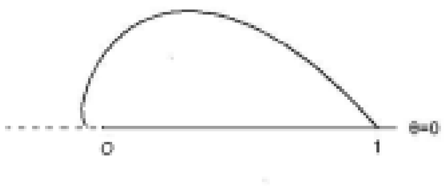
1	(i)	$\begin{pmatrix} a-4 & 2 \\ 3 & 0 \end{pmatrix}$	B1		Two elements correct
			B1	2	Remaining elements correct
	(ii)	$4a-6$	B1		Correct determinant
			M1		Equate det A to 0 and solve
		$a=\frac{3}{2}$	A1	3	Obtain correct answer a. e. f.
			$\boxed{5}$		
2	(i)	u^3-3u^2+3u-1	B1		Correct unsimplified expansion of $(u-1)^3$
			M1		Substitute for x
		$2u^3-6u^2+9u-8=0$	A1	3	Obtain correct **equation**
	(ii)		M1		Use $(\pm)\frac{\text{d}}{\text{a}}$ of new equation
		4	A1ft	2	Obtain correct answer from their equation
			$\boxed{5}$		
3		$x-\text{i}y$	B1		Conjugate known
			M1		Equate real and imaginary parts
		$x+2y=12 \quad 2x+y=9$	A1		Obtain both equations, OK with factor of i
			M1		Solve pair of equations
		$z=2+5\text{i}$	A1	5	Obtain correct answer as a complex number **S.C.** Solving $z+2\text{i}z=12+9\text{i}$ can get max 4/5, not first B1
			$\boxed{5}$		
4			M1		Express as sum of three series
			M1		Use standard results
		$\frac{1}{4}n^2(n+1)^2-\frac{1}{6}n(n+1)(2n+1)-n(n+1)$	A1		Obtain correct unsimplified answer
			M1		Attempt to factorise
			A1		Obtain at least factor of $n(n+1)$
		$\frac{1}{12}n(n+1)(n+2)(3n-7)$	A1	6	Obtain fully factorised correct answer
			$\boxed{6}$		

		Mark		Guidance
5 (i)		B1		Rotation 90° (about origin)
		B1	**2**	Anticlockwise
(ii)	***Either***	M1		Show image of unit square after reflection in $y = -x$
	$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$	A1		Deduce reflection in x-axis
		B1ft		Each column correct
		B1ft	**4**	ft for matrix of their transformation
	Or	M1		Post multiply by correct reflection matrix
		A1		Obtain correct answer
		B1B1		State reflection, in x-axis
			S.C.	If pre-multiplication, M0 but B1 B1 Available for correct description of their matrix
		$\boxed{6}$		
6 (i)		B1		State or use 5 + i as a root
		M1		Use $\sum \alpha\beta = 6$
	$x = -2$	A1	**3**	Obtain correct answer
(ii)	***Either***	M1		Use $p = -\sum \alpha$
	$p = -8$	A1ft		Obtain correct answer, from their root
		M1		Use $q = -\alpha\beta\gamma$
	$q = 52$	A1ft	**4**	Obtain correct answer, from their root
	Or	M1		Attempt to find quadratic factor
		M1		Attempt to expand quadratic and linear
		A1A1		Obtain correct answers
	Or	M1		Substitute (5 – i)into equation
		M1		Equate real and imaginary parts
		A1		Obtain correct answer for p
		A1ft		Obtain correct answer for q, ft their p
		$\boxed{7}$		
7 (i)		B1	**1**	Obtain **given** answer correctly
(ii)		M1		Express at least 1st two and last term using **(i)**
		A1		All terms correct
		M1		Show that correct terms cancel
	$1 - \frac{1}{(n+1)^2}$	A1	**4**	Obtain correct answer, in terms of n
(iii)	$\frac{1}{4}$	B1		Sum to infinity seen or implied
		B1	**2**	Obtain correct answer
				S.C. -¾ scores B1
		$\boxed{7}$		

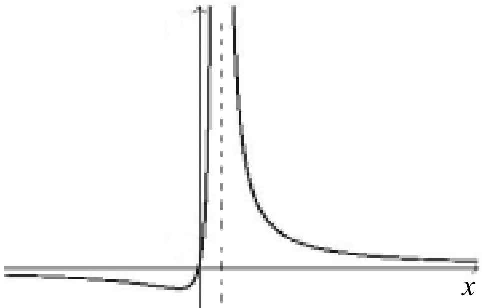
8	**(i)**	M1		Attempt to equate real and imaginary parts of $(x + iy)^2$ & $5 - 12i$
	$x^2 - y^2 = 5$ and $xy = -6$	A1		Obtain both results, a.e.f
		M1		Obtain quadratic in x^2 or y^2
		M1		Solve to obtain $x = (\pm)3$ or $y = (\pm)2$
	$\pm(3 - 2i)$	A1	**5**	Obtain correct answers as complex nos
	(ii)	B1ft		Circle with centre at their square root
		B1		Circle passing through origin
		B1ft		2nd circle centre correct relative to 1st
		B1	**4**	Circle passing through origin
		$\boxed{9}$		
9	**(i)**	M1		Show correct expansion process for 3×3 or multiply adjoint by **A**
		M1		Correct evaluation of any 2×2 at any stage
	det **A** = $\Delta = 6a - 6$	A1		Obtain correct answer
	$\mathbf{A}^{-1} = \frac{1}{\Delta}\begin{pmatrix} 3a-1 & a+1 & -4 \\ 1 & 2a-1 & -2 \\ -3 & -3 & 6 \end{pmatrix}$	M1		Show correct process for adjoint entries
		A1		Obtain at least 4 correct entries in adjoint
		B1		Divide by their determinant
		A1	**7**	Obtain completely correct answer
	(ii) $\frac{1}{\Delta}\begin{pmatrix} 5a-7 \\ 4a-5 \\ 3 \end{pmatrix}$	M1		Attempt product of form $\mathbf{A}^{-1}\mathbf{C}$ or eliminate to get 2 equations and solve
		A1A1A1 ft all 3		Obtain correct answer
			4	**S.C.** if det now omitted, allow max A2 ft
		$\boxed{11}$		
10	**(i)**			
		B1		Correct $\mathbf{M}^2$ seen
	$\mathbf{M}^2 = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$ $\mathbf{M}^3 = \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}$	M1		Convincing attempt at matrix multiplication for $\mathbf{M}^3$
		A1	**3**	Obtain correct answer
	(ii) $\mathbf{M}^n = \begin{pmatrix} 1 & 2n \\ 0 & 1 \end{pmatrix}$	B1ft	**1**	State correct form, consistent with **(i)**

10 (iii)	M1		Correct attempt to multiply **M** & $\mathbf{M}^k$ or v.v.
	A1		Obtain element $2(k+1)$
	A1		Clear statement of induction step, from correct working
	B1	**4**	Clear statement of induction conclusion, following their working
(iv)	B1		Shear
	DB1		x-axis invariant
	DB1	**3**	e.g. $(1, 1) \rightarrow (21, 1)$ or equivalent using scale factor or angles
	$\boxed{11}$		

4726 Further Pure Mathematics 2

1	(i)	Get 0.876096, 0.876496, 0.876642	B1√	For any one correct or √ from wrong answer; radians only
			B1	All correct
	(ii)	Subtract correctly (0.00023(0), 0.000084)	B1√	On their answers
		Divide their errors as e_4/e_3 only	M1	May be implied
		Get 0.365(21…)	A1	Cao
2	(i)	Find $f'(x) = 1/(1+(1+x)^2)$	M1	Quoted or derived; may be simplified or left as $\sec^2 y\ dy/dx = 1$
		Get $f(0) = ¼\pi$ and $f'(0) = ½$	A1√	On their $f'(0)$; allow $f(0)$=0.785 but not 45
		Attempt $f''(x)$	M1	Reasonable attempt at chain/quotient rule or implicit differentiation
		Correctly get $f''(0) = -½$	A1	A.G.
	(ii)	Attempt Maclaurin as $af(0)+bf'(0)+cf''(0)$	M1	Using their $f(0)$ and $f'(0)$
		Get $¼\pi + ½x - ¼x^2$	A1	Cao; allow 0.785
3	(i)	Attempt gradient as $\pm f(x_1)/(x_2 - x_1)$	M1	Allow reasonable y-step/x-step
		Equate to gradient of curve at x_1	M1	Allow ±
		Clearly arrive at A.G.	A1	Beware confusing use of ±
		SC Attempt equation of tangent	M1	As $y - f(x_1) = f'(x_1)(x - x_1)$
		Put $(x_2, 0)$ into their equation	M1	
		Clearly arrive at A.G.	A1	
	(ii)	Diagram showing at least one more tangent	B1	
		Description of tangent meeting x-axis, used as next starting value	B1	
	(iii)	Reasonable attempt at N-R	M1	Clear attempt at differentiation
		Get 1.60	A1	Or answer which rounds
4	(i)	State r =1 and $\theta = 0$.	B1	May be seen or implied
			B1	Correct shape, decreasing r (not through O)
	(ii)	Use $½\int r^2\ d\theta$ with $r = e^{-2\theta}$ seen or implied	M1	Allow $½\int e^{4\theta}\ d\theta$
		Integrate correctly as $-\frac{1}{8}e^{-4\theta}$	A1	
		Use limits in correct order	M1	In their answer
		Use $r_1^2 = e^{-4\theta}$ etc.	M1	May be implied
		Clearly get $k=\frac{1}{8}$	A1	

5	(i)	Use correct definitions of cosh and sinh	B1	
		Attempt to square and subtract	M1	On their definitions
		Clearly get A.G.	A1	
		Show division by $\cosh^2$	B1	Or clear use of first result
	(ii)	Rewrite as quadratic in sech and attempt to solve	M1	Or quadratic in cosh
		Eliminate values outside $0 < \text{sech} \le 1$	B1	Or eliminate values outside $\cosh \ge 1$ (allow positive)
		Get $x = \ln(2+\sqrt{3})$	A1	
		Get $x = -\ln(2+\sqrt{3})$ or $\ln(2-\sqrt{3})$	A1	
6	(i)	Attempt at correct form of P.F.	M1	Allow $Cx/(x^2+1)$ here; not $C = 0$
		Rewrite as 4= $A(1 + x)(1 + x^2) + B(1 - x)(1 + x^2) + (Cx + D)(1 - x)(1 + x)$	M1 √	From their P.F.
		Use values of x/equate coefficients	M1	
		Get $A = 1$, $B = 1$	A1	cwo
		Get $C = 0$, $D = 2$	A1	
				SC Use of cover-up rule for A,B M1 If both correct A1 cwo
	(ii)	Get $A\ln(1 + x) - B\ln(1 - x)$	M1	Or quote from List of Formulae
		Get $D\tan^{-1}x$	B1	
		Use limits in their integrated expressions	M1	
		Clearly get A.G.	A1	
7	(i)	LHS = sum of areas of rectangles, area = 1x y-value from $x = 1$ to $x = n$	B1	
		RHS = Area under curve from $x = 0$ to n	B1	
	(ii)	Diagram showing areas required	B1	
		Use sum of areas of rectangles	B1	
		Explain/show area inequality with limits in integral clearly specified	B1	
	(iii)	Attempt integral as $kx^{4/3}$	M1	
		Limits gives 348(.1) and 352(.0)	A1	Allow one correct
		Get 350	A1	From two correct values only

8	**(i)**	Get $x = 1$, $y = 0$	B1,B1	
	(ii)	Rewrite as quadratic in x Use $b^2 - 4ac \geq 0$ for all real x Get correct inequality State use of k>0 to A.G.	M1 M1 A1 A1	($x^2y - x(2y + k) + y = 0$) Allow >, = here $4ky + k^2 \geq 0$ SC Use differentiation (parts **(ii)** and **(iii)**) Attempt prod/quotient rule M1 Solve = 0 for $x = -1$ A1 Use x =−1 only (reject x=1), $y = -\frac{1}{4}k$ A1 Fully justify minimum B1 Attempt to justify for all x M1 Clearly get A.G. A1
	(iii)	Replace $y = -\frac{1}{4}k$ in quadratic in x Get $x = -1$ only  	M1 A1 B1 B1	 Through origin with minimum at (−1, −¼k) seen or given in the answer Correct shape (asymptotes and approaches) **SC** (Start again) Differentiate and solve dy/dx = 0 for at least one x-value, independent of k M1 Get $x = -1$ only A1
9	**(i)**	Rewrite tanh y as $(e^y - e^{-y})/(e^y + e^{-y})$ Attempt to write as quadratic in e^{2y} Clearly get A.G.	B1 M1 A1	Or equivalent
	(ii)	**(a)** Attempt to diff. and solve = 0 Get tanh $x = b/a$ Use (−1) < tanh x < 1 to show $b < a$	M1 A1 B1	SC Use exponentials M1 Get $e^{2x} = (a + b)/(a - b)$ A1 Use $e^{2x} > 0$ to show $b < a$ B1 SC Write $x = \tanh^{-1}(b/a)$ M1 $= \frac{1}{2}\ln((1 + b/a)/(1 - b/a))$ A1 Use ( ) > 0 to show $b < a$ B1
		(b) Get tanh x =1/a from part **(ii)(a)** Replace as ln from their answer Get $x = \frac{1}{2}\ln((a + 1)/(a - 1))$ Use $e^{\frac{1}{2}\ln((a+1)/(a-1))} = \sqrt{((a + 1)/(a - 1))}$ Clearly get A.G. Test for minimum correctly	B1 M1 A1 M1 A1 B1	 At least once **SC** Use of $y = \cosh x(a - \tanh x)$ and $\cosh x = 1/\text{sech}x = 1/\sqrt{(1 - \tanh^2 x)}$

