RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE **A2 7890 - 2**

Advanced Subsidiary GCE **AS 3890 - 2**

Mark Schemes for the Units

January 2007

3890-2/7890-2/MS/R/07J

OCR (Oxford, Cambridge and RSA Examinations) is a unitary awarding body, established by the University of Cambridge Local Examinations Syndicate and the RSA Examinations Board in January 1998. OCR provides a full range of GCSE, A level, GNVQ, Key Skills and other qualifications for schools and colleges in the United Kingdom, including those previously provided by MEG and OCEAC. It is also responsible for developing new syllabuses to meet national requirements and the needs of students and teachers.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annersley
NOTTINGHAM
NG15 0DL

Telephone: 0870 870 6622
Facsimile: 0870 870 6621
E-mail: publications@ocr.org.uk

CONTENTS

Advanced GCE Mathematics (7890)
Advanced GCE Pure Mathematics (7891)
Advanced GCE Further Mathematics (7892)

Advanced Subsidiary GCE Mathematics (3890)
Advanced Subsidiary GCE Pure Mathematics (3891)
Advanced Subsidiary GCE Further Mathematics (3892)

MARK SCHEME ON THE UNITS

Mark Scheme 4721
January 2007

1	$\frac{5}{2-\sqrt{3}}$ x $\frac{2+\sqrt{3}}{2+\sqrt{3}}$ $= \frac{5(2+\sqrt{3})}{4-3}$ $= 10+5\sqrt{3}$	M1 A1 A1 3 **3**	Multiply top and bottom by $\pm(2+\sqrt{3})$ $(2+\sqrt{3})(2-\sqrt{3}) = 1$ (may be implied) $10+5\sqrt{3}$
2(i) (ii)	1 $\frac{1}{2}\times 2^4$ $= 8$	B1 1 M1 M1 A1 3 **4**	 $2^{-1} = \frac{1}{2}$ **<u>or</u>** $32^{\frac{1}{5}} = 2$ **<u>or</u>** $2^5 = 32$ soi $32^{\frac{4}{5}} = 2^4$ or 16 seen or implied 8
3(i) (ii)	$3x-15 \le 24$ $3x \le 39$ $x \le 13$ **or** $x-5 \le 8$ M1 $x \le 13$ A1 $5x^2 > 80$ $x^2 > 16$ $x > 4$ or $x < -4$	M1 A1 2 M1 B1 A1 3 **5**	Attempt to simplify expression by multiplying out brackets $x \le 13$ Attempt to simplify expression by dividing through by 3 Attempt to rearrange inequality or equation to combine the constant terms $x > 4$ fully correct, not wrapped, not 'and' **SR** B1 for $x \ge 4$, $x \le -4$

4	Let $y = x^{\frac{1}{3}}$ $y^2 + 3y - 10 = 0$ $(y-2)(y+5) = 0$ $y = 2, y = -5$ $x = 2^3, x = (-5)^3$ $x = 8, x = -125$	*M1 DM1 A1 DM1 A1 ft 5 **5**	Attempt a substitution to obtain a quadratic or factorise with $\sqrt[3]{x}$ in each bracket Correct attempt to solve quadratic Both values correct Attempt cube Both answers correctly followed through **SR** B2 $x = 8$ from T & I
5 (i)		M1 A1 2	Reflection in either axis Correct reflection in x axis
(ii)	(1, 3)	B1 B1 2	Correct x coordinate Correct y coordinate **SR** B1 for (3, 1)
(iii)	Translation 2 units in negative x direction	B1 B1 2 **6**	
6 (i)	$2(x^2 - 12x + 40)$ $= 2\left[(x-6)^2 - 36 + 40\right]$ $= 2\left[(x-6)^2 + 4\right]$ $= 2(x-6)^2 + 8$	B1 B1 M1 A1 4	$a = 2$ $b = 6$ $80 - 2b^2$ or $40 - b^2$ or $80 - b^2$ or $40 - 2b^2$ (their b) $c = 8$
(ii)	$x = 6$	B1 ft 1	
(iii)	$y = 8$	B1 ft 1 **6**	

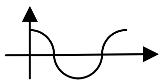
7(i)	$\frac{dy}{dx}=5$	B1 1	
(ii)	$y=2x^{-2}$	B1	x^{-2} soi
	$\frac{dy}{dx}=-4x^{-3}$	B1	$-4x^{c}$
		B1 3	kx^{-3}
(iii)	$y=10x^2-14x+5x-7$	M1	Expand the brackets to give an expression of form ax^2+bx+c $(a\neq 0, b\neq 0, c\neq 0)$
	$y=10x^2-9x-7$	A1	Completely correct (allow 2 x-terms)
	$\frac{dy}{dx}=20x-9$	B1 ft B1 ft 4	1 term correctly differentiated Completely correct (2 terms)
		8	
8 (i)	$\frac{dy}{dx}=9-6x-3x^2$	*M1	Attempt to differentiate y or $-y$ (at least one correct term)
		A1	3 correct terms
	At stationary points, $9-6x-3x^2=0$	M1	Use of $\frac{dy}{dx}=0$ (for y or $-y$)
	$3(3+x)(1-x)=0$ $x=-3$ or $x=1$	DM1 A1	Correct method to solve 3 term quadratic $x=-3,\ 1$
	$y=0,\ 32$	A1ft 6	$y=0,\ 32$ (1 correct pair www A1 A0)
(ii)	$\frac{d^2y}{dx^2}=-6x-6$	M1	Looks at sign of $\frac{d^2y}{dx^2}$, derived correctly from $k\frac{dy}{dx}$, or other correct method
	When $x=-3, \frac{d^2y}{dx^2}>0$	A1	$x=-3$ minimum
	When $x=1, \frac{d^2y}{dx^2}<0$	A1 3	$x=1$ maximum
(iii)	$-3<x<1$	M1	Uses the x values of both turning points in inequality/inequalities
		A1 2	Correct inequality or inequalities. Allow $\leq$
		11	

9 (i)	Gradient = 4	B1	Gradient of 4 soi
	$y - 7 = 4(x - 2)$	M1	Attempts equation of straight line through (2, 7) with any gradient
	$y = 4x - 1$	A1 3	
(ii)	$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$	M1	Use of correct formula for d or d^2 (3 values correctly substituted)
	$= \sqrt{(2-^-1)^2 + (7-^-2)^2}$		
	$= \sqrt{3^2 + 9^2}$	A1	$\sqrt{3^2 + 9^2}$
	$= \sqrt{90}$		
	$= 3\sqrt{10}$	A1 3	Correct simplified surd
(iii)	Gradient of AB = 3	B1	
	Gradient of perpendicular line = $-\frac{1}{3}$	B1 ft	**SR** Allow B1 for $-\frac{1}{4}$
	Midpoint of AB = $\left(\frac{1}{2}, \frac{5}{2}\right)$	B1	
	$y - \frac{5}{2} = -\frac{1}{3}\left(x - \frac{1}{2}\right)$	M1	Attempts equation of straight line through their midpoint with any non-zero gradient
	$x + 3y - 8 = 0$	A1	$y - \frac{5}{2} = \frac{-1}{3}(x - \frac{1}{2})$
		A1 6	$x + 3y - 8 = 0$
		12	

10 (i)	Centre (-1, 2) $(x+1)^2 - 1 + (y-2)^2 - 4 - 8 = 0$ $(x+1)^2 + (y-2)^2 = 13$ Radius $\sqrt{13}$	B1 M1 A1 3	Correct centre Attempt at completing the square Correct radius <u>Alternative method:</u> Centre ($-g, -f$) is (-1, 2) B1 $g^2 + f^2 - c$ M1 Radius = $\sqrt{13}$ A1
(ii)	$(2)^2 + (k-2)^2 = 13$ $(k-2)^2 = 9$ $k - 2 = \pm 3$ $k = -1$	M1 M1 A1 3	Attempt to substitute x = -3 into circle equation Correct method to solve quadratic $k = -1$ (negative value chosen)
(iii)	EITHER $y = 6 - x$ $(x+1)^2 + (6 - x - 2)^2 = 13$ $(x+1)^2 + (4 - x)^2 = 13$ $x^2 + 2x + 1 + 16 - 8x + x^2 = 13$ $2x^2 - 6x + 4 = 0$ $2(x-1)(x-2) = 0$ $x = 1 , 2$ $\therefore y = 5 , 4$ OR $x = 6 - y$ $(6 - y + 1)^2 + (y-2)^2 = 13$ $(7 - y)^2 + (y-2)^2 = 13$ $49 - 14y + y^2 + y^2 - 4y + 4 = 13$ $2y^2 - 18y + 40 = 0$ $2(y-4)(y-5) = 0$ $y = 4 , 5$ $\therefore x = 2 , 1$	M1 M1 A1 M1 A1 A1 6 **12**	Attempt to solve equations simultaneously Substitute into their circle equation for x/y or attempt to get an equation in 1 variable only Obtain correct 3 term quadratic Correct method to solve quadratic of form $ax^2 + bx + c = 0$ ($b \neq 0$) Both x values correct Both y values correct <u>or</u> one correct pair of values **www** B1 second correct pair of values B1 **SR** <u>**T & I**</u> M1 A1 One correct x (or y) value A1 Correct associated coordinate

Mark Scheme 4722
January 2007

1	$15+19d=72$ Hence $d = 3$ $S_n = {}^{100}/_2\ \{(2\times15)+(99\times3)\}$ $= 16350$	M1 A1 M1 A1	 **4**	Attempt to find d, from $a + (n-1)d$ or $a + nd$ Obtain $d = 3$ Use correct formula for sum of n terms Obtain 16350
			$\boxed{4}$	
2	**(i)** $46\times\frac{\pi}{180}=0.802$ / 0.803	M1 A1	 **2**	Attempt to convert to radians using π and 180 (or 2π & 360) Obtain 0.802 / 0.803, or better
	(ii) 8 x 0.803 = 6.4 cm	B1	**1**	State 6.4, or better
	(iii) ½ x 8^2 x 0.803 = 25.6 / 25.7 cm^2	M1 A1	 **2**	Attempt area of sector using $\frac{1}{2}r^2\theta$ or $r^2\theta$, with θ in radians Obtain 25.6 / 25.7, or better
			$\boxed{5}$	
3	**(i)** $\int(4x-5)\,dx=2x^2-5x+c$	M1 A1	 **2**	Obtain at least one correct term Obtain at least $2x^2-5x$
	(ii) $y = 2x^2 - 5x + c$ $7=2\times3^2-5\times3+c\Rightarrow c=4$ So equation is $y=2x^2-5x+4$	B1√ M1 A1	 **3**	State or imply y = their integral from **(i)** Use (3,7) to evaluate c Correct final equation
			$\boxed{5}$	
4	**(i)** area = $\frac{1}{2}\times5\sqrt{2}\times8\times\sin60^o$ $=\frac{1}{2}\times5\sqrt{2}\times8\times\frac{\sqrt{3}}{2}$ $=10\sqrt{6}$	B1 M1 A1	 **3**	State or imply that $\sin 60^o=\frac{\sqrt{3}}{2}$ or exact equiv Use $\frac{1}{2}ac\sin B$ Obtain $10\sqrt{6}$ only, from working in surds
	(ii) $AC^2=(5\sqrt{2})^2+8^2-2\times5\sqrt{2}\times8\times\cos60^o$ $AC = 7.58$ cm	M1 A1 A1	 **3**	Attempt to use the correct cosine formula Correct unsimplified expression for AC^2 Obtain $AC = 7.58$, or better
			$\boxed{6}$	
5	**(a) (i)** $\log_3\frac{4x+7}{x}$	B1	**1**	Correct single logarithm, as final answer, from correct working only
	(ii) $\log_3\frac{4x+7}{x}=2$ $\frac{4x+7}{x}=9$ $4x+7=9x$ $x=1.4$	 B1 M1 A1	 **3**	 State or imply $2=\log_3 9$ Attempt to solve equation of form f(x) = 8 or 9 Obtain x = 1.4, or exact equiv
	(b) $\int_3^9\log_{10}x\,dx\approx\frac{1}{2}\times3\times(\log_{10}3+2\log_{10}6+\log_{10}9)$ ≈ 4.48	B1 M1 A1 A1	 **4**	State, or imply, the 3 correct y-values only Attempt to use correct trapezium rule Obtain correct unsimplified expression Obtain 4.48, or better
			$\boxed{8}$	