4727 Further Pure Mathematics 3

1	METHOD 1			
	line segment between l_1 and l_2 = $\pm[4, -3, -9]$	B1		For correct vector
	$\mathbf{n} = [1, -1, 2] \times [2, 3, 4] = (\pm)[-2, 0, 1]$	M1* A1		For finding vector product of direction vectors
	distance $= \dfrac{\lvert [4, -3, -9] \cdot [-2, 0, 1] \rvert}{\left(\sqrt{2^2+0^2+1^2}\right)} = \dfrac{17}{\left(\sqrt{5}\right)}$	M1 (*dep)		For using numerator of distance formula
	$\neq 0$, so skew	A1	**5**	For correct scalar product and correct conclusion
	METHOD 2 lines would intersect where			
	$\left.\begin{array}{l} 1+s=-3+2t \\ -2-s=1+3t \\ -4+2s=5+4t \end{array}\right\} \Rightarrow \left\{\begin{array}{l} s-2t=-4 \\ s+3t=-3 \\ 2s-4t=9 \end{array}\right.$	B1		For correct parametric form for either line
		M1*		For 3 equations using 2 different parameters
		A1		
		M1 (*dep)		For attempting to solve to show (in)consistency
	$\Rightarrow$ contradiction, so skew	A1		For correct conclusion
		$\boxed{5}$		
2 (i)	$\left(a+b\sqrt{5}\right)\left(c+d\sqrt{5}\right)$	M1		For using product of 2 distinct elements
	$= ac+5bd+(bc+ad)\sqrt{5} \in H$	A1	**2**	For correct expression
(ii)	$(e=)\ 1$ *OR* $1+0\sqrt{5}$	B1	**1**	For correct identity
(iii)	*EITHER* $\dfrac{1}{a+b\sqrt{5}} \times \dfrac{a-b\sqrt{5}}{a-b\sqrt{5}}$	M1		For correct inverse as $\left(a+b\sqrt{5}\right)^{-1}$ and multiplying top and bottom by $a-b\sqrt{5}$
	OR $\left(a+b\sqrt{5}\right)\left(c+d\sqrt{5}\right)=1 \Rightarrow \left\{\begin{array}{l} ac+5bd=1 \\ bc+ad=0 \end{array}\right.$			*OR* for using definition and equating parts
	inverse $= \dfrac{a}{a^2-5b^2} - \dfrac{b}{a^2-5b^2}\sqrt{5}$	A1	**2**	For correct inverse. Allow as a single fraction
(iv)	5 is prime *OR* $\sqrt{5} \notin \mathbb{Q}$	B1	**1**	For a correct property (or equivalent)
		$\boxed{6}$		
3	Integrating factor $= e^{\int 2\,dx} = e^{2x}$	B1		For correct IF
	$\Rightarrow \dfrac{d}{dx}\left(ye^{2x}\right) = e^{-x}$	M1		For $\dfrac{d}{dx}(y.\text{their IF}) = e^{-3x}$. their IF
	$\Rightarrow y\,e^{2x} = -e^{-x}(+c)$	A1		For correct integration both sides
	$(0, 1) \Rightarrow c = 2$	M1		For substituting $(0, 1)$ into their GS and solving for c
		A1√		For correct c f.t. from their GS
	$\Rightarrow y = -e^{-3x} + 2e^{-2x}$	A1	**6**	For correct solution
		$\boxed{6}$		
4 (i)	$(z=)\ 2, -2,\ 2i, -2i$	M1		For at least 2 roots of the form $k\{1,\ i\}$ **AEF**
		A1	**2**	For correct values

(ii)	$\frac{w}{1-w}=2,-2,\ 2\mathrm{i},-2\mathrm{i}$	M1	For $\frac{w}{1-w}$ = any one solution from **(i)**
	$w=\frac{z}{1+z}$	M1	For attempting to solve for w, using any solution or in general
	$w=\frac{2}{3},\ 2$	B1 A1	For any one of the 4 solutions For both real solutions
	$w=\frac{4}{5}\pm\frac{2}{5}\mathrm{i}$	A1 **5**	For both complex solutions **SR** Allow B1√ and one A1√ from $k \neq 2$
		7	
5 (i)	$\mathbf{AB}=k\left[\frac{2}{3}\sqrt{3},0,-\frac{2}{3}\sqrt{6}\right]$, $\mathbf{BC}=k\left[-\sqrt{3},1,0\right]$, $\mathbf{CA}=k\left[\frac{1}{3}\sqrt{3},-1,\frac{2}{3}\sqrt{6}\right]$	B1 B1	For any one edge vector of ΔABC For any other edge vector of ΔABC
	$\mathbf{n}=k_1\left[\frac{2}{3}\sqrt{6},\frac{2}{3}\sqrt{18},\frac{2}{3}\sqrt{3}\right]=k_2\left[1,\sqrt{3},\frac{1}{2}\sqrt{2}\right]$	M1 M1	For attempting to find vector product of any two edges For substituting A, B or C into $\mathbf{r.n}$
	substitute A, B or C $\Rightarrow x+\sqrt{3}y+\frac{1}{2}\sqrt{2}z=\frac{2}{3}\sqrt{3}$	A1 **5**	For correct equation **AG**
			SR For verification only allow M1, then A1 for 2 points and A1 for the third point
(ii)	Symmetry in plane OAB or Oxz or $y=0$	B1* B1 (*dep)**2**	For quoting symmetry or reflection For correct plane Allow "in y coordinates" or "in y axis" **SR** For symmetry implied by reference to opposite signs in y coordinates of C and D, award B1 only
(iii)	$\cos\theta=\frac{\left\lvert\left[1,\sqrt{3},\frac{1}{2}\sqrt{2}\right].\left[1,-\sqrt{3},\frac{1}{2}\sqrt{2}\right]\right\rvert}{\sqrt{1+3+\frac{1}{2}}\sqrt{1+3+\frac{1}{2}}}$	M1 A1	For using scalar product of normal vectors For correct scalar product
	$=\frac{\left\lvert 1-3+\frac{1}{2}\right\rvert}{\frac{9}{2}}=\frac{\frac{3}{2}}{\frac{9}{2}}=\frac{1}{3}$	M1 A1 **4**	For product of both moduli in denominator For correct answer. Allow $-\frac{1}{3}$
		11	
6 (i)	$\left(m^2+16=0\Rightarrow\right)\ m=\pm 4\mathrm{i}$	M1	For attempt to solve correct auxiliary equation (may be implied by correct CF)
	CF = $A\cos 4x+B\sin 4x$	A1 **2**	For correct CF (**AEtrig** but not $A\mathrm{e}^{4\mathrm{i}x}+B\mathrm{e}^{-4\mathrm{i}x}$ only)
(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x}=p\sin 4x+4px\cos 4x$	M1 A1	For differentiating PI twice, using product rule For correct $\frac{\mathrm{d}y}{\mathrm{d}x}$
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=8p\cos 4x-16px\sin 4x$	A1√	For unsimplified $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. f.t. from $\frac{\mathrm{d}y}{\mathrm{d}x}$
	$\Rightarrow 8p\cos 4x=8\cos 4x$	M1	For substituting into DE
	$\Rightarrow p=1$	A1	For correct p
	$\Rightarrow (y=)A\cos 4x+B\sin 4x+x\sin 4x$	B1√ **6**	For using GS = CF + PI, with 2 arbitrary constants in CF and none in PI

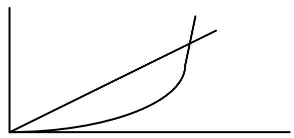
(iii)	$(0, 2) \Rightarrow A = 2$	B1√	For correct A. f.t. from their GS
	$\frac{dy}{dx} = -4A\sin 4x + 4B\cos 4x + \sin 4x + 4x\cos 4x$	M1	For differentiating their GS
	$x = 0,\ \frac{dy}{dx} = 0 \Rightarrow B = 0$	M1	For substituting values for x and $\frac{dy}{dx}$ to find B
	$\Rightarrow y = 2\cos 4x + x\sin 4x$	A1 **4**	For stating correct solution **CAO** including $\boxed{y=}$
		$\boxed{12}$	
7 (i)	$\cos 6\theta = 0 \Rightarrow 6\theta = k \times \frac{1}{2}\pi$	M1	For multiples of $\frac{1}{2}\pi$ seen or implied
	$\Rightarrow \theta = \frac{1}{12}\pi\{1, 3, 5, 7, 9, 11\}$	A1	A1 for any 3 correct
		A1 **3**	A1 for the rest, and no extras in $0 < \theta < \pi$
(ii)	METHOD 1		
	$\text{Re}(c + \text{i}s)^6 = \cos 6\theta = c^6 - 15c^4s^2 + 15c^2s^4 - s^6$	M1	For expanding $(c + \text{i}s)^6$ at least 4 terms and 2 binomial coefficients needed
		A1	For 4 correct terms
	$\cos 6\theta = c^6 - 15c^4(1 - c^2) + 15c^2(1 - c^2)^2 - (1 - c^2)^3$	M1	For using $s^2 = 1 - c^2$
	$\Rightarrow \cos 6\theta = 32c^6 - 48c^4 + 18c^2 - 1$	A1	For correct expression for $\cos 6\theta$
	$\Rightarrow \cos 6\theta = \left(2c^2 - 1\right)\left(16c^4 - 16c^2 + 1\right)$	A1 **5**	For correct result **AG** (may be written down from correct $\cos 6\theta$)
	METHOD 2		
	$\text{Re}(c + \text{i}s)^3 = \cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$	M1	For expanding $(c + \text{i}s)^3$ at least 2 terms and 1 binomial coefficient needed
		A1	For 2 correct terms
	$\Rightarrow \cos 6\theta = \cos 2\theta\left(\cos^2 2\theta - 3\sin^2 2\theta\right)$	M1	For replacing θ by 2θ
	$\Rightarrow \cos 6\theta = \left(2\cos^2\theta - 1\right)\left(4\left(2\cos^2\theta - 1\right)^2 - 3\right)$	A1	For correct expression in $\cos\theta$ (unsimplified)
	$\Rightarrow \cos 6\theta = \left(2c^2 - 1\right)\left(16c^4 - 16c^2 + 1\right)$	A1	For correct result **AG**
(iii)	METHOD 1		
	$\cos 6\theta = 0$	M1	For putting $\cos 6\theta = 0$
	$\Rightarrow$ 6 roots of $\cos 6\theta = 0$ satisfy $16c^4 - 16c^2 + 1 = 0$ and $2c^2 - 1 = 0$	A1	For association of roots with quartic and quadratic
	But $\theta = \frac{1}{4}\pi, \frac{3}{4}\pi$ satisfy $2c^2 - 1 = 0$	B1	For correct association of roots with quadratic
	EITHER Product of 4 roots *OR* $c = \pm\frac{1}{2}\sqrt{2 \pm \sqrt{3}}$	M1	For using product of 4 roots *OR* for solving quartic
	$\Rightarrow \cos\frac{1}{12}\pi \cos\frac{5}{12}\pi \cos\frac{7}{12}\pi \cos\frac{11}{12}\pi = \frac{1}{16}$	A1 **5**	For correct value (may follow A0 and B0)

	METHOD 2			
	$\cos 6\theta = 0$	M1		For putting $\cos 6\theta = 0$
	$\Rightarrow$ 6 roots of $\cos 6\theta = 0$ satisfy $32c^6 - 48c^4 + 18c^2 - 1 = 0$	A1		For association of roots with sextic
	Product of 6 roots $\Rightarrow$	M1		For using product of 6 roots
	$\cos\frac{1}{12}\pi \cdot \frac{1}{\sqrt{2}} \cdot \cos\frac{5}{12}\pi \cos\frac{7}{12}\pi \cdot \frac{-1}{\sqrt{2}} \cdot \cos\frac{11}{12}\pi = -\frac{1}{32}$	B1		For using $\cos\left\{\frac{3}{12}\pi, \frac{9}{12}\pi\right\} = \left\{\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right\}$
	$\cos\frac{1}{12}\pi \cos\frac{5}{12}\pi \cos\frac{7}{12}\pi \cos\frac{11}{12}\pi = \frac{1}{16}$	A1		For correct value
			13	
8 (i)	$g(x) = \dfrac{1}{2 - 2 \cdot \dfrac{1}{2-2x}} = \dfrac{2-2x}{2-4x} = \dfrac{1-x}{1-2x}$	M1		For use of $\mathrm{f\,f}(x)$
		A1		For correct expression **AG**
	$gg(x) = \dfrac{1 - \dfrac{1-x}{1-2x}}{1 - 2 \cdot \dfrac{1-x}{1-2x}} = \dfrac{-x}{-1} = x$	M1		For use of $gg(x)$
		A1	4	For correct expression **AG**
(ii)	Order of f = 4	B1		For correct order
	order of g = 2	B1	2	For correct order
(iii)	METHOD 1			
	$y = \dfrac{1}{2-2x} \Rightarrow x = \dfrac{2y-1}{2y}$	M1		For attempt to find inverse
	$\Rightarrow f^{-1}(x) = h(x) = \dfrac{2x-1}{2x}$ *OR* $1 - \dfrac{1}{2x}$	A1	2	For correct expression
	METHOD 2			
	$f^{-1} = f^3 = \mathrm{f\,g}$ or $\mathrm{g\,f}$	M1		For use of $\mathrm{f\,g}(x)$ or $\mathrm{g\,f}(x)$
	$\mathrm{f\,g}(x) = h(x) = \dfrac{1}{2 - 2\left(\dfrac{1-x}{1-2x}\right)} = \dfrac{1-2x}{-2x}$	A1		For correct expression
(iv)	(table below)	M1		For correct row 1 and column 1
		A1		For e, f, g, h in a latin square
		A1		For correct diagonal e - g - e - g
		A1	4	For correct table
			12	