Question		Answer	Marks	Total	Guidance
6	(i)	$(1+4x)^7 = 1 + 28x + 336x^2 + 2240x^3$	B1		Obtain $1 + 28x$
			M1		Attempt binomial expansion of at least 1 more term, with each term the product of binomial coeff and power of $4x$
			A1		Obtain $336x^2$
			A1	**4**	Obtain $2240x^3$
	(ii)	$28a + 1008 = 1001$	M1		Multiply together two relevant pairs of terms
		Hence $a = -\frac{1}{4}$	A1√		Obtain $28a + 1008 = 1001$
			A1	**3**	Obtain $a = -\frac{1}{4}$
				$\boxed{7}$	
7	(i)	**(a)** 	B1		Correct shape of $k\cos x$ graph
			B1	**2**	(90, 0), (270, 0) and (0, 2) stated or implied
		(b) $\cos x = 0.4$	M1		Divide by 2, and attempt to solve for x
		$x = 66.4^{\circ}, 294^{\circ}$	A1		Correct answer of 66.4° / 1.16 rads
			A1√	**3**	Second correct answer only, in degrees, following their x
	(ii)	$\tan x = 2$	M1		Use of $\tan x = \frac{\sin x}{\cos x}$ (or square and use $\sin^2 x + \cos^2 x \equiv 1$)
		$x = 63.4^{\circ}, -117^{\circ}$	A1		Correct answer of 63.4° / 1.56 rads
			A1√	**3**	Second correct answer only, in degrees, following their x
				$\boxed{8}$	
8	(i)	$-8 - 36 - 14 + 33 = -25$	M1		Substitute $x = -2$, or attempt complete division by $(x + 2)$
			A1	**2**	Obtain -25, as final answer
	(ii)	$27 - 81 + 21 + 33 = 0$ **A.G.**	B1	**1**	Confirm f(3) = 0, or equiv using division
	(iii)	$x = 3$	B1		State $x = 3$ as a root at any point
		$f(x) = (x-3)(x^2 - 6x - 11)$	M1		Attempt complete division by $(x - 3)$ or equiv
			A1		Obtain $x^2 - 6x + k$
			A1		Obtain completely correct quotient
		$x = \dfrac{6 \pm \sqrt{36+44}}{2}$	M1		Attempt use of quadratic formula, or equiv, to find roots
		$= 3 \pm 2\sqrt{5}$ or $3 \pm \sqrt{20}$	A1	**6**	Obtain $3 \pm 2\sqrt{5}$ or $3 \pm \sqrt{20}$
				$\boxed{9}$	
9	(i)	$u_5 = 1.5 \times 1.02^4$	M1		Use $1.5r^4$, or find u_2, u_3, u_4
		$= 1.624$ tonnes **A.G.**	A1	**2**	Obtain 1.624 or better
	(ii)	$\dfrac{1.5(1.02^N - 1)}{1.02 - 1} \le 39$	M1		Use correct formula for S_N
			A1		Correct unsimplified expressions for S_N
		$(1.02^N - 1) \le (39 \times 0.02 \div 1.5)$	M1		Link S_N to 39 and attempt to rearrange
		$(1.02^N - 1) \le 0.52$			
		Hence $1.02^N \le 1.52$	A1	**4**	Obtain given inequality convincingly, with no sign errors
	(iii)	$\log 1.02^N \le \log 1.52$	M1		Introduce logarithms on both sides and use $\log a^b = b \log$
		$N \log 1.02 \le \log 1.52$	A1		Obtain $N \log 1.02 \le \log 1.52$ (ignore linking sign)
		$N \le 21.144..$	M1		Attempt to solve for N
		$N = 21$ trips	A1	**4**	Obtain $N = 21$ only
				$\boxed{10}$	

10 (i)	$0 = 1 - \frac{3}{\sqrt{9}}$	B1	1	Verification of (9, 0), with at least one step shown
(ii)	$\int_9^a 1 - 3x^{-\frac{1}{2}} \mathrm{dx} = \left[x - 6\sqrt{x}\right]_9^a$	M1		Attempt integration – increase in power for at least 1 term
		A1		For second term of form $kx^{\frac{1}{2}}$
		A1		For correct integral
	$= \left(a - 6\sqrt{a}\right) - \left(9 - 6\sqrt{9}\right)$	M1		Attempt F(a) – F(9)
	$= a - 6\sqrt{a} + 9$	A1		Obtain $a - 6\sqrt{a} + 9$
	$a - 6\sqrt{a} + 9 = 4$	M1		Equate expression for area to 4
	$a - 6\sqrt{a} + 5 = 0$	M1		Attempt to solve 'disguised' quadratic
	$\left(\sqrt{a} - 1\right)\left(\sqrt{a} - 5\right) = 0$			
	$\sqrt{a} = 1, \sqrt{a} = 5$	A1		Obtain at least $\sqrt{a} = 5$
	$a = 1$, $a = 25$			
	but $a > 9$, so $a = 25$	A1	9	Obtain $a = 25$ only
			$\boxed{10}$	

Mark Scheme 4723
January 2007

1		Attempt use of quotient rule to find derivative	M1		allow for numerator 'wrong way round'; or attempt use of product rule
		Obtain $\dfrac{2(3x-1)-3(2x+1)}{(3x-1)^2}$	A1		or equiv
		Obtain $-\frac{5}{4}$ for gradient	A1		or equiv
		Attempt eqn of straight line with numerical gradient	M1		obtained from their $\frac{dy}{dx}$; tangent not normal
		Obtain $5x+4y-11=0$	A1	5	or similar equiv
2	(i)	Attempt complete method for finding $\cot\theta$	M1		rt-angled triangle, identities, calculator, …
		Obtain $\frac{5}{12}$	A1	2	or exact equiv
	(ii)	Attempt relevant identity for $\cos 2\theta$	M1		$\pm 2\cos^2\theta \pm 1$ or $\pm 1 \pm 2\sin^2\theta$ or $\pm(\cos^2\theta - \sin^2\theta)$
		State correct identity with correct value(s) substituted	A1		
		Obtain $-\frac{119}{169}$	A1	3	correct answer only earns 3/3
3	(a)	Sketch reasonable attempt at $y=x^5$	*B1		accept non-zero gradient at O but curvature to be correct in first and third quadrants
		Sketch straight line with negative gradient	*B1		existing at least in (part of) first quadrant
		Indicate in some way single point of intersection	B1	3	dep *B1 *B1
	(b)	Obtain correct first iterate	B1		allow if not part of subsequent iteration
		Carry out process to find at least 3 iterates in all	M1		
		Obtain at least 1 correct iterate after the first	A1		allow for recovery after error; showing at least 3 d.p. in iterates
		Conclude 2.175	A1	4	answer required to precisely 3 d.p.

[$0 \to 2.21236 \to 2.17412 \to 2.17480 \to 2.17479$;
$1 \to 2.19540 \to 2.17442 \to 2.17480 \to 2.17479$;
$2 \to 2.17791 \to 2.17473 \to 2.17479 \to 2.17479$;
$3 \to 2.15983 \to 2.17506 \to 2.17479 \to 2.17479$]

4	(i)	Obtain derivative of form $k(4t+9)^{-\frac{1}{2}}$	M1		any constant k
		Obtain correct $2(4t+9)^{-\frac{1}{2}}$	A1		or (unsimplified) equiv
		Obtain derivative of form $k\,e^{\frac{1}{2}x+1}$	M1		any constant k different from 6
		Obtain correct $3e^{\frac{1}{2}x+1}$	A1	4	or equiv
	(ii)	Either: Form product of two derivatives	M1		numerical or algebraic
		Substitute for t and x in product	M1		using $t=4$ and calculated value of x
		Obtain 39.7	A1	3	allow ±0.1; allow greater accuracy
		Or: Obtain $k(4t+9)^n e^{\frac{1}{2}(4t+9)^{\frac{1}{2}}+1}$	M1		differentiating $y=6e^{\frac{1}{2}(4t+9)^{\frac{1}{2}}+1}$
		Obtain correct $6(4t+9)^{-\frac{1}{2}}e^{\frac{1}{2}(4t+9)^{\frac{1}{2}}+1}$	A1		or equiv
		Substitute $t=4$ to obtain 39.7	A1	(3)	allow ±0.1; allow greater accuracy
5	(i)	Obtain $R=\sqrt{17}$ or 4.12 or 4.1	B1		or greater accuracy
		Attempt recognisable process for finding α	M1		allow for sin/cos confusion
		Obtain $\alpha = 14$	A1	3	or greater accuracy 14.036…

(ii)	Attempt to find at least one value of $\theta + \alpha$	M1		
	Obtain or imply value 61	A1√		following R value; or value rounding to 61
	Obtain 46.9	A1		allow ±0.1; allow greater accuracy
	Show correct process for obtaining second angle	M1		
	Obtain –75	A1	**5**	allow ±0.1; allow greater accuracy; max of 4/5 if extra angles between –180 and 180

---

6 (i)	Obtain integral of form $k(3x+2)^{\frac{1}{2}}$	M1		any constant k
	Obtain correct $\frac{2}{3}(3x+2)^{\frac{1}{2}}$	A1		or equiv
	Substitute limits 0 and 2 and attempt evaluation	M1		for integral of form $k(3x+2)^n$
	Obtain $\frac{2}{3}(8^{\frac{1}{2}} - 2^{\frac{1}{2}})$	A1	**4**	or exact equiv suitably simplified
(ii)	State or imply $\pi\int \frac{1}{3x+2}\,\mathrm{d}x$ or unsimplified version	B1		allow if $\mathrm{d}x$ absent or wrong
	Obtain integral of form $k\ln(3x+2)$	M1		any constant k involving π or not
	Obtain $\frac{1}{3}\pi\ln(3x+2)$ or $\frac{1}{3}\ln(3x+2)$	A1		
	Show correct use of ln a – ln b property	M1		
	Obtain $\frac{1}{3}\pi\ln 4$	A1	**5**	or (similarly simplified) equiv

---

7 (i)	State a in x-direction	B1		or clear equiv
	State factor 2 in x-direction	B1	**2**	or clear equiv
(ii)	Show (largely) increasing function crossing x-axis	M1		with correct curvature
	Show curve in first and fourth quadrants only	A1	**2**	not touching y-axis and with no maximum point; ignore intercept
(iii)	Show attempt at reflecting negative part in x-axis	M1		
	Show (more or less) correct graph	A1√	**2**	following their graph in **(ii)** and showing correct curvatures
(iv)	Identify $2a$ as asymptote or $2a + 2$ as intercept	B1		allow anywhere in question
	State $2a < x \le 2a + 2$	B1	**2**	allow < or ≤ for each inequality

---

8 (i)	Obtain $-2x\mathrm{e}^{-x^2}$ as derivative of e^{-x^2}	B1		
	Attempt product rule	*M1		allow if sign errors or no chain rule
	Obtain $8x^7\mathrm{e}^{-x^2} - 2x^9\mathrm{e}^{-x^2}$	A1		or (unsimplified) equiv
	<u>Either</u>: Equate first derivative to zero and attempt solution	M1		dep *M; taking at least one step of solution
	Confirm 2	A1	**5**	AG
	<u>Or</u>: Substitute 2 into derivative and show attempt at evaluation	M1		
	Obtain 0	A1	**(5)**	AG; necessary correct detail required

(ii)	Attempt calculation involving attempts at y values	M1		with each of 1, 4, 2 present at least once as coefficients
	Attempt $k(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4)$	M1		with attempts at five y values corresponding to correct x values
	Obtain $\frac{1}{6}(0 + 4 \times 0.00304 + 2 \times 0.36788$ $+ 4 \times 2.70127 + 4.68880)$	A1		or equiv with at least 3 d.p. or exact values
	Obtain 2.707	A1	**4**	or greater accuracy; allow ±0.001
(iii)	Attempt 4(y value) – 2(part **(ii)**)	M1		or equiv
	Obtain 13.3	A1	**2**	or greater accuracy; allow ±0.1

---

9 (i)	State $-2 \le y \le 2$	B1		allow <; any notation
	State $y \le 4$	B1	**2**	allow <; any notation
(ii)	Show correct process for composition	M1		right way round
	Obtain or imply 0.959 and hence 2.16	A1		AG; necessary detail required
	Obtain g(0.5) = 3.5	B1		or (unsimplified) equiv
	Observe that 3.5 not in domain of f	B1	**4**	or equiv
(iii)	Relate quadratic expression to at least one end of range of f	M1		or equiv
	Obtain both of $4 - 2x^2 < -2$ and $4 - 2x^2 > 2$	A1		or equiv; allow any sign in each (< or ≤ or > or ≥ or =)
	Obtain at least two of the x values $-\sqrt{3}, -1, 1, \sqrt{3}$	A1		
	Obtain all four of the x values	A1		
	Attempt solution involving four x values	M1		to produce at least two sets of values
	Obtain $x < -\sqrt{3}$, $-1 < x < 1$, $x > \sqrt{3}$	A1	**6**	allow ≤ instead of < and/or ≥ instead of >