	e	f	g	h
e	e	f	g	h
f	f	g	h	e
g	g	h	e	f
h	h	e	f	g

4728 Mechanics 1

1 i	$v = 4.2 + 9.8 \times 1.5$ $v = 18.9$ ms^{-1}.	M1 A1 [2]	Uses $v = u + gt$ 18.9(15) from g = 9.81
ii	$s = 4.2 \times 1.5 + 9.8 \times 1.5^2/2$ or $18.9^2 = 4.2^2 + 2 \times 9.8s$ $s = 17.325$ m	M1 A1 [2]	Uses $s = ut + gt^2/2$ or $v^2 = u^2 + 2gs$ Accept 17.3
iii	$v^2 = 4.2 + 2 \times 9.8 \times (17.3(25) - 5)$ $v = 16.1$ ms^{-1}	M1 A1 [2]	$18.9^2 = u^2 + 2 \times 9.8 \times 5$ $u = 16.1$ ms^{-1}. Accept answers close to 16.1 from correct working
2 i	Resolves a force in 2 perpendicular directions Uses Pythagoras $R^2 = (12+19\cos60)^2 + (19\sin60)^2$ R = 27.1 N $\{R = \sqrt{((19+12\cos60)^2 + (12\sin60)^2)} = 27.1\}$	M1 DM1 A1 A1 A1 [5]	*Diagram for vector addition/subtraction* *Uses Cosine Rule* $R^2 = 12^2 + 19^2 - 2 \times 12 \times 19\cos120$ *R = 27.1*
ii	Trig on a valid triangle for correct angle $\tan\theta = (19\sin60)/(12 + 19\cos60)$ etc Angle is 37.4°, 37.5°	M1 A1 A1 [3]	Either Pythagoras or vector add/sub triangle $\sin\theta/19 = \sin120/(27.1)$ *etc*
3ia **ib**	$\pm(9m + 2 \times 0.8)$ $\{\pm(3.5\times0.8-2\times0.8)\}$ $\pm(-3.5m + 3.5 \times 0.8)$ $\{\pm(9m + 3.5m)\}$ $\pm(9m + 2 \times 0.8) = \pm(-3.5m + 3.5 \times 0.8)$ m = 0.096 kg $\pm 0.096(9\pm3.5)$ *OR* $\pm 0.8(3.5 - 2)$ ± 1.2 kgms^{-1}	B1 B1 M1 A1 [4] M1 A1ft [2]	Before mom, or mom change Q, OK with g After mom, or mom change P, OK with g Equates moms, or changes, accept with g Do not award if g used Using before & after speeds of P or Q, no g ft 12.5 × cv(0.096)
ii	(0.8+0.4)v or 0.8v + 0.4v $3.5 \times 0.8 + 0.4 \times 2.75 = (0.8+0.4)v$ $v = 3.25$ ms^{-1}	M1 A1 A1 [3]	Using Q and R common speed after, no g 2.8 +1.1 = 1.2v
4ia **ib**	0.3gcos 60 and 0.3gsin60 0.4gcos60 and 0.4gsin60 Calculates either relevant difference Perp = 0.1gcos60 and Para = +/- 0.1gsin60 $0.1g\sin60 = \mu 0.1g\cos60$ $= 1.73$ ($= \sqrt{3}$)	B1 B1 M1 A1 [4] M1 A1 [2]	Accept use of “m = 0.1 kg” for M1 and 0.1gcos60 (B1) 0.1gsin60 (B1) = 0.49 and = 0.849 (accept 0.85 and 0.84) $F = \mu R$, F>R>0 From correct R, F values

4 ii	 $0.5g - T = 0.5a$ $T - 0.4g = 0.4a$ $a = 1.09\ ms^{-2}$ $T = 4.36$ N	M1 A1 B1 B1 [4]	N2L for either particle no resolving, at least 1 unknown Formula round the pulley, M0A0. But award M1 for $T-0.4g = 0.4 \times 1.09$ etc later Both equations correct
5 i	$11 = 3 + 20a$ $(a = 0.4)$ $8 = 3 + (11-3)t/20$ $t = 12.5$	M1 M1 A1 [3]	Uses $v = u + at$, no zero terms Their a>0. $t/20 = (8-3)/(11-3)$ is M1M1
ii	$s(A,20) = 8 \times 20\ (=160)$ $s(B,20) = (3 + 11) \times 20/2 =$ $3 \times 20 + 0.4 \times 20^2/2\ (=140)$ $8T = (3+11) \times 20/2 + 11 \times (T-20)$ or $(160 - 140) = 11t - 8t$ $T = 26\ 2/3$	B1 B1 M1 A1 A1 [5]	Or $s(A) = 8T$ or as stage of $s(B)=(3+11)\times20/2 + 11\times(T-20)$ 3 part equation balancing distances Accept 26.6 or 26.7
iii		B1 B1 B1 [3]	Linear rising graph (for A) starting at B's start Non-linear rising graph for B below A's initially. Accept 2 straight lines as non-linear. Single valued graphs graphs intersect and continue
6 i	$a = 2 \times 0.006t - 0.18$ $a = 0.012t - 0.18$	M1 A1 [2]	Differentiates v (not v/t) Award for unsimplified form, accept +c, not +k
ii	$0.012t - 0.18 = 0$ $t = 15$ $0.006 \times 15^2 - 0.18 \times 15 + k = 0.65$ $k = 2$ AG	M1* A1 D*M1 A1 A1 [5]	Sets a = 0, and solves for t Substitutes t(v(min)) in v(t)
iii	$s = 0.006t^3/3 - 0.18t^2/2 + 2t\ (+c)$ $(s = 0.002t^3 - 0.09t^2 + 2t\ (+c))$ $t = 0, s = 0$ hence $c = 0$ $L = 0.002 \times 28.4^3 - 0.09 \times 28.4^2 + 2 \times 28.4$ $L = 30.0$ m	M1A1 B1 M1 A1 [5]	Integrates v (not multiplies by t). Award if +c omitted, accept kt Explicit, not implied (or uses limits 0, 28.4) Substitutes 28.4 or 14.2 in s(t), (and k=2) Accept a r t 30(.0), accept +c

7 i	(Fr =) 0.15 × 600gcos10 (Wt cmpt =) 600gsin10 600 × 0.11 = T - 0.15 × 600gcos10 - 600gsin10 (66 = T - 868.6 – 1021) T = 1960 N	B1 B1 M1 A1 A1 [5]	Implied by Fr = 0.15×600gcos10 (=868.6..) N2L. T with at least 1 resolved forces and 600 × 0.11 1955.6..
ii a	a(up) = +/-(600gsin10+.15×600gcos10)/600 a(up) = +/-3.15 ms^{-2} AG	M1 A1 [2]	2 resolved forces and 600a or "unit mass" Disregard sign, accept 3.149
b	UP $v^2 = 2 \times 0.11 \times 10$ v = 1.48 when cable breaks t = 1.48/3.149 (t = 0.471 time for log to come to rest) s = $1.48^2/(2 \times 3.149)$ s = 0.349 distance for log to come to rest DOWN a(down) = (600gsin10-0.15×600gcos10)/600 10+0.349= $0.254t^2/2$ t = 9.025 T = (9.025 + 0.471) = 9.5 s	M1 A1 M1 M1 A1 B1 M1 A1 A1 [9]	 Correct, need not be accurate Or 1.48 = 0 + 3.15t Correct, need not be accurate = 0.254 Needs a< 3.15, s>10. Or V^2= 2×0.254× (10+0.349) [V= 2.29..], V=0.254t Correct, need not be accurate Accept 9.49

4729 Mechanics 2

1	$75\times9.8\times40$ $(75\times9.8\times40)\div120$ 245 W	B1 M1 A1 **[3]**	Average Speed = $40\div120$ $(75\times9.8)\times$(Average speed) **3**
2 (i)	$v^2 = 2\times9.8\times3$ or $2\times9.8\times1.8$ $v_1 = \sqrt{6g}$ or $\sqrt{58.8}$ or $\frac{7}{5}\sqrt{30}$ or 7.67 $v_2 = \sqrt{3.6g}$ or $\sqrt{35.28}$ or $\frac{21}{5}\sqrt{2}$ or 5.94 $I = \pm 0.2(5.94 + 7.67)$ 2.72	M1 A1 A1 M1 A1ft **[5]**	Kinematics or energy Speed of impact (±) Speed of rebound (±) +ve, ft on v_1 and v_2
(ii)	e = 5.94/7.67 0.775 or $\frac{\sqrt{15}}{5}$	M1 A1ft **[2]**	Allow 0.774, ft on v_1 and v_2 **7**
3 (i)	$\bar{u} = 0.2$ (from vertex) or 0.8 or 0.1 $0.5\bar{d} = 0.2\times\bar{u} + 0.3\times0.65$ $\bar{d} = 0.47$	B1 M1 A1 A1 **[4]**	com of conical shell **AG**
(ii)	s = 0.5 Tsin80°× 0.5 = 0.47 × 0.5 × 9.8 T = 4.68 N	B1 M1 A1 A1 **[4]**	slant height, may be implied **8**
4 (i)	D – 400 = 700 × 0.5 D = 750 N	M1 A1 **[2]**	3 terms
(ii)	P = 750 × 12 9 000 W or 9 kW	M1 A1ft **[2]**	
(iii)	P/35 = 400 14 000 W or 14 kW	M1 A1 **[2]**	
(iv)	D = 14000/12 3500/3 = 400 + 700×9.8sinθ θ = 6.42°	B1ft M1 A1 A1 **[4]**	May be implied 3 terms Their P/12 **10**

5	$16 - 12 = 2x + 3y$	M1	
	$4 = 2x + 3y$	A1	aef
	$½.2(8)^2 + ½.3(4)^2$ or $½.2x^2 + ½..3y^2$ or $±½.2(8^2 - x^2)$ or $±½.3(4^2 - y^2)$	B1	
	$½.2(8)^2 + ½.3(4)^2 - ½.2x^2 - ½...3y^2 = 81$	M1	
	$2x^2 + 3y^2 = 14$	A1	aef
	Attempt to eliminate x or y from a linear and a quadratic equation	M1	
	$15y^2 - 24y - 12 = 0$ or $10x^2 - 16x - 26 = 0$	A1	aef
	Attempt to solve a three term quadratic	M1	
	$x = -1$ (or $x = 2.6$)	A1	
	$y = 2$ (or $y = -2/5$)	A1	
	$x = -1$ and $y = 2$ only	A1	
	speeds 1, 2 away from each other	A1 **[12]**	**12**
6 (i)	$30^2 = V_1^2 \sin^2 \theta_1 - 2\times9.8\times250$	M1	$½m\ V_1^2 = ½m\ 50^2 + m\times9.8\times250$
	$V_1^2 \sin^2 \theta_1 = 5800$ AEF	A1	
	$V_1 \cos \theta_1 = 40$	B1	
	$V_1 = 86.0$	A1	**AG**
	$\theta_1 = 62.3°$	A1 **[5]**	**AG**
(ii)	$0 = \sqrt{5800}\ t_p - 4.9t_p^2$	M1	$30 = V_1 \sin \theta_1 - 9.8t$
	$t_p = 15.5$	A1	$t = 4.71$
	$-\sqrt{5800} = 30 - 9.8t_q$	M1	
	$t_q = 10.8$	A1 **[4]**	
(iii)	$R = 40\times15.5$	M1	
	$R = 621$	A1	(620, 622)
	$V_2 \cos \theta_2 \times 10.8 = 621$	B1	$V_2 \cos \theta_2 = 57.4$
	$0 = V_2 \sin \theta_2 \times 10.8 - 4.9 \times 10.8^2$	M1	
	$V_2 \sin \theta_2 = 53.1$ or 53.0	A1	(52.9,53.1)
	Method to find a value of V_2 or θ_2	M1	
	$\theta_2 = 42.8°$	A1	42.6° to 42.9°
	$V_2 = 78.2$ m s^{-1} or 78.1 m s^{-1}	A1 **[8]**	or 78.1° **17**
7 (i)	$\cos\theta = 3/5$ or $\sin\theta = 4/5$ or $\tan\theta = 4/3$ or $\theta = 53.1°$	B1	θ = angle to vertical
	$R\cos\theta = 0.2\times9.8$	M1	
	$R = 3.27$ N or $49/15$	A1 **[3]**	
(ii)	$r = 4$	B1	
	$R\sin\theta = 0.2\times4\times\omega^2$	M1	
		A1	
	$\omega = 1.81$ rad s^{-1}	A1 **[4]**	

(iii)	φ = 26.6° or $\sin\varphi = \frac{1}{\sqrt{5}}$ or $\cos\varphi = \frac{2}{\sqrt{5}}$ or tan φ = 0.5	B1	φ = angle to horizontal
	T = 0.98 or 0.1g	B1	
	Ncosθ = Tsinφ + 0.2×9.8	M1	Vertically, 3 terms
	N×3/5 = 0.438 + 1.96	A1	
	N = 4.00	A1	may be implied
	Nsinθ + Tcosφ = 0.2×4×ω^2	M1	Horizontally, 3 terms
	4×4/5 + 0.98cos 26.6° = 0.8ω^2	A1	
	ω = 2.26 rad s^{-1}	A1 **[8]**	**15**