Mark Scheme 4724
January 2007

1	Factorise numerator and denominator	M1		or Attempt long division
	Num = $(x+6)(x-4)$ or denom = $x(x-4)$	A1		Result = $1+\dfrac{6x-24}{x^2-4x}$
	Final answer = $\dfrac{x+6}{x}$ or $1+\dfrac{6}{x}$	A1	3	$= 1+\dfrac{6}{x}$
2	Use parts with $u = \ln x, \mathrm{d}v = x$	M1		& give 1st stage in form $\mathrm{f}(x) +/- \int \mathrm{g}(x)(\mathrm{d}x)$
	Obtain $\frac{1}{2}x^2\ln x - \int \frac{1}{x}.\frac{1}{2}x^2(\mathrm{d}x)$	A1		or $\frac{1}{2}x^2\ln x - \int \frac{1}{2}x(\mathrm{d}x)$
	$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$ (+c)	A1		
	Use limits correctly	M1		
	Exact answer $2\ln 2 - \frac{3}{4}$	A1	5	AEF ISW
3	(i) Find $\boldsymbol{a}-\boldsymbol{b}$ or $\boldsymbol{b}-\boldsymbol{a}$ irrespective of label	M1		(expect $11\mathbf{i} - 2\mathbf{j} - 6\mathbf{k}$ or $-11\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$)
	Method for magnitude of any vector	M1		
	$\sqrt{161}$ or $12.7(12.688578)$	A1	3	
	(ii) Using $(\overline{\mathrm{AO}}$ or $\overline{\mathrm{OA}})$ and $(\overline{AB}$ or $\overline{\mathrm{BA}})$	B1		Do not class angle AOB as MR
	$\cos\theta = \dfrac{\text{scalar product of any two vectors}}{\text{product of their moduli}}$	M1		
	43 or better (42.967…), 0.75 or better (0.7499218..)	A1	3	If 137 obtained, followed by 43, award A0 Common answer 114 probably $\to$ B0 M1 A0
4	Attempt to connect $\mathrm{d}x$ and $\mathrm{d}u$	M1		but not just $\mathrm{d}x = \mathrm{d}u$
	For $\mathrm{d}u = 2\,\mathrm{d}x$ AEF correctly used	A1		sight of $\frac{1}{2}$ ($\mathrm{d}u$) necessary
	$\int u^8 + u^7\,(\mathrm{d}u)$	A1		or $\int u^7(u+1)(\mathrm{d}u)$
	Attempt new limits for u at any stage (expect 0,1)	M1		or re-substitute & use ($\frac{5}{2}$,3)
	$\frac{17}{72}$	A1	5	**AG** WWW
	S.R. If M1 A0 A0 M1 A0, award S.R. B1 for answer $\frac{68}{72}, \frac{34}{36}$ or $\frac{17}{18}$			ISW
5	(i) Show clear knowledge of binomial expansion	M1		$-3x$ should appear but brackets can be missing; $-\frac{1}{3}.-\frac{4}{3}$ should appear, not $-\frac{1}{3}.\frac{2}{3}$
	$= 1+x$	B1		Correct first 2 terms; not dep on M1
	$+2x^2$	A1		
	$+\frac{14}{3}x^3$	A1	4	
	(ii) Attempt to substitute $x+x^3$ for x in (i)	M1		Not just in the $\frac{14}{3}x^3$ term
	Clear indication that $(x+x^3)^2$ has no term in x^3	A1		
	$\frac{17}{3}$	√A1	3	f.t. $\mathrm{cf}(x)$ + $\mathrm{cf}(x^3)$ in part (i)
6	(i) $2x+1 = / \equiv A(x-3)+B$	M1		
	$A = 2$	A1		
	$B = 7$	A/B 1	3	Cover-up rule acceptable for B1
	(ii) $\int \frac{1}{x-3}(\mathrm{d}x) = \ln(x-3)$ or $\ln\lvert x-3\rvert$	B1		Accept A or $\frac{1}{A}$ as a multiplier
	$\int \frac{1}{(x-3)^2}(\mathrm{d}x) = -\frac{1}{x-3}$	B1		Accept B or $\frac{1}{B}$ as a multiplier
	6 + 2 ln 7 Follow-through $\dfrac{6}{7}B + A\ln 7$	√B2	4	

7	$\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$	B1		
	$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$	B1		
	$4x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$	B1		
	Put $\frac{dy}{dx} = 0$	*M1		
	Obtain $4x + y = 0$ AEF	A1		and no other (different) result
	Attempt to solve simultaneously with eqn of curve	dep*M1		
	Obtain $x^2 = 1$ or $y^2 = 16$ from $4x + y = 0$	A1		
	$(1,-4)$ and $(-1,4)$ and no other solutions	A1	8	Accept $(\pm 1, \mp 4)$ but not $(\pm 1, \pm 4)$
8	(i) Use $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ and $-\frac{1}{m}$ for grad of normal	M1		or change to cartesian.,diff & use $-\frac{1}{m}$
	$= -p$ **AG** WWW	A1	2	Not $-t$.
	(ii) Use correct formula to find gradient of line	M1		
	Obtain $\frac{2}{p+q}$ **AG** WWW	A1	2	Minimum of denom = $2(p-q)(p+q)$
	(iii) State $-p = \frac{2}{p+q}$	M1		Or find eqn normal at P & subst $(2q^2, 4q)$
	Simplify to $p^2 + pq + 2 = 0$ **AG** WWW	A1	2	With sufficient evidence
	(iv) (8,8) $\rightarrow t$ or p or $q = 2$ only	B1		No possibility of -2
	Subst $p = 2$ in eqn (iii) to find q_1	M1		Or eqn normal,solve simult with cartes/param
	Subst $p = q_1$ in eqn (iii) to find q_2	M1		Ditto
	$q_2 = \frac{11}{3} \rightarrow \left(\frac{242}{9}, \frac{44}{3}\right)$	A1	4	No follow-through; accept (26.9 , 14.7)
9	(i) Separate variables as $\int \sec^2 y\, dy = 2\int \cos^2 2x\, dx$	M1		seen or implied
	LHS = $\tan y$	A1		
	RHS; attempt to change to double angle	M1		
	Correctly shown as $1 + \cos 4x$	A1		
	$\int \cos 4x\, dx = \frac{1}{4}\sin 4x$	A1		
	Completely correct equation (other than +c)	A1		$\tan y = x + \frac{1}{4}\sin 4x$
	+c on either side	A1	7	<u>not</u> on both sides unless c_1 and c_2
	(ii) Use boundary condition	M1		provided a sensible outcome would ensue
	c (on RHS) = 1	A1		or $c_2 - c_1 = 1$; not fortuitously obtained
	Substitute $x = \frac{1}{6}\pi$ into their eqn, produce $y = 1.05$	A1	3	or 4.19 or 7.33 etc. Radians only
10	(i) For (either point) + t(diff between posn vectors)	M1		"**r** =" not necessary for the M mark
	r = (either point) + t(**i** -2**j** – 3**k** or –**i** + 2**j** + 3**k**)	A1	2	**…** but it is essential for the A mark
	(ii) **r** = s(**i** + 2**j** – **k**) or (**i** + 2**j** – **k**) + s(**i** + 2**j** – **k**)	B1		Accept any parameter, including t
	Eval scalar product of **i**+2**j**-**k** & their dir vect in (i)	M1		
	Show as (1x1 or 1)+(2x-2 or -4)+(-1x-3 or 3)	A1		This is just one example of numbers involved
	= 0 <u>and</u> state perpendicular **AG**	A1	4	
	(iii) For at least two equations with diff parameters	M1		e.g. $5 + t = s$, $2 - 2t = 2s$, $-9 - 3t = -s$
	Obtain $t = -2$ or $s = 3$ (possibly -3 or 2 or -2)	A1		Check if $t = 2, 1$ or -1
	Subst. into eqn AB or OT and produce 3**i** + 6**j** – 3**k**	A1	3	
	(iv) Indicate that $\lvert \overrightarrow{OC} \rvert$ is to be found	M1		where C is their point of intersection
	$\sqrt{54}$;f.t. $\sqrt{a^2 + b^2 + c^2}$ from a**i** + b**j** + c**k** in (iii)	√A1	2	

<u>In the above question, accept any vectorial notation</u>
t and s may be interchanged, and values stated above need to be treated with caution.
In (iii), if the point of intersection is correct, it is more than likely that the whole part is correct – but check.