4730 Mechanics 3

1	$0.4(3\cos60^o - 4) = -I\cos\theta$ (= -1) $0.4(3\sin60^o) = I\sin\theta$ (= 1.03920) [$\tan\theta = -1.5\sqrt{3}/(1.5-4)$; $I^2 = 0.4^2[(1.5-4)^2 + (1.5\sqrt{3})^2]$] θ = 46.1 or I = 1.44 I = 1.44 or θ = 46.1	M1 A1 A1 M1 A1 M1 A1ft [7]	For using I = Δmv in one direction SR: Allow B1 (max 1/3) for $3\cos60^o - 4 = -I\cos\theta$ and $3\sin60^o = I\sin\theta$ For eliminating I or θ (allow following SR case) Allow for θ (only) following SR case. For substituting for θ or for I (allow following SR case) ft incorrect θ or I; allow for θ (only) following SR case.
	Alternatively $I^2 = 1.2^2 + 1.6^2 - 2\times1.2\times1.6\cos60^o$ or $'V'^2 = 3^2 + 4^2 - 2\times3\times4\cos60^o$ I = 1.44 $\frac{\sin\theta}{3(or 1.2)} = \frac{\sin 60}{\sqrt{13(or 2.08)}}$ or $\frac{\sin\alpha}{4(or 1.6)} = \frac{\sin 60}{\sqrt{13(or 2.08)}} and \theta = 120 - \alpha$ θ = 46.1	M1 A1 M1 A1 M1 A1ft A1 [7]	For use of cosine rule For correct use of factor 0.4 (= m) For use of sine rule α must be angle opposite 1.6; (α = 73.9) ft value of I or 'V'
2	2a + 3b = 2×4 b – a = 0.6×4 [2(b - 2.4) + 3b = 8] b = 2.56 v = 2.56	M1 A1 M1 A1 M1 A1 B1ft [7]	For using the principle of conservation of momentum For using NEL For eliminating a ft v = b
3(i)	$2W(a\cos45^o) = T(2a)$ $W = \sqrt{2}\,T$	M1 A1 A1 [3]	For using 'mmt of 2W = mmt of T' AG
(ii)	Components (H, V) of force on BC at B are $H = -T/\sqrt{2}$ and $V = T/\sqrt{2} - 2W$ W(a cosα) + H(2a sinα) = V(2a cosα) [$W\cos\alpha - T\sqrt{2}\sin\alpha = T\sqrt{2}\cos\alpha - 4W\cos\alpha$] $T\sqrt{2}\sin\alpha = (5W - T\sqrt{2})\cos\alpha$ tan α = 4	B1 M1 A1 M1 A1ft A1 [6]	 For taking moments about C for BC For substituting for H and V and reducing equation to the form X sinα = Y cosα

	Alternatively for part **(ii)** anticlockwise mmt = $W(a\cos\alpha) + 2W(2a\cos\alpha + a\cos45^o)$ $= T[2a\cos(\alpha - 45^o) + 2a]$ $[5W\cos\alpha + \sqrt{2}W =$ $T(\sqrt{2}\cos\alpha + \sqrt{2}\sin\alpha) + 2]$ $T\sqrt{2}\sin\alpha = (5W - T\sqrt{2})\cos\alpha$ $\tan\alpha = 4$	 M1 A1 A1 M1 A1ft A1 [6]	 For taking moments about C for the whole For reducing equation to the form X sinα = Y cosα
4(i)	$[-0.2(v + v^2) = 0.2a]$ $[v\,dv/dx = -(v + v^2)$ $[1/(1 + v)]\,dv/dx = -1$	M1 M1 A1 [3]	For using Newton's second law For using a = v dv/dx AG
(ii)	 $\ln(1 + v) = -x\ (+ C)$ $\ln(1+ v) = -x + \ln3$ $[(1 + dx/dt)/3 = e^{-x} \Rightarrow dx/dt = 3e^{-x} - 1$ $\Rightarrow e^x\,dx/dt = 3 - e^x]$ $[-e^x/(3 - e^x)]\,dx/dt = -1$	M1 A1 A1 M1 A1 [5]	For integrating For transposing for v and using v = dx/dt AG
(iii)	$[\ln(3 - e^x) = -t + \ln2]$ $\ln(3 - e^x) = -t + \ln2$ Value of t is 1.96 (or $\ln\{2 \div (3 - e)\}$	M1 A1 A1 [3]	For integrating and using x(0) = 0
5(i)	 Loss of EE = $120(0.5^2 - 0.3^2)/(2\times1.6)$ and gain in PE = 1.5×4 v = 0 at B and loss of EE = gain in PE (= 6) $\Rightarrow$distance AB is 4m	M1 A1 M1 A1 [4]	For using EE = $\lambda x^2/2L$ and PE = Wh For comparing EE loss and PE gain AG
(ii)	$[120e/1.6 = 1.5]$ e = 0.02 Loss of EE = $120(0.5^2 - 0.02^2)/(2\times1.6)$ (or $120(0.3^2 - 0.02^2)/(2\times1.6)$) Gain in PE = 1.5(2.1- 1.6 – 0.02) (or 1.5(1.9 + 1.6 + 0.02) loss) [KE at max speed = 9.36 – 0.72 (or 3.36 + 5.28)] $\frac{1}{2}(1.5/9.8)v^2 = 9.36 - 0.72$ Maximum speed is 10.6 ms^{-1}	M1 A1 B1ft B1ft M1 A1 A1 [7]	For using T = mg and T = $\lambda x/L$ ft incorrect e only ft incorrect e only For using KE at max speed = Loss of EE – Gain (or + loss) in PE
	First alternative for **(ii)** x is distance AP $[\frac{1}{2}(1.5/9.8)v^2 + 1.5x + 120(0.5 - x)^2/3.2 =$ $120\times0.5^2/3.2]$ KE and PE terms correct EE terms correct $v^2 = 470.4x - 490x^2$ $[470.4 - 980x = 0]$ x = 0.48 Maximum speed is 10.6 ms^{-1}	 M1 A1 A1 A1 M1 A1 A1	 For using energy at P = energy at A For attempting to solve $dv^2/dx = 0$

	Second alternative for **(ii)** [120e/1.6 = 1.5] e = 0.02 [1.5 – 120(0.02 + x)/1.6 = 1.5 $\ddot{x}$ /g] n = $\sqrt{490}$ a = 0.48 Maximum speed is 10.6 ms^{-1}	M1 A1 M1 M1 A1 A1 A1	For using T = mg and T = λx/L For using Newton's second law For obtaining the equation in the form $\ddot{x} = -n^2x$, using (AB – L – e_{equil}) for amplitude and using v_{max} = na.
6(i)	PE gain by P = 0.4g × 0.8 sin θ PE loss by Q = 0.58g × 0.8 θ ½ (0.4 + 0.58)v^2 = g × 0.8(0.58 θ –0.4sin θ) v^2 = 9.28 θ - 6.4sin θ	B1 B1 M1 A1ft A1 [5]	 For using KE gain = PE loss AEF
(ii)	 0.4g sin θ – R = 0.4v^2/0.8 [0.4g sin θ – R = 4.64 θ – 3.2 sin θ] R = 7.12 sin θ – 4.64 θ	M1 A1 M1 A1 [4]	For applying Newton's second law to P and using a = v^2/r For substituting for v^2 AG
(iii)	R(1.53) = 0.01(48...), R(1.54) = -0.02(9...) or simply R(1.53) > 0 and R(1.54) < 0 R(1.53) × R(1.54) < 0 ➔1.53 < α < 1.54	M1 A1 M1 A1 [4]	For substituting 1.53 and 1.54 into R(θ) For using the idea that if R(1.53) and R(1.54) are of opposite signs then R is zero (and thus P leaves the surface) for some value of θ between 1.53 and 1.54. AG
7(i)	T_{AP} = 19.6e/1.6 and T_{BP} = 19.6(1.6-e)/1.6 0.5g sin30° + 12.25(1.6 – e) = 12.25e Distance AP is 2.5m	M1 A1 M1 A1ft A1 [5]	For using T = λe/L For resolving forces parallel to the plane
(ii)	Extensions of AP and BP are 0.9 + x and 0.7 – x respectively 0.5g sin 30° + 19.6(0.7 – x)/1.6 – 19.6(0.9 + x)/1.6 = 0.5 $\ddot{x}$ $\ddot{x}$ = -49x Period is 0.898 s	B1 B1ft B1 M1 A1 [5]	 AG For stating k < 0 and using T = 2π/ $\sqrt{-k}$
(iii)	 $2.8^2 = 49(0.5^2 – x^2)$ x^2 = 0.09 x = 0.3 and -0.3	M1 A1ft A1 A1ft [4]	For using $v^2 = \omega^2(A^2 – x^2)$ where ω^2 = -k ft incorrect value of k May be implied by a value of x ft incorrect value of k or incorrect value of x^2 (stated)

4732 Probability & Statistics 1

Note: "(3 sfs)" means "answer which rounds to ... to 3 sfs". If correct ans seen to $\geq$ 3sfs, ISW for later rounding
Penalise over-rounding only once in paper.

1 (i)	attempts at threading indep prob of succeeding in threading const	B1 B1 2	in context in context
(ii) (a)	$0.7^4 \times 0.3$ = 0.0720 (3sf)	M1 A1 2	 Condone 0.072
(b)	0.7^5 = 0.168 (3 sfs)	M2 A1 3	or $1-(0.3+0.7\times0.3+0.7^2\times0.3+0.7^3\times0.3+0.7^4\times0.3)$ M1 for one term omitted or extra or wrong or $1-0.7^5$ or$(0.3+...+0.7^4\times0.3)$ or 0.3, 0.7 muddle or 0.7^4 or 0.7^6 alone. 0.6 not 0.7 M0 in **(a)** M1 in **(b)** 1/3,2/3 used M1in **(a)** M1 in **(b)**
(iii)	likely to improve with practice hence independence unlikely or prob will increase each time	B1 B1 2	or thread strands gradually separate 1st B1 must be in context. hence independence unlikely or prob will decrease each time or similar Allow 'change'
Total		**[9]**	
2 (i) (a)	Use of correct midpts $\Sigma lf \div \Sigma f$ (= 706 ÷ 40) = 17.65 $\Sigma l^2 f$ (= 13050.5) $\sqrt{\frac{"13050.5"}{40} - "17.65"^2}$ (= √14.74) = 3.84 (3 sfs)	B1 M1 A1 M1 M1 A1 6	11,14,18,25.5 l within class, $\geq$ three lf seen [17.575,17.7] $\geq$ three l^2f seen ÷40,-mean²,√.Dep>0. ∑(l-17.65)²f, at least 3 M1,÷40,√ M1,3.84 A1. ÷ 4 ⇒ max B1M0A0M1M0A0
(b)	mid pts used or data grouped or exact values unknown oe	 B1 1	not "orig values were guesses"
(ii)	20 ÷ 5 = 4	M1 A1 2	condone 20 ÷ [4,5] or ans 5
(iii)	20.5th value requ'd and 1st two classes contain 14 values 16 – 20	 M1 B1 2	condone 20th oe or third class oe
(iv) (a)	increase	B1 1	
(b)	decrease	B1 1	
Total		**[13]**	
3 (i)	$S_{hm} = 0.2412$ $S_{hh} = 0.10992$ $S_{mm} = 27.212$ $r = \frac{S_{hm}}{\sqrt{(S_{hh}S_{mm})}}$ = 0.139 (3 sfs)	 B1 M1 A1 3	Allow x or ÷ 5 any one S correct ft their Ss
(ii)	Small, low or not close to 1 or close to 0 oe pts not close to line oe	B1 ft B1	1st B1 about value of r 2nd B1 about diag
(iii)	none or unchanged or "0.139" oe	B1 1	
(iv)	Larger oe	B1 1	
Total		**[7]**	

4 (i)	$(0\times\frac{1}{2}) + 1\times\frac{1}{4} + 2\times\frac{1}{8} + 3\times\frac{1}{8}$ $= \frac{7}{8}$ or 0.875 oe $(0\times\frac{1}{2}) + 1\times\frac{1}{4} + 2^2\times\frac{1}{8} + 3^2\times\frac{1}{8}$ $(= 1\frac{7}{8})$ $-(\text{“}\frac{7}{8}\text{”})^2$ $= \frac{71}{64}$ or 1.11 (3 sfs) oe	M1 A1 M1 M1 A1 5	$\geq$ 2 non-zero terms seen <u>If ÷3 or 4 M0M0M1(poss)</u> $\geq$ 2 non-zero terms seen dep +ve result M1 all4 $(x-0.875)^2$ terms seen. M1 mult p,$\sum$ A1 1.11
(ii)	Bin stated or implied 0.922 (3 sfs)	M1 A1 2	Eg table or $\frac{1}{4}^n\times\frac{3}{4}^m$ (n+m=10,n,m$\neq$ 1) or10C4 or 5(or 4 or 6) terms correct
(iii)	$n = 10$ & $p = \frac{1}{8}$ stated or implied ${}^{10}C_4\times\frac{7}{8}^6\times\frac{1}{8}^4$ = 0.0230 (3 sfs)	M1 M1 A1 3	condone 0.023
Total		**[10]**	
5 (i)	$\frac{6}{14}\times\frac{5}{13}\times\frac{3}{12}$ × 3! oe $= \frac{45}{182}$ or 0.247 (3 sfs)oe	M1 M1 A1 3	${}^6C_1\times{}^5C_1\times{}^3C_1$ $\div\,{}^{14}C_3$ With repl M0M1A0
(ii)	$\frac{6}{14}\times\frac{5}{13}\times\frac{4}{12}+\frac{5}{14}\times\frac{4}{13}\times\frac{3}{12}+\frac{3}{14}\times\frac{2}{13}\times\frac{1}{12}$ $= \frac{31}{364}$ or 0.0852 (3 sf)	M2 A1 3	${}^6C_3+{}^5C_3+{}^3C_3$ M1 for any one $(\div\,{}^{14}C_3)$M1 all 9 numerators correct. With repl M1$(6/14)^3+(5/14)^3+(3/14)^3$
Total		**[6]**	
6 (a)	A: diag or explanation showing pts close to st line, always increasing B:Diag or expl based on r=1=>pts on st line =>r(s)=1	B1 B1 B1 3	. Diag or expl based on r(s)$\neq$ 1=>pts not on st line =>r$\neq$ 1 r=1=>pts on st line&r(s)$\neq$ 1=>pts not on st line B1B1 r=1=>r(s)=1 B2
(b)	$\bar{y}$ = 2.4 × 4.5 + 3.7 = 14.5 4.5 = 0.4 × “14.5”- c c = 1.3 a’=x-b’y :-14.5 M1A1; then a’=4.5-0.4x14.5=-1.3 M1A1	M1 A1 M1 A1 4	Attempt to sub expression for y x=0.96x+1.48-c oe sub x=4.5 and solve c=1.3 14.5 M1A1.(y-3.7)/2.4=0.4y-c and sub14.5 M1 c=1.3 A1
Total		**[7]**	
7 (i)	$^{25}/_{37}$	B2 2	B1 num, B1 denom 25/37xp B1
(ii)	$\frac{15}{23}$ seen or implied $\times\frac{39}{59}$ seen or implied $= \frac{585}{1357}$ or 0.431 (3 sfs) oe	M1 M2 A1 4	M1 num, M1 denom Allow M1 for 39/59x or + wrong p
Total		**[6]**	