Mark Scheme 4725
January 2007

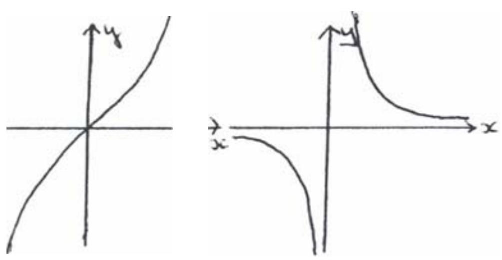
1.	(i) $a = -3$	B1	1	State correct value
	(ii) $2a - 3 = 7$ or $3a - 6 = 9$	M1		Sensible attempt at multiplication
	$a = 5$	A1	2	Obtain correct answer
			3	
2.		M1		Attempt to equate real and imaginary parts of $(x + iy)^2$ and $15 + 8i$
	$x^2 - y^2 = 15$ and $xy = 4$	A1 A1		Obtain each result
		M1		Eliminate to obtain a quadratic in x^2 or y^2
		DM1		Solve to obtain $x = (\pm)4$, or $y = (\pm)1$
	$\pm(4 + i)$	A1	6	Obtain only correct two answers as complex numbers
			6	
3.		M1		Expand to obtain $r^3 - r$
		M1		Consider difference of two standard results
	$\frac{1}{4}n^2(n+1)^2 - \frac{1}{2}n(n+1)$	A1		Obtain correct unfactorised answer
		M1		Attempt to factorise
		A1		Obtain factor of $\frac{1}{4}n(n+1)$
	$\frac{1}{4}n(n-1)(n+1)(n+2)$	A1	6	Obtain correct answer
			6	
4.	(i)	B1		Circle
		B1		Centre (1, -1)
		B1	3	Passing through (0, 0)
	(ii)	B1		Sketch a concentric circle
		B1		Inside (i) and touching axes
		B1	3	Shade between the circles
5.	(i)	B1	1	Show given answer correctly

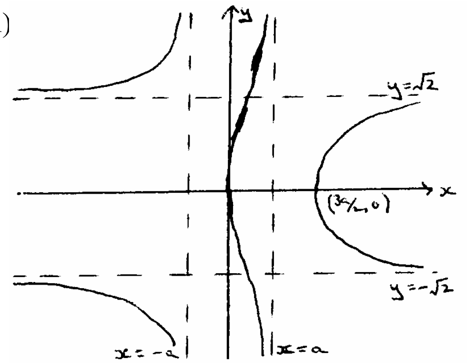
	(ii)	M1		Attempt to solve quadratic equation or substitute $x + iy$ and equate real and imaginary parts
	$-1 \pm i\sqrt{3}$	A1		Obtain answers as complex numbers
		A1	3	Obtain correct answers, simplified
	(iii)	B1		Correct root on x axis, co-ords. shown
		B1		Other roots in 2nd and 3rd quadrants
		B1	3	Correct lengths and angles or co-ordinates or complex numbers shown
			7	
6.	(i)	B1		Correct expression for u_{n+1}
		M1		Attempt to expand and simplify
	$u_{n+1} - u_n = 2n + 4$	A1	3	Obtain given answer correctly
	(ii)	B1		State $u_1 = 4$ (or $u_2 = 10$)and is divisible by 2
		M1		State induction hypothesis true for u_n
		M1		Attempt to use result in (ii)
		A1		Correct conclusion reached for u_{n+1}
		A1	5	Clear,explicit statement of induction conclusion
			8	
7.	(i) $\alpha + \beta = -5 \quad \alpha\beta = 10$	B1 B1	2	State correct values
	(ii) $\alpha^2 + \beta^2 = 5$	M1		Use $(\alpha + \beta)^2 - 2\alpha\beta$
		A1	2	Obtain given answer correctly, using value of -5
	(iii)	B1		Product of roots = 1
		M1		Attempt to find sum of roots
		A1		Obtain $\frac{5}{10}$ or equivalent
	$x^2 - \frac{1}{2}x + 1 = 0$	B1ft	4	Write down required quadratic equation, or any multiple.
			8	

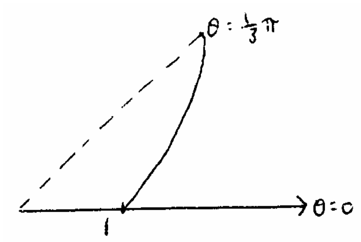
8.	(i)	M1		Factor of $r!$ or $(r + 1)!$ seen
		A1		Factor of $(r + 1)$ found
	$(r + 1)^2 r!$	A1	3	Obtain given answer correctly
	(ii)	M1		Express terms as differences using
		A1		(i)
		M1		At least 1st two and last term correct
	$(n + 2)! - 2!$	A1	4	Show that pairs of terms cancel
	(iii)	B1ft	1	Obtain correct answer in any form
			8	Convincing statement for non-converging, ft their (ii)
9.	(i) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$	M1		For at least two correct images
		A1	2	For correct diagram, co-ords.clearly written down
	(ii) 90^0 clockwise, centre origin	B1 B1		Or equivalent correct description
	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	B1	3	Correct matrix, not in trig form
	(iii) Stretch parallel to x-axis, s.f. 3	B1 B1		Or equivalent correct description, but must be a stretch for 2nd B1
	$\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$	B1 B1	4 **9**	Each correct column

10.	(i)	M1		Show correct expansion process for 3 x 3
		M1		
	Δ = det **D** = $3a - 6$	A1		Correct evaluation of any 2 x 2 det
		M1		Obtain correct answer
		A1		Show correct process for adjoint entries
		B1		
	$\mathbf{D}^{-1} = \frac{1}{\Delta}\begin{pmatrix} 3 & -2 & 4 \\ -3 & a & -2a \\ -3 & a & a-6 \end{pmatrix}$	A1	7	Obtain at least 4 correct entries in adjoint
				Divide by their determinant
				Obtain completely correct answer
	(ii) $\frac{1}{\Delta}\begin{pmatrix} 5 \\ 2a-9 \\ 5a-15 \end{pmatrix}$	M1		Attempt product of form $\mathbf{D}^{-1}\mathbf{C}$, or eliminate to get 2 equations and solve
		A1A1A1 ft all 3	4	Obtain correct answers, ft their inverse
			11	