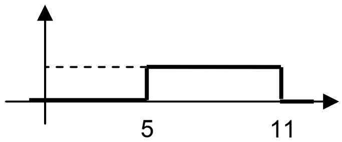
8 (i)	$^{5!}/_2$ = 60	M1 A1 2	Allow 5P3
(ii)	4! = 24	M1 A1 2	Allow 2×4!
(iii)	$^2/_5 \times {}^3/_4$ or3/5×2/4 × 2 = $^3/_5$ oe	M1 M1 A1 3	allow M1 for $^2/_5 \times {}^3/_5 \times 2$ or $^{12}/_{25}$ or (6×3!)÷**(i)** M2 or 3!÷**(i),**6÷**(i)**,(6+6)÷**(i)**,6k÷**(i)** or 6×6 or 36 or 1-correct answer M1 (k,integer ≤ 5)
Total		**[7]**	
9 (i)	p^2	B1 1	
(ii)	$(q^2p)^2$ oe =AG	B1 1	
(iii)	r=q² a/(1-r) used $(S_\infty =)\ \dfrac{p^2}{1-q^2}$ $= \dfrac{p^2}{1-(1-p)^2}$ p/(2-p) AG	B1 M1 A1 M1 A1 5	May be implied With a=p²and r=q²or q ⁴ Attempt to simplify using p+q=1 correctly. Dep on r = q² or q⁴ $\dfrac{(1-q)^2}{(1-q)(1+q)}$ or p²/p(1+q) Correctly obtain given answer showing at least one intermediate step.
P2Total		**[7]**	

Total 72 marks

4733 Probability & Statistics 2

Penalise over-specified answers (> 6 SF) first time but only once per paper.
Use A or C to annotate "over-assertive" or "no context" respectively

1		$\hat{\mu} = \bar{x} = 15.16$	B1		15.16 or 15.2 as answer only
		$\hat{\sigma}^2 = \frac{5}{4}s^2$	M1		Use $\frac{\Sigma x^2}{5} - \bar{x}^2$ [=1.0904]
			M1		Multiply by 5/4, or equiv for single formula
		= **1.363**	A1	4	Final answer 1.36 or 1.363 only, *not* isw
2	(i)	Not all equally likely – those in range 0 to 199 more likely to be chosen	M1 A1	2	Not all equally likely stated or implied Justified by reference to numbers, no spurious reasons
	(ii)	Ignore random numbers greater than 799, or 399	B1	1	Any valid resolution of this problem, no spurious reasons
3		B(60, 0.35) ≈ N(21, 13.65)	M1		B(60, 0.35) stated or implied
		$\Phi\left(\frac{18.5-21}{\sqrt{13.65}}\right) = \Phi(-0.6767)$	M1		N(21, ...)
			A1		Variance or SD = 13.65
		$= 1 - 0.7507$	M1		Standardise, their np and $\sqrt{npq}$ or npq, wrong or no cc
			A1		Both $\sqrt{npq}$ and cc correct
		= **0.2493**	A1	6	Answer, a.r.t. 0.249
4		$H_0: \mu = 60$; $H_1: \mu < 60$	B2		Both correct, B2 B1 for one error, but not x, t, $\bar{x}$ or $\bar{t}$
		(α) $z = \frac{58.9-60}{\sqrt{5^2/80}} = -1.967$	M1 A1		Standardise 58.9 & √80, allow – or √ errors z, art –1.97 or p in range [0.024, 0.025]
		< -1.645	B1		Explicit comparison with –1.645 or 0.05, or +1.645 or 0.95 if 1.967 or 0.976 used
	or:	(β) $c = 60 - 1.645 \times \frac{5}{\sqrt{80}} = 59.08$ 58.9 < 59.08	M1 B1 A1✓		60 – z×5/√80, any $z = \Phi^{-1}$, allow √ errors or ±, not just +; z = 1.645 and compare 58.9 59.1 or better, ✓on wrong z
		Reject H_0	M1		Correct first conclusion, needs essentially correct method including √80 or 80
		Significant evidence that people underestimate time	A1✓	7	Contextualised, uncertainty acknowledged SR: μ = 58.9: B0M1A0B1 max 2/7 SR: 2-tail: max 5/7
5	(i)	$H_0: \lambda = 11.0$ $H_1: \lambda > 11.0$	B2		Allow μ. Both correct, B2 One error: B1, but not C, x etc
		(α) P(≥ 19) = 1 – 0.9823	M1		Find P(≥ 19) [or P(< 19) if later 0.95]
		= 0.0177	A1		art 0.0177 [0.9823, ditto]
		< 0.05	B1		Compare 0.05 [0.95 if consistent], needs M1
		(β) CR ≥ 18,	M1		CR or CV 16/17/18/19 stated or clearly implied, but not <
		P(≥ 18) = 0.0322	A1		18 and 0.0322 both seen, allow 0.9678
		19 > 18	B1		Explicit comparison with 19, needs M1
		Reject H_0	M1		Needs essentially correct method & comparison
		Significant evidence of an increase in number of customers	A1✓	7	Contextualised, uncertainty acknowledged SR: Normal, or P(= 19) or P(≤ 19) or P(> 19): First B2 only.
	(ii)	Can't deduce cause-and-effect, or there may be other factors	B1	1	Conclusion needed. No spurious reasons. If "DNR" in **(i)**, "couldn't deduce even if…"

6	(i)	(a) Probabilities don't total 1	B1	**1**	Equivalent statement
		(b) P(> 70) must be < P(> 50)	B1	**1**	Equivalent statement
		(c) P(> 50) = 0.3 ⇒ μ < 50 P(< 70) = 0.3 ⇒ μ > 70	B1	**1**	Any relevant valid statement, e.g. "P(< 50) = 0.7 but P(< 50) must be < P(< 70)"
	(ii)	μ = 60 by symmetry	B1		μ = 60 obtained at any point, allow from Φ
		$\frac{10}{\sigma} = \Phi^{-1}(0.7) = 0.524(4)$	M1		One standardisation, equate to Φ^{-1}, not 0.758
		σ = 10/0.5243	B1		$\Phi^{-1} \in$ [0.524, 0.5245] seen
		= **19.084**	A1	**4**	σ in range [19.07, 19.1], e.g. 19.073
7	(i)		M1		Horizontal line
			A1	**2**	Evidence of truncation *[no need for labels]*
	(ii)	μ = 8	B1		8 only, cwd
		$\int_5^{11} \frac{1}{6}t^2 dt = \left[\frac{1}{18}t^3\right]_5^{11}$ [= 67]	M1		Attempt $\int kt^2\,dt$, limits 5 and 11 seen
			B1		k = 1/6 stated or implied
		$-\ 8^2$	M1		Subtract their (non-zero) mean^2
		= **3**	A1	**5**	Answer 3 only, *not* from MF1
	(iii)	N(8, 3/48)	M1		Normal stated or implied
		$1-\Phi\left(\frac{8.3-8}{\sqrt{3/48}}\right) = 1 - \Phi(1.2)$	A1		Mean 8
			A1		Variance their (non-zero) **(ii)**/48
		= 1 – 0.8848	M1		Standardise, $\sqrt{n}$, ignore sign or √ errors. cc: M0
		= **0.1151**	A1		Answer, art 0.115
		Normal distribution only approx.	B1	**6**	Any equivalent comment, e.g. CLT used
8	(i)	P(≤ 4) = 0.0473	M1		P(≤ r) from B(10, 0.7), r = 3/4/5, *not* N
		Therefore CR is ≤ 4	B1		"≤ 4" stated, not just "4", nothing else
		P(Type I error) = **4.73%**	A1	**3**	Answer, art 0.0473 or 4.73%, must be stated
	(ii)	B(10, 0.4) and find P(> 4)	M1		Must be this, *not* isw, ✓on **(i)**
		1 – P(≤ 4)	M1		Allow for 0.6177 or 0.1622
		= **0.3669**	A1	**3**	Answer, art 0.367
	(iii)	0.5 × 0.3669	M1		0.5 × **(ii)**
		= **0.18345**	A1✓	**2**	Ans correct to 3 SF, e.g. 0.184 from 0.367

9	(i)	$1 - P(\le 7) = 1 - 0.9881$ $= \mathbf{0.0119}$	M1 A1	 **2**	Allow for 0.0038 or 0.0335 Answer, a.r.t. 0.0119
	(ii)	Po(12) $P(\le 14) - P(\le 12)$ [0.7720 – 0.5760] $= \mathbf{0.196}$	M1 M1 A1	 **3**	Po(12) stated or implied Formula, 2 consecutive correct terms, or tables, e.g. .0905 or .3104 or .1629 Answer, art 0.196
	(iii)	$Po(60) \approx N(60, 60)$ $\Phi\left(\frac{69.5-60}{\sqrt{60}}\right) = \Phi(1.226)$ $= \mathbf{0.8899}$	M1 A1 M1 A1 A1	 **5**	N(60, …) Variance or SD 60 Standardise, λ & $\sqrt{\lambda}$, allow λ or wrong or no cc $\sqrt{\lambda}$ and cc both correct Answer 0.89 or a.r.t. 0.890
	(iv)	(a) $1 - e^{-3m}(1 + 3m)$	M1 A1	 **2**	M1 for one error, e.g. no “1 –”, or extra term, or 0^{th} term missing; answer, aesf
		(b) $m = 1.29$, $p = 0.89842$ $m = 1.3$, $p = 0.9008$ Straddles 0.9, therefore solution between 1.29 and 1.3	M1 A1 A1 A1	 **4**	Substitute 1.29 or 1.3 into appropriate fn Comp | 0.9 | 0.1 | 0 1.29 | 0.898 | 0.10158 | –.00158 1.3 | 0.901 | 0.09918 | .0008146 Explicit comparison with relevant value, & conclusion, needs both ps correct
	or	Method for iteration; 1.296… 1.2965or better; conclusion stated	M1A1 A1A1		Can be implied by at least 1.296… Need at least 4 dp for M1A2

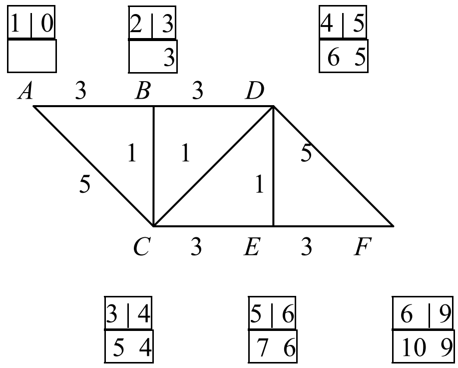
4734 Probability & Statistics 3

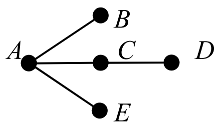
1(i)	$\int_{-a}^{0}\frac{2}{5}\,dx+\int_{0}^{\infty}\frac{2}{5}e^{-2x}\,dx=1$ $2a/5\ +1/5\ =1$ $a=2$	M1 A1 A1 **3**	Sum of probabilities =1
(ii)	$\int_{-2}^{0}\frac{2}{5}x\,dx+\int_{0}^{\infty}\frac{2}{5}xe^{-2x}\,dx$ $\int_{-a}^{0}\frac{2}{5}x\,dx=-\frac{a^2}{5}$ $\int_{0}^{\infty}\frac{2}{5}xe^{-2x}\,dx=\left[-\frac{1}{5}xe^{-2x}\right]+\left[-\frac{1}{10}e^{-2x}\right]$ = - 0.7	M1 A1 √ M1 A1 A1 **5** **[8]**	Σ ∫ x f(x) dx √ a By parts with 1 part correct Both parts correct CAO
2(i)	4 cartons: Total, $Y\sim$ N(2016, 36) P($Y\le 2000$)= Φ(- 16/√36) = 0.00383	B1B1 M1 A1 **4**	Mean and variance
(ii)	E(V) = 0 Var(V) = 36 + 16×9 = 180	B1 M1 A1 **3**	 CWO
(iii)	0.5	B1 **1** **[8]**	
3(i)	Normal distribution Mean $\mu_1-\mu_2$; variance $2.47/n_1+\ 4.23/n_2$	B1 B1B1 **3**	
(ii)	H_0: $\mu_1=\mu_2$, H_1:$\mu_1\neq\mu_2$ (9.65 – 7.23)/√(2.47/5+4.23/10) =2.527 > 2.326 Reject H_0 There is sufficient evidence at the 2% significance level that the means differ	B1 M1 B1 A1 M1 A1 **6**	 Or find critical region Numerator Compare with critical value **SR1:If no specific comparison but CV and conclusion correct B1. Same in Q5,6,7** **SR2: From CI: 2.42±$z\sigma$ M1, σ correct** **z = 2.326 B1, (0.193, 4.647) A1** **0 in not in CI ; reject H_0 etc M1A1 Total 6** **Conclusions not over-assertive in Q3, 5, 6**
(iii)	Any relevant comment.	B1 **1** **[10]**	e.g sample sizes too small for CLT to apply