Mark Scheme 4726
January 2007

1 (i) f(O) = In 3 f	Bl
f'(0) = $^1/_3$	Bl
f''(O) = - $^1/_9$ *A.G.*	B1 Clearly derived
(ii) Reasonable attempt at Maclaurin	Ml Form In3 + $ax + bx^2$, with a,b related to f " f'
$f(x) = \ln 3 + {}^1/_3x - {}^1/_{18}x^2$	*Al* √*J* On their values off' and f''
	SR Use *In(3+x)* = In3 + In(1 + $^1/_3$ *x)* Ml Use Formulae Book to get In3 + *Y3X - Y2(VJX)2* = In3 + *Y3X - 1/lgX2* Al
2 (i) f(0.8) = - 0.03, f(0.9) = +0.077 (accurately e.g. accept -0.02 t0 -0.04)	B1
Explain (change of sign, graph etc.)	B1
	SR Use x = √*J(tan^{-l}x)* and compare x to √J(tan^{-l} x) for $x=0.8$, 0.9 B 1
	Explain "change in sign" B 1
(ii) Differentiate two terms	B1 Get $2x$ - I *I(* 1 +x^2*)*
Use correct form of Newton-Ra ph son with 0.8, using their f *'(x)*	Ml 0.8 - *f(0.8)/f '(0.8)*
Use their N-R to give one more approximation to 3 d.p. minimum	Ml√
Get x = 0.835	Al 3d.p. - accept answer which rounds
3 (i) Show area of rect. = $^1/_4$ ($e^{1/16} + e^{1/4} + e^{9/16} + e^{1}$)	Ml Or numeric equivalent
Show area = 1.7054	Al At least 3 d.p. correct
Explain the < 1.71 in terms of areas	Bl AG. Inequality required
(ii) Identify areas for > sign	B1 Inequality or diagram required
Show area of rect. = $^1/_4$ ($e^o + e^{lll6} + e^{1/4} + e^{9/16}$)	Ml Or numeric evidence
Get A > 1.27	Al cao; or answer which rounds down
4 (i) 	BI Correct shape for sinh x
	B1 Correct shape for cosech x
	B1 Obvious point $(dy/dx \neq O)$/asymptotes clear
(ii) Correct definition of sinh x	B1 May be implied
Invert and mult. by e^X to AG.	B1 Must be clear; allow $2/(e^X - e^{-X})$ as mimimum simplification
Sub. $u = e^X$ and $du = e^X dx$	M1 Or equivalent, all x eliminated and not $dx = du$
Replace to $2/(u^2 - 1)\,du$	Al
Integrate to *aln((u - I)/(u* + 1)	A1√ Use formulae book, PT, or atanh^{-1}u
Replace u	Al No need for c

5 (i) Reasonable attempt at parts Get	M1 Involving second integral Al
xnsin x - ∫ sin x. nx^{n-1} dx	M1
Attempt parts again Accurately	Al
Clearly derive AG.	A1 Indicate $(^1/_2\pi)^n$ and 0 from limits
(ii) Get $I_4 = (^1/_2\pi)^4 - 12I_2$ or $I_2 = (^1/_2\pi)^2 - 2I_0$	B1
Show clearly $I_0 = 1$	B1 May use I_2
Replace their values in relation Get	M1
$I_4 = {}^1/_{16}\pi^4 - 3\pi^2 + 24$	A1 cao
6 (i) $x = \pm a$, $y = 2$	B1, B1, B1 Must be =; no working needed
(ii) 	B1 Two correct labelled asymptotes ∥ Ox and approaches B1 Two correct labelled asymptotes ∥ Oy and approaches B1 Crosses at $(^3/_2a,0)$ (and (0,0) - may be implied B1 90° where it crosses Ox; smoothly B1 Symmetry in Ox
7 (i) Write as $A/t + B/t^2 + (Ct + D)/(t^2 + 1)$	M1 Allow $(At+B)/t^2$; justify $B/t^2 + D/(l + t^2)$ if only used
Equate $At(t^2+1) + B(t^2+1) + (Ct+D)t^2$ to $1 - t^2$	M1√
Insert t values / equate coeff.	M1 Lead to at least two constant values
Get A = C = 0, B = L D =-2	Al SR Other methods leading to correct PF can earn 4 marks; 2 M marks for reasonable method going wrong
(ii) Derive or quote cos x in terms of t	Bl
Derive or quote dx = 2 $dt/(1 + t^2)$	B1
Sub. in to correct P.F.	M1 Allow $k\ (l-t^2)/((t^2(l+t^2))$ or equivalent
Integrate to $-1/t$ -2 $\tan^{-1}t$	Al√ From their k
Use limits to clearly get AG.	Al
8 (i) Get $(e^y - e^{-y})/(e^y + e^{-y})$	B1 Allow $(e^{2Y}-1)/(e^{2y}+ 1)$ or if x used
(ii) Attempt quad. in e^Y	M1 Multiply by e^Y and tidy
Solve for e^Y	M1
Clearly get AG.	Al
(iii) Rewrite as tanh $x = k$	M1 SR Use hyp def^n to get quad. in e^X M I
Use (ii) for x = ½ 1n 7 or equivalent	Al Solve e^{2x} = 7 for x to $^1/_2$ 1n 7 Al
(iv) Use of log laws	Bl One used correctly
Correctly equate 1n A = 1n B to $A = B$	M1 Or 1n(^{A}I$_B$) = 0
Get $x = \pm\ ^3/_5$	Al

9 (i)		B1 Shape for correct θ; ignore other θ Used; start at $(r,0)$ B1 θ=0, r=1 and increasing r
(ii)	U se correct formula with correct r	B1
	$\int \sec^2 x \, dx = \tan x$ used	B1
	Quote $\int 2 \sec x \tan x \, dx = 2\sec x$	B1 Or sub. correctly
	Replace $\tan^2 x$ by $\sec^2 x - 1$ to integrate	M1
	Reasonable attempt to integrate 3 terms And to use limits correctly	M1
	Get $\sqrt{3} + 1 - \frac{1}{6}\pi$	Al Exact only
(iii)	Use $x = r\cos\theta$, $y = r\sin\theta$, $r = (x^2+y^2)^{1/2}$	M1
	Reasonable attempt to eliminate r,θ	M1
	Get $y = (x-1)\sqrt{(x^2+y^2)}$	A1 Or equivalent

Mark Scheme 4727
January 2007

1 **(i)** Attempt to show no closure $3\times3=1,\ 5\times5=1$ *OR* $7\times7=1$	M1 A1	For showing operation table or otherwise For a convincing reason
OR Attempt to show no identity Show $a\times e=a$ has no solution	M1 A1 **2**	For attempt to find identity *OR* for showing operation table For showing identity is not 3, not 5, and not 7 by reference to operation table or otherwise
(ii) $(a=)\ 1$	B1 **1**	For value of a stated
(iii) *EITHER*: $\{e, r, r^2, r^3\}$ is cyclic, **(ii)** group is not cyclic	B1*	For a pair of correct statements
OR: $\{e, r, r^2, r^3\}$ has 2 self-inverse elements, **(ii)** group has 4 self-inverse elements	B1*	For a pair of correct statements
OR: $\{e, r, r^2, r^3\}$ has 1 element of order 2 **(ii)** group has 3 elements of order 2	B1*	For a pair of correct statements
OR: $\{e, r, r^2, r^3\}$ has element(s) of order 4 **(ii)** group has no element of order 4	B1*	For a pair of correct statements
Not isomorphic	B1 (dep*) **2** $\boxed{5}$	For correct conclusion
2 *EITHER*: $[3, 1, -2]\times[1, 5, 4]$ $\Rightarrow \mathbf{b}=k[1, -1, 1]$ e.g. put x *OR* y *OR* $z=0$ and solve 2 equations in 2 unknowns Obtain $[0, 2, -1]$ *OR* $[2, 0, 1]$ *OR* $[1, 1, 0]$	M1 A1 M1 M1 A1	For attempt to find vector product of both normals For correct vector identified with **b** For giving a value to one variable For solving the equations in the other variables For a correct vector identified with **a**
OR: Solve $3x+y-2z=4,\ x+5y+4z=6$ e.g. $y+z=1$ *OR* $x-z=1$ *OR* $x+y=2$ Put x *OR* y *OR* $z=t$ $[x, y, z]=[t, 2-t, -1+t]$ *OR* $[2-t, t, 1-t]$ *OR* $[1+t, 1-t, t]$ Obtain $[0, 2, -1]$ *OR* $[2, 0, 1]$ *OR* $[1, 1, 0]$ Obtain $k[1, -1, 1]$	 M1 M1 M1 A1 A1 **5** $\boxed{5}$	 For eliminating one variable between 2 equations For solving in terms of a parameter For obtaining a parametric solution for x, y, z For a correct vector identified with **a** For correct vector identified with **b**
3 **(i)** $z=\dfrac{6\pm\sqrt{36-144}}{2}$ $z=3\pm3\sqrt{3}\,\mathrm{i}$ Obtain $(r=)\ 6$ Obtain $(\theta=)\ \frac{1}{3}\pi$	M1 A1 A1 A1 **4**	For using quadratic equation formula or completing the square For obtaining cartesian values **AEF** For correct modulus For correct argument
(ii) *EITHER*: 6^{-3} *OR* $\frac{1}{216}$ seen $z^{-3}=6^{-3}(\cos(-\pi)\pm\mathrm{i}\sin(-\pi))$ Obtain $-\frac{1}{216}$	B1√ M1 A1	f.t. from their r^{-3} For using de Moivre with $n=\pm3$ For correct value
OR: $z^3=6z^2-36z=6(6z-36)-36z$ 216 seen Obtain $-\frac{1}{216}$	M1 B1 A1 **3** $\boxed{7}$	For using equation to find z^3 Ignore any remaining z terms For correct value