4(i)	$G(y)=P(Y \le y)=P(1/(1+V) \le y)$ $=P(V \ge 1/y - 1)$ $= 1 - F(1/y - 1)$ $=\begin{cases} 0 & y \le 0, \\ 8y^3 & 0 < y \le 1/2, \\ 1 & y > 1/2. \end{cases}$ $g(y) = \begin{cases} 24y^2 & 0 < y \le 1/2, \\ 0 & \text{otherwise.} \end{cases}$	M1 A1 A1 A1 B1 M1 A1 **7**	 Use of F $8y^3$ obtained correctly Correct range. Condone omission of $y \le 0$ For $G'(y)$ Correct answer with range √
	$\int 24y^2/y^2\,dy$ with limits =12	M1 A1 **2** **[9]**	 With attempt at integration
5(i)	Use $p_s \pm z\sqrt{(p_s q_s/200)}$ z=1.645 $s = \sqrt{(0.135 \times 0.865/200)}$ (0.0952, 0.1747)	M1 B1 A1 A1 **4**	 Or /199 (0.095, 0.175) to 3DP
(ii)	H_0: $p_1 - p_2 = 0$, H_1: $p_1 - p_2 > 0$ $\dfrac{27/200 - 8/100}{\sqrt{35/300 \times 265/300 \times (200^{-1} + 100^{-1})}}$ = 1.399 > 1.282 Reject H_0. There is sufficient evidence at the 10% significance level that the proportion of faulty bars has reduced	B1 M1 B1 A1 A1 M1 A1 **7** **[11]**	Or equivalent Correct form. Pooled estimate of p = 35/300 Correct form of sd OR: $P(z \ge 1.399) = 0.0809 < 0.10$ **SR: No pooled estimate: B1M1B0B0 A1 for 1.514, M1A1 Max 5/7**
6(i)	Assumes that decreases have a normal distn H_0:μ_{O-F} = 0.2 (or ≥), H_1:μ_{O-F} > 0.2 O-F: 0.6 0.4 0.2 0.1 0.3 0.2 0.4 0.3 $\bar{D} = 0.3125$ $s^2 = 0.024107$ $(0.3125-0.2)/\sqrt{(0.024107/8)}$ =2.049 > 1.895 Reject H_o – there is sufficient evidence at the 5% significance level that the reduction is more than 0.2	B1 B1 M1 B1 A1 M1 A1 M1 A1 **9**	 B1 Use paired differences t-test Must have /8 OR: $P(t \ge 2.049)$= 0.0398 <0.05 Allow M1 from t_{14} = 1.761 **SR: 2-sample test:B1B1M0B1A0 M1 using 1.761 A0 Max 4/9**
(ii)	$0.3125 \pm t\sqrt{(0.024107/8)}$ t = 2.365 (0.1827, 0.4423)	M1 B1 A1 **3** **[12]**	Allow with z but with /8 Rounding to (0.283, 0.442)

7(i)	H_0:Vegetable preference is independent of gender H_1: All alternatives E-Values 26 16.25 22.75 22 13.75 19.25 $\chi^2 = 5^2(26^{-1} + 22^{-1}) + 7.25^2(16.25^{-1} + 13.75^{-1})$ $+ 2.25^2(22.75^{-1} + 19.25^{-1})$ $=9.641$ $9.64 > 5.991$ Reject H_0, (there is sufficient evidence at the 5% that) vegetable preference and gender are not independent	 B1 M1 A1 M1 A1 A1 M1 A1 **8**	 For both hypotheses At least one correct All correct Correct form of any one All correct ART 9.64 OR: $P(\geq 9.641)=0.00806 <0.05$
(ii)	(H_0: Vegetables have equal preference H_1: All alternatives) Combining rows: 48 30 42 E-Values: 40 40 40 $\chi^2 = (8^2+10^2 + 2^2)/40$ $= 4.2$ $4.2 < 4.605$ Do not reject H_0, there is insufficient evidence at the 10% significance level of a difference in the proportion of preferred vegetables	 M1 A1 M1 A1 M1 A1 **6** **[14]**	 OR:$P(\geq 4.2) = 0.122 > 0.10$ AEF in context

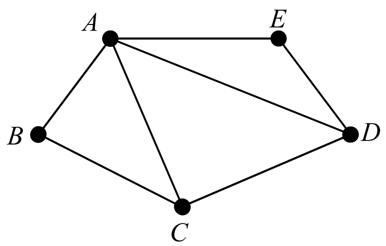
4736 Decision Mathematics 1

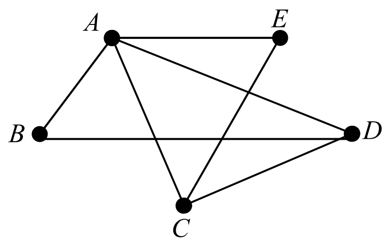
				TO BE ANSWERED ON INSERT	
1	(i)	 Path: *A – B – C – D – E – F* Weight: 9	M1 A1 B1 B1 B1	Evidence of updating at *C*, *D*, *E* or *F* All temporary labels correct, with no extras All permanent labels correct cao cao	[5]
	(ii)	Total weight of all arcs = 25 Only odd nodes are *B* and *E*. Least weight path joining *B* to *E* is *B – C – E* = 3. Weight: 28 Route: (example) *A–B–D–F–E–C–B–C–D–E – D – C - A*	B1 M1 A1 B1	Total weight = 25 (may be implied from weight) *B* to *E* = 3 28 (cao) A valid closed route that uses *BC*, *CD* and *DE* twice and all other arcs once	[4]
	(iii)	*A – B – E – F* Graph is now Eulerian, so no need to repeat arcs	B1 B1	cao Eulerian (or equivalent)	[2]
				Total =	**11**

2	**(i)**	A graph cannot have an odd number of odd vertices (nodes)	B1	Or equivalent (eg 3×5 = 15 ⇒ 7½ arcs) Not from a diagram of a specific case	**[1]**
	(ii)	It has exactly two odd nodes eg *C A B C D E A D*	B1 B1	2 odd nodes A valid semi-Eulerian trail	**[2]**
	(iii)	$AE = 2$ $AC = 3$ $AB = 5$ $CD = 7$  Weight = 17	 B1 B1 B1	 Correct tree (vertices must be labelled) Order of choosing arcs in a valid application of Prim, starting at *A* (working shown on a network or matrix) 17	**[3]**
	(iv)	Lower bound = 29 $A - E - D - F - C - B - A$ = 34 $F - C - A - E - D$ and $F - D - C - A - E$ Vertex *B* is missed out	B1 M1 A1 B1	29 or 12 + their tree weight from **(iii)** $A - E - D - F - C -$ 34, from correct working seen Correctly explaining why method fails, need to have explicitly considered both cases	**[4]**
				Total =	**10**

For reference

(ii)

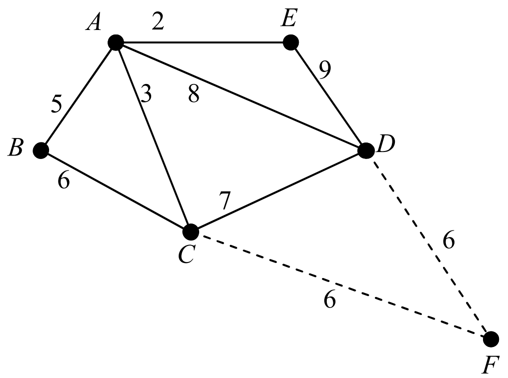


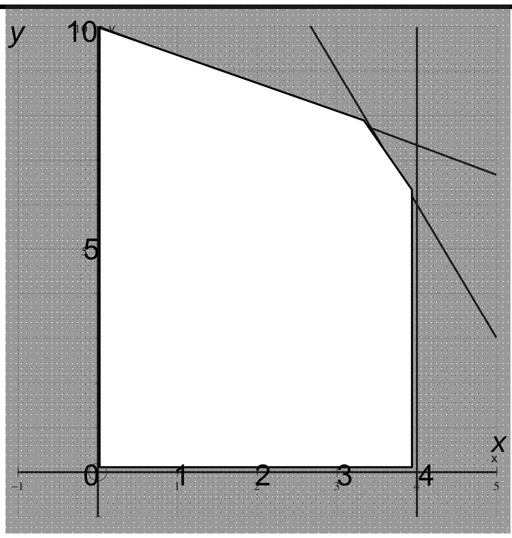


(iii) (iv)

	A	B	C	D	E
A	-	5	3	8	2
B	5	-	6	-	-
C	3	6	-	7	-
D	8	-	7	-	9
E	2	-	-	9	-

$CF = 6$
$DF = 6$



3 (i)	x = number of clients who use program X y = number of clients who use program Y	B1	Number of clients on X and Y, respectively	**[1]**
(ii)	Spin cycle: $30x + 10y \le 180$ $\Rightarrow 3x + y \le 18$ Rower: $10x \le 40$ $\Rightarrow x \le 4$ Free weights: $20x + 30y \le 300$ $\Rightarrow 2x + 3y \le 30$	B1 B1 B1	$3x + y \le 18$, or equivalent, simplified $x \le 4$, or equivalent, simplified $2x + 3y \le 30$, or equivalent, simplified Allow use of slack variables instead of inequalities	**[3]**
(iii)	Both must take non-negative integer values	B1	Non-negative <u>and</u> integer Accept $x + y \le 12$ as an alternative answer	**[1]**
(iv)		B1 M1 A1	Axes scaled and labelled appropriately (on graph paper) Boundaries of their three constraints shown correctly (non-negativity may be missed) Correct graph with correct shading or feasible region correct and clearly identified (non-negativity may be missed) (cao)	**[3]**
	Checking vertices or using a profit line $(4, 6) \to 72$ $(3\frac{3}{7}, 7\frac{5}{7}) \to 77\frac{1}{7}$ or $(24/7, 54/7) \to 77\frac{1}{7}$ $(0, 10) \to 60$ $(4, 0) \to 36$ Checking other feasible integer points near (non-integer) optimum for continuous problem $(3, 8) \to 75$ Put 3 clients on program X, 8 on program Y and 1 on program Z	M1 M1 A1	Follow through their graph if possible $x = 3.4$, $y = 7.7$ may be implied from (3, 8) Could be implied from identifying point (3, 8) in any form cao, in context and including program Z	**[3]**
			Total =	**11**

4	(i)	<table><tr><td>*A*</td><td>*A*</td><td>*A*</td><td>*A*</td><td>*A*</td><td>*A*</td><td>*A*</td><td>*A*</td><td>*A*</td><td>*A*</td></tr><tr><td>*A*</td><td>*A*</td><td>*A*</td><td>*A*</td><td>*A*</td><td>*D*</td><td>*D*</td><td>*D*</td><td>*D*</td><td>*C*</td></tr><tr><td>*C*</td><td>*C*</td><td>*B*</td><td>*B*</td><td>*B*</td><td>*B*</td><td>*B*</td><td>*B*</td><td>*B*</td><td>*B*</td></tr></table>	B1	15 *A*'s, 4 *D*'s, 3 *C*'s, 8*B*'s (but not just *A D C B*)	
		<table><tr><td>Box 1</td><td>*A A A A A*</td></tr><tr><td>Box 2</td><td>*A A A A A*</td></tr><tr><td>Box 3</td><td>*A A A A A*</td></tr><tr><td>Box 4</td><td>*D D D D C C C B*</td></tr><tr><td></td><td>*B B B B B B B*</td></tr></table>	M1 M1 A1	Three boxes each containing *A A A A A* (or shown using weights) A box containing all the rest Completely correct, including order of packing into boxes	
		Cannot fit all the items into box 4 There is only room for one *B* in a box	B1	Any identification of a (specific) volume conflict	**[5]**
	(ii)	<table><tr><td>*B*</td><td>*B*</td><td>*B*</td><td>*B*</td><td>*B*</td><td>*B*</td><td>*B*</td><td>*B*</td><td>*C*</td><td>*C*</td></tr><tr><td>*C*</td><td>*D*</td><td>*D*</td><td>*D*</td><td>*D*</td><td>*A*</td><td>*A*</td><td>*A*</td><td>*A*</td><td>*A*</td></tr><tr><td>*A*</td><td>*A*</td><td>*A*</td><td>*A*</td><td>*A*</td><td>*A*</td><td>*A*</td><td>*A*</td><td>*A*</td><td>*A*</td></tr></table>	B1	8 *B*'s, 3 *C*'s, 4 *D*'s, 15 *A*'s (but not just *B C D A*)	
		<table><tr><td>Box 1</td><td>*B D A A*</td></tr><tr><td>Box 2</td><td>*B D A A*</td></tr><tr><td>Box 3</td><td>*B D A A*</td></tr><tr><td>Box 4</td><td>*B D A A*</td></tr><tr><td>Box 5</td><td>*B A A A A A A*</td></tr><tr><td>Box 6</td><td>*B A*</td></tr><tr><td>Box 7</td><td>*B*</td></tr><tr><td>Box 8</td><td>*B*</td></tr><tr><td>Box 9</td><td>*C C C*</td></tr></table>	M1 M1 A1	Four boxes each containing *B D A A* (in any order) Using exactly 9 boxes, the first eight of which each contain a *B* (with or without other items) and the ninth contains three *C*'s. Completely correct, including order of packing into boxes	
		Box 5 is over the weight limit More than five A's is too heavy for one box	B1	Any identification of a (specific) weight conflict	**[5]**
	(iii)	Items may be the wrong shape for the boxes eg too tall	B1	Reference to shape, height, etc. but not practical issues connected with the food	**[1]**
				Total =	**11**

For reference

Item type	*A*	*B*	*C*	*D*
Number to be packed	15	8	3	4
Length (cm)	10	40	20	10
Width (cm)	10	30	50	40
Height (cm)	10	20	10	10
Volume (cm^3)	1 000	24 000	10 000	4 000
Weight (g)	1 000	250	300	400

<table>
<tr><td>5</td><td>(i)</td><td>Minimise $2a - 3b + c + 18$
$\Rightarrow$ minimise $2(20-x) - 3(10-y) + (8-z) + 18$
$\Rightarrow$ minimise $-2x + 3y - z$
$\Rightarrow$ maximise $2x - 3y + z$ (given)

$a + b - c \geq 14$
$\Rightarrow (20-x) + (10-y) - (8-z) \geq 14$
$\Rightarrow x + y - z \leq 8$ (given)

$-2a + 3c \leq 50$
$\Rightarrow -2(20-x) + 3(8-z) \leq 50$
$\Rightarrow 2x - 3z \leq 66$ (given)

$10 + 4a \geq 5b$
$\Rightarrow 10 + 4(20-x) \geq 5(10-y)$
$\Rightarrow 4x - 5y \leq 40$ (given)

$a \leq 20 \Rightarrow 20-x \leq 20 \Rightarrow x \geq 0$
$b \leq 10 \Rightarrow 10-y \leq 10 \Rightarrow y \geq 0$
$c \leq 8 \Rightarrow 8-z \leq 8 \Rightarrow z \geq 0$</td><td>B1

M1

A1</td><td>(Constant has no effect on slope of objective)
Replacing a, b and c in objective to get $-2x + 3y - z$
(Condone omission of conversion to maximisation here)

Replacing a, b and c in the first three constraints to get given expressions

Showing how $a \leq 20$, $b \leq 10$, $c \leq 8$ give $x \geq 0$, $y \geq 0$, $z \geq 0$</td><td>[3]</td></tr>
<tr><td></td><td>(ii)</td><td><table>
<tr><td>P</td><td>x</td><td>y</td><td>z</td><td>s</td><td>t</td><td>u</td><td>RHS</td></tr>
<tr><td>1</td><td>-2</td><td>3</td><td>-1</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>1</td><td>1</td><td>-1</td><td>1</td><td>0</td><td>0</td><td>8</td></tr>
<tr><td>0</td><td>2</td><td>0</td><td>-3</td><td>0</td><td>1</td><td>0</td><td>66</td></tr>
<tr><td>0</td><td>4</td><td>-5</td><td>0</td><td>0</td><td>0</td><td>1</td><td>40</td></tr>
</table></td><td>M1
A1</td><td>Constraint rows correct, with three slack variable columns
Objective row correct</td><td>[2]</td></tr>
<tr><td></td><td></td><td>x and z columns have negative entries in objective row, but z column has no positive entries in constraint rows, so pivot on x col
$8 \div 1 = 8$; $66 \div 2 = 33$; $40 \div 4 = 10$
Least ratio is $8 \div 1$, so pivot on 1 from x col</td><td>M1

A1</td><td>Choosing to pivot on x column
(may be implied from pivot choice)

Calculations seen or referred to and correct pivot choice made (cao)</td><td>[2]</td></tr>
<tr><td></td><td></td><td>New row 2 = row 2
New row 1 = row 1 + 2(new row 2)
New row 3 = row 3 – 2(new row 2)
New row 4 = row 4 – 4(new row 2)</td><td>M1

A1</td><td>Pivot row unchanged (may be implied) or follow through for their +ve pivot

Calculations for other rows shown (cao)</td><td>[2]</td></tr>
<tr><td></td><td></td><td><table>
<tr><td>P</td><td>x</td><td>y</td><td>z</td><td>s</td><td>t</td><td>u</td><td>RHS</td></tr>
<tr><td>1</td><td>0</td><td>5</td><td>-3</td><td>2</td><td>0</td><td>0</td><td>16</td></tr>
<tr><td>0</td><td>1</td><td>1</td><td>-1</td><td>1</td><td>0</td><td>0</td><td>8</td></tr>
<tr><td>0</td><td>0</td><td>-2</td><td>-1</td><td>-2</td><td>1</td><td>0</td><td>50</td></tr>
<tr><td>0</td><td>0</td><td>-9</td><td>4</td><td>-4</td><td>0</td><td>1</td><td>8</td></tr>
</table></td><td>M1

A1</td><td>An augmented tableau with three basis columns, non-negative values in final column and value of objective having not decreased
Correct tableau after one iteration (cao)</td><td>[2]</td></tr>
<tr><td></td><td></td><td>$x = 8$, $y = 0$, $z = 0$

$x = 8 \Rightarrow a = 20 - 8 = 12$
$y = 0 \Rightarrow b = 10 - 0 = 10$
$z = 0 \Rightarrow c = 8 - 0 = 8$</td><td>B1

M1

A1</td><td>Non-negative values for x, y and z from their tableau

Putting their values for x, y and z into $a = 20 - x$, $b = 10 - y$ and $c = 8 - z$

Correct values for a, b and c, from their non-negative x, y and z</td><td>[3]</td></tr>
<tr><td></td><td>(iii)</td><td>$x \leq 20$, $y \leq 10$ and $z \leq 8$</td><td>M1
A1</td><td>20, 10, 8
Correct inequalities for x, y and z</td><td>[2]</td></tr>
<tr><td colspan="5">Total =</td><td>16</td></tr>
</table>

<table>
<tr><td colspan="6" align="center">TO BE ANSWERED ON INSERT</td></tr>
<tr><td>6</td><td>(i)</td><td>10
½n(n-1)</td><td>B1
B1</td><td>10
1+2+…+(n-1) seen, or equivalent
Check that sum stops at n-1 not n</td><td>[2]</td></tr>
<tr><td></td><td>(ii)(a)</td><td>9
1
2
3
45</td><td>B1

M1

A1</td><td>Their 10 minus 1

1, 2 and 3

45 following from method mark earned cao</td><td>[3]</td></tr>
<tr><td></td><td>(b)</td><td>1+2+3+…+(N-1)
= ½N(N-1), where N = ½n(n-1)
= ¼n(n-1)(½n(n-1) – 1) (given)</td><td>M1

A1</td><td>1+2+3+…+(N-1) or ½N(N-1),
where N = ½n(n-1)
Convincingly achieving the given result</td><td>[2]</td></tr>
<tr><td></td><td>(iii)</td><td>
<table>
<tr><td>M1
Vertices in tree</td><td>M2
Arcs in tree</td><td>M3
Vertices not in tree</td><td>M4
Sorted list</td></tr>
<tr><td></td><td></td><td>ABCDE</td><td rowspan="5">$\cancel{D \mid 2 \mid E}$
$\cancel{A \mid 3 \mid E}$
$\cancel{A \mid 4 \mid C}$
$\cancel{C \mid 5 \mid D}$
$\cancel{B \mid 6 \mid E}$
$\cancel{B \mid 7 \mid C}$
$\cancel{A \mid 8 \mid B}$
$\cancel{C \mid 9 \mid E}$</td></tr>
<tr><td>D E</td><td>$D \mid 2 \mid E$</td><td>A B C</td></tr>
<tr><td>D E A</td><td>$D \mid 2 \mid E$
$A \mid 3 \mid E$</td><td>B C</td></tr>
<tr><td>D E A C</td><td>$D \mid 2 \mid E$
$A \mid 3 \mid E$
$A \mid 4 \mid C$</td><td>B</td></tr>
<tr><td>DEACB</td><td>$D \mid 2 \mid E$
$A \mid 3 \mid E$
$A \mid 4 \mid C$
$B \mid 6 \mid E$</td><td></td></tr>
</table>
</td><td>

M1

M1

M1

A1</td><td>(Order of entries in M1, M2 and M3 does not matter)

Arc $\boxed{A \mid 3 \mid E}$ is added to M2, A is added to M1 and deleted from M3

Arc $\boxed{A \mid 4 \mid C}$ is added to M2, C is added to M1 and deleted from M3

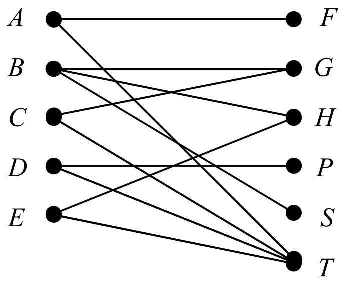
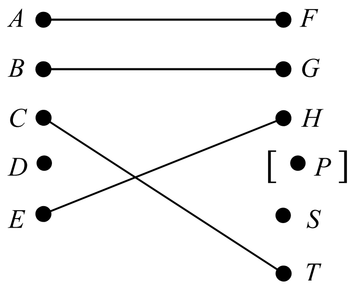
Arc $\boxed{C \mid 5 \mid D}$ is not added to M2 and arc $\boxed{B \mid 6 \mid E}$ is added to M2

cao (lists M1, M2 and M3 totally correct, ignore what is done in list M4).</td><td>[4]</td></tr>
<tr><td></td><td>(iv)</td><td>$30 \times \left(\frac{500}{100}\right)^4$
= 18750 seconds</td><td>M1

A1</td><td>Or equivalent

cao, with units
(312 min 30 sec or 5 hours 12 min 30 sec)</td><td>[2]</td></tr>
<tr><td colspan="5" align="right">Total =</td><td>13</td></tr>
</table>

4737 Decision Mathematics 2

1	(i)		B1	Bipartite graph correct	[1]
	(ii)		B1	A new bipartite graph showing the pairings AF, BG, CT and EH but not DS	
		$D = T - C = G - B = S$	M1	This alternating path written down, not read off from labels on graph	
		Andy = food Beth = science Chelsey = geography Dean = television Elly = history	A1 B1	$B = S$, $C = G$ and $D = T$ written down $A = F$, $E = H$ written down	[4]
	(iii)	Andy = food Beth = television Chelsey = geography Dean = politics Elly = history	B1	$A = F$, $C = G$, $D = P$ and $E = H$ (cao) ($B = T$ may be omitted)	
		Science did not arise	B1	S (cao)	[2]
				Total =	7

2

Add a dummy row

	P	R	S	T
April	30	28	32	25
May	32	34	32	35
June	40	40	39	38
Dummy	40	40	40	40

B1 — Adding a dummy row of all equal values

Reduce rows

5	3	7	0
0	2	0	3
2	2	1	0
0	0	0	0

Columns are already reduced

M1 — Substantially correct attempt to reduce matrix (condone 1 numerical slip)

A1 — Correct reduced cost matrix from reducing rows first and statement of how table was formed, including reference to columns (cao) **[3]**

Incomplete matching, cross through zeros

5	3	7	0
0	2	0	3
2	2	1	0
0	0	0	0

B1 — Cross through zeros using minimum number of lines

Augment by 1

4	2	6	0
0	2	0	4
1	1	0	0
0	0	0	1

B1 — Correct augmented matrix and statement of how table was formed (cao) **[2]**

Complete matching

	P	R	S	T
April	4	2	6	0
May	0	2	0	4
June	1	1	0	0
Dummy	0	0	0	1

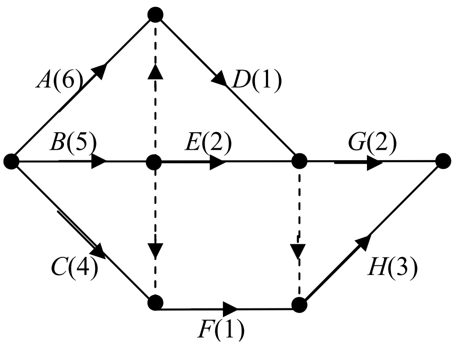
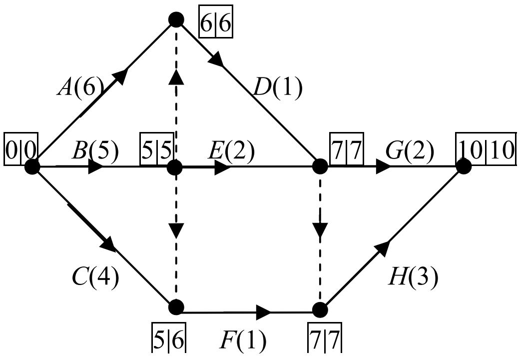
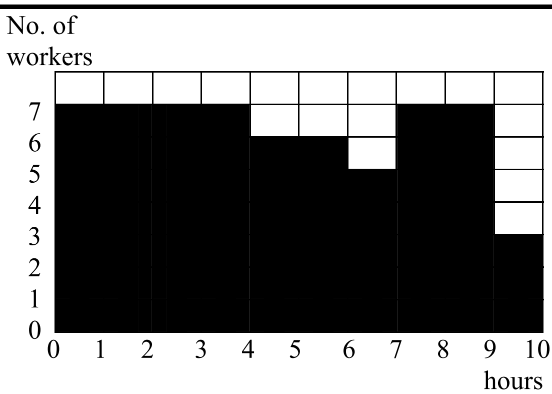
April = Tall Trees £2500
May = Palace £3200
June = Sunnyside £3900

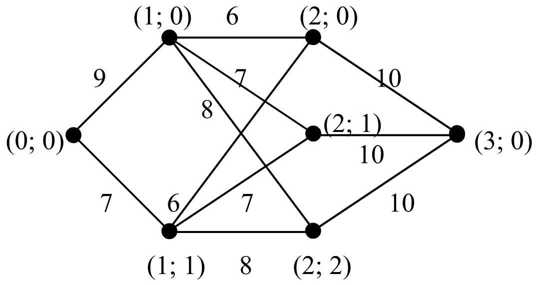
B1 — A = T, M = P, J = S (cao)

Total cost = £9600

B1 — £9600 (cao) with units **[2]**

Total = 7

3	(i)		M1 A1	Durations not necessary Correct structure, even without directions shown Activities must be labelled Completely correct, with exactly three dummies and all arcs directed	[2]
	(ii)		M1 M1 A1ft	Follow through their activity network if possible Substantially correct attempt at forward pass (at most 1 independent error) Substantially correct attempt at backward pass (at most 1 independent error) Both passes wholly correct	[3]
		Minimum project completion time = 10 hours Critical activities A, B, D, E, H	B1 M1 A1	10 hours (with units) cao Either B, E, H or A, D, H (possibly with other critical activities, but C, F, G not listed). Not follow through. A, B, D, E, H (and no others) cao	[3]
	(iii)		M1 A1	On graph paper A plausible resource histogram with no holes or overhangs Axes scaled and labelled and histogram completely correct, cao	[2]
	(iv)	1 hour	B1	Accept 1 (with units missing) cao	[1]
	(v)	No need to change start times for A, B, C, D and E Activities G and H cannot happen at the same time, so they must follow one another This causes a 2 hour delay	M1 A1	G and H cannot happen together (stated, not just implied from a diagram) 2 cao	[2]
		F could be delayed until 1 hour before H starts H should be started as late as possible $\Rightarrow$ a maximum delay of 3 hours	B1 B1	Diagram or explaining that for max delay on F need H to happen as late as possible 3 cao	[2]
				Total =	**15**

4	(i)		B1 M1 A1	Correct structure (vertex labels and graph correct) Assigning weights to their graph (no more than 1 error or no more than 2 arcs missing/extra) Completely correct network	[3]
	(ii)	Maximin	B1	cao	[1]
	(iii)	(table below)	B1 B1 B1	Four or five columns, including 'stage', 'state' and 'action' Stage and state columns completed correctly Action column completed correctly	[3]
			M1 A1	Min values correct for stage 1 Suboptimal maximin values correct for stages 2 and 1 (follow through their network if possible, no more than 2 arcs missing/extra)	[2]
			M1 A1	Min values correct for stage 0 Maximin value for stage 0 (follow through their network if possible, no more than 2 arcs missing/extra)	[2]
		Weight of heaviest truck = 8 tonnes Maximin route = (0; 0) – (1; 0) – (2; 2) – (3; 0)	B1 B1	8, cao Correct route, or in reverse	[2]
				Total =	**13**

Stage	State	Action	Working	Suboptimal maximin
2	0	0	10	10
	1	0	10	10
	2	0	10	10
1	0	0	min(6,10) = 6	
		1	min(7,10) = 7	
		2	min(8,10) = 8	8
	1	0	min(6,10) = 6	
		1	min(7,10) = 7	
		2	min(8,10) = 8	8
0	0	0	min(9,8) = 8	8
		1	min(7,8) = 7	

SR		Special ruling for working forwards			
	(iii)	(table below)	B1 B0 B0	Four or five columns, including 'stage', 'state' and 'action'	[3]
			M1 A0	No follow through from incorrect networks Min values correct for stage 2 and suboptmal maximin values correct for stages 1 and 2 (cao)	[2]
			M1 A0	No follow through from incorrect networks Correct min values for stage 3 and maximin value for stage 3 (cao)	[2]
		Weight of heaviest truck = 8 tonnes Maximin route = (0; 0) – (1; 0) – (2; 2) – (3; 0)	B1 B1	8, cao Correct route, or in reverse	[2]
				Maximum = B1 M1 M1 B1 B1 = 5 marks out of 9	

Stage	State	Action	Working	Suboptimal maximin
1	0	0	9	9
	1	0	7	7
2	0	0	min(9, 6) = 6	6
		1	min(7, 6) = 6	
	1	0	min(9, 7) = 7	7
		1	min(7, 7) = 7	
	2	0	min(9, 8) = 8	8
		1	min(7, 8) = 7	
3	0	0	min(6,10) = 6	
		1	min(7,10) = 7	
		2	min(8,10) = 8	8

<table>
<tr><td>5</td><td>(i)</td><td>$$\begin{array}{rc|ccc|c} & & & \text{Conan} & & \\ & & G & H & I & \text{row min} \\ \hline & D & -1 & -4 & 2 & -4 \\ \text{Robbie} & E & 3 & 1 & -4 & -4 \\ & F & 1 & -1 & 1 & -1 \quad * \\ \hline \text{col max} & & 3 & 1 & 2 & \\ & & & * & & \end{array}$$
Play-safe for Robbie is fairy
Play-safe for Conan is hag

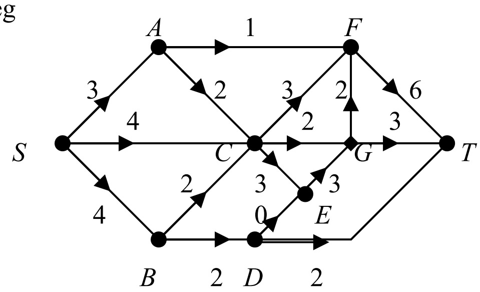
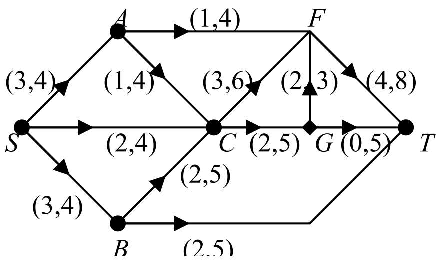
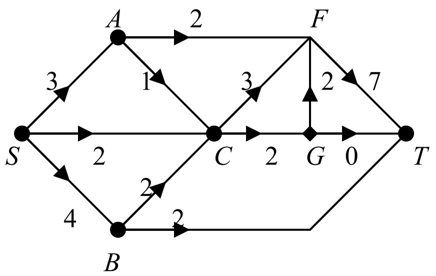
Robbie should choose the elf</td><td>M1
M1
A1
A1
B1</td><td>Calculating row minima (cao)
Calculating column maxima (or their negatives) (cao)
Fairy or F (not just -1 or identifying row)
Hag or H (not just $\pm$ 1 or identifying column)
Follow through their play-safe for Conan
Elf or E</td><td>[5]</td></tr>
<tr><td></td><td>(ii)</td><td>Dwarf: $\frac{1}{3}[(-1) + (-4) + (2)] = -1$
Elf: $\frac{1}{3}[(3) + (1) + (-4)] = 0$
Fairy: $\frac{1}{3}[(1) + (-1) + (1)] = \frac{1}{3}$</td><td>M1
A1</td><td>$D = -1$ or $F = \frac{1}{3}$ or -3, 0, 1
All three correct</td><td>[2]</td></tr>
<tr><td></td><td>(iii)</td><td>Goblin: $3p + (1-p) = 1 + 2p$
Hag: $p - (1-p) = 2p - 1$
Imp: $-4p + (1-p) = 1 - 5p$

$2p - 1 = 1 - 5p$
$\Rightarrow p = \frac{2}{7}$</td><td>M1
A1

M1
A1</td><td>Any one correct (in any form)
All three correct (in any form)

Appropriate equation seen for their expressions
$\frac{2}{7}$ or 0.286 (or better) from method seen</td><td>[2]

[2]</td></tr>
<tr><td></td><td>(iv)</td><td>4 is added throughout the table to make all the entries non-negative
If Conan chooses the goblin, this gives an expected value (in the new table) of $3x + 7y + 5z$</td><td>B1
B1</td><td>Add 4 to remove negative values
Expected value when Conan chooses the goblin</td><td>[2]</td></tr>
<tr><td></td><td>(v)</td><td>$z = \frac{5}{7} \Rightarrow m \leq 5.571,\ m \leq 3.571,\ m \leq 3.571$
$\Rightarrow m \leq 3.571\ (3\frac{4}{7})\ (\frac{25}{7})$
Hence, maximum value for M is 3.571 – 4
= -0.429 or $-\frac{3}{7}$</td><td>M1
M1
A1</td><td>Using $z = \frac{5}{7}$ to find a value for m (or implied)
Subtacting 4 from their m value
cao</td><td>[3]</td></tr>
<tr><td colspan="5" align="right">Total =</td><td>16</td></tr>
</table>

6	**(i)**	α = 12 litres per second β = 15 litres per second	B1 B1	12 15	**[2]**
	(ii)	At least 3 litres per second must flow into *A*, so *AC* and *AF* cannot both have flows of 1	B1	At least 3 flows along *SA*	**[1]**
	(iii)	At most 4 litres per second can flow into *B*, and at least 4 must flow out, so *BC* and *BD* must have flows of 2 Hence, only 2 litres per second flows into *D* and at least 2 litres per second must flow out, so *DE* and *DT* must both be at their lower capacities	B1 B1	At *B*: flow in $\leq$ 4 (and flow out $\geq$ 4) hence given flows in *BC* and *BD* Stating that flow into *D* is 2 and hence given flows in *DE* and *DT*	**[2]**
	(iv)	Flow across {*S*, *A*, *B*, *C*}, {*D*, *E*, *F*, *G*, *T*} $\geq$ 11 (so 10 litres per second is impossible)	M1 A1	Or any equivalent reasoning (eg flow through *C*) Wholly convincing argument	**[2]**
		Minimum = 11 eg 	M1 A1	11 Showing that 11 is possible (check *C*)	**[2]**
		Maximum = 12 No more than 12 can cross cut α and 12 is possible, eg augment flow shown above by 1 litre per second along *SAFT*	M1 A1	12 Showing that 12 is possible but 13 is not	**[2]**
	(v)	 e.g 	B1 M1 A1	A correct reduced network (vertex *E* and all arcs incident on *E* deleted), including arc capacities Or putting E_{in} and E_{out} with a capacity of 0 between them Or giving *CE*, *EG* and *DE* upper and lower capacities of 0 On same diagram or a new diagram *SA* = 3, *SC* = 2, *SB* = 4, *BC* = 2 and *BT* = 2 (and nothing through *E*, if shown) A valid flow of 9 litres per second through the network	**[3]**
				Total =	**14**

Grade Thresholds

Advanced GCE Mathematics (3890-2, 7890-2)
January 2010 Examination Series

Unit Threshold Marks

7892		Maximum Mark	A	B	C	D	E	U
4721	Raw	72	56	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
4722	Raw	72	61	53	46	39	32	0
	UMS	100	80	70	60	50	40	0
4723	Raw	72	51	43	36	29	22	0
	UMS	100	80	70	60	50	40	0
4724	Raw	72	55	47	39	32	25	0
	UMS	100	80	70	60	50	40	0
4725	Raw	72	62	54	46	38	31	0
	UMS	100	80	70	60	50	40	0
4726	Raw	72	53	46	39	32	25	0
	UMS	100	80	70	60	50	40	0
4727	Raw	72	55	47	40	33	26	0
	UMS	100	80	70	60	50	40	0
4728	Raw	72	52	44	36	28	21	0
	UMS	100	80	70	60	50	40	0
4729	Raw	72	56	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
4730	Raw	72	51	44	37	30	24	0
	UMS	100	80	70	60	50	40	0
4732	Raw	72	54	47	40	33	26	0
	UMS	100	80	70	60	50	40	0
4733	Raw	72	62	53	44	35	26	0
	UMS	100	80	70	60	50	40	0
4734	Raw	72	58	50	42	35	28	0
	UMS	100	80	70	60	50	40	0
4736	Raw	72	47	40	34	28	22	0
	UMS	100	80	70	60	50	40	0
4737	Raw	72	51	45	39	33	28	0
	UMS	100	80	70	60	50	40	0

Specification Aggregation Results

Overall threshold marks in UMS (ie after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
3890	300	240	210	180	150	120	0
3891	300	240	210	180	150	120	0
3892	300	240	210	180	150	120	0
7890	600	480	420	360	300	240	0
7891	600	480	420	360	300	240	0
7892	600	480	420	360	300	240	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
3890	28.2	53.1	73.0	87.2	96.4	100	1385
3892	39.2	61.7	79.2	92.5	97.5	100	126
7890	30.8	60.1	83.8	95.0	99.3	100	459
7892	21.1	60.5	84.2	100	100	100	43

For a description of how UMS marks are calculated see:
http://www.ocr.org.uk/learners/ums/index.html

Statistics are correct at the time of publication.

List of abbreviations

Below is a list of commonly used mark scheme abbreviations. The list is not exhaustive.

AEF	Any equivalent form of answer or result is equally acceptable
AG	Answer given (working leading to the result must be valid)
CAO	Correct answer only
ISW	Ignore subsequent working
MR	Misread
SR	Special ruling
SC	Special case
ART	Allow rounding or truncating
CWO	Correct working only
SOI	Seen or implied
WWW	Without wrong working
Ft or √	Follow through (allow the A or B mark for work correctly following on from previous incorrect result.)

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