4 (i) $(y = xz \Rightarrow)\ \dfrac{dy}{dx} = x\dfrac{dz}{dx} + z$	B1	For a correct statement
$x\dfrac{dz}{dx} + z = \dfrac{x^2(1-z^2)}{x^2 z} = \dfrac{1}{z} - z$	M1	For substituting into differential equation and attempting to simplify to a variables separable form
$x\dfrac{dz}{dx} = \dfrac{1}{z} - 2z = \dfrac{1-2z^2}{z}$	A1 **3**	For correct equation **AG**
(ii) $\displaystyle\int \frac{z}{1-2z^2}\,dz = \int \frac{1}{x}\,dx \Rightarrow -\tfrac{1}{4}\ln(1-2z^2) = \ln cx$	M1 M1* A1	For separating variables and writing integrals For integrating both sides to ln forms For correct result (*c* not required here)
$1-2z^2 = (cx)^{-4}$	A1√	For exponentiating their ln equation including a constant (this may follow the next M1)
$\dfrac{x^2-2y^2}{x^2} = \dfrac{c^{-4}}{x^4}$	M1 (dep*)	For substituting $z = \dfrac{y}{x}$
$x^2(x^2-2y^2) = k$	A1 **6** $\boxed{9}$	For correct solution properly obtained, including dealing with any necessary change of constant to *k* as given **AG**
5 (i) (a) e, p, p^2	B1	For correct elements
(b) e, q, q^2	B1 **2**	For correct elements **SR** If the answers to parts **(i)** and **(iv)** are reversed, full credit may be earned for both parts
(ii) $p^3 = q^3 = e \Rightarrow (pq)^3 = p^3q^3 = e$ $\Rightarrow$ order 3 $(pq^2)^3 = p^3q^6 = p^3(q^3)^2 = e \Rightarrow$ order 3	M1 A1 A1 **3**	For finding $(pq)^3$ or $(pq^2)^3$ For correct order For correct order **SR** For answer(s) only allow B1 for either or both
(iii) 3	B1 **1**	For correct order and no others
(iv) e, pq, p^2q^2 *OR* $e, pq, (pq)^2$	B1 B1	For stating *e* and either *pq* or p^2q^2 For all 3 elements and no more
e, pq^2, p^2q *OR* $e, pq^2, (pq^2)^2$ *OR* $e, p^2q, (p^2q)^2$	B1 B1 **4** $\boxed{10}$	For stating *e* and either pq^2 or p^2q For all 3 elements and no more

6 **(i)** (CF $m=-3\Rightarrow$) Ae^{-3x}	B1 **1**	For correct CF
(ii) $(y=)\ px+q$	B1	For stating linear form for PI (may be implied)
$\Rightarrow p+3(px+q)=2x+1$	M1	For substituting PI into DE (needs y and $\frac{dy}{dx}$)
$\Rightarrow p=\frac{2}{3},\quad q=\frac{1}{9}$	A1 A1	For correct values
$\Rightarrow$ GS $y=Ae^{-3x}+\frac{2}{3}x+\frac{1}{9}$	A1√	For correct GS. f.t. from their CF + PI
		SR Integrating factor method may be used, but CF must be stated somewhere to earn the mark in **(i)**
I.F. $e^{3x}\quad\Rightarrow \frac{d}{dx}\left(ye^{3x}\right)=(2x+1)e^{3x}$	B1	For stating integrating factor
$\Rightarrow ye^{3x}=\frac{1}{3}e^{3x}(2x+1)-\int\frac{2}{3}e^{3x}dx$	M1	For attempt at integrating by parts the right way round
$\Rightarrow ye^{3x}=\frac{2}{3}xe^{3x}+\frac{1}{3}e^{3x}-\frac{2}{9}e^{3x}+A$	A2 *	For correct integration, including constant Award A1 for any 2 algebraic terms correct
$\Rightarrow$ GS $y=Ae^{-3x}+\frac{2}{3}x+\frac{1}{9}$	A1√ **5**	For correct GS. f.t. from their * with constant
(iii) *EITHER* $\frac{dy}{dx}=-3Ae^{-3x}+\frac{2}{3}$	M1	For differentiating their GS
$\Rightarrow -3A+\frac{2}{3}=0$	M1	For putting $\frac{dy}{dx}=0$ when $x=0$
$y=\frac{2}{9}e^{-3x}+\frac{2}{3}x+\frac{1}{9}$	A1	For correct solution
OR $\frac{dy}{dx}=0,\ x=0\ \Rightarrow 3y=1$	M1	For using original DE with $\frac{dy}{dx}=0$ and $x=0$ to find y
$\Rightarrow \frac{1}{3}=A+\frac{1}{9}$	M1	For using their GS with y and $x=0$ to find A
$y=\frac{2}{9}e^{-3x}+\frac{2}{3}x+\frac{1}{9}$	A1 **3**	For correct solution
(iv) $y=\frac{2}{3}x+\frac{1}{9}$	B1√ **1**	For correct function. f.t. from linear part of **(iii)**
	$\boxed{10}$	

7 **(i)** *EITHER*: (**AG** is **r** =) $[6, 4, 8] + tk[1, 0, 1]$ *or* $[3, 4, 5] + tk[1, 0, 1]$	B1	For a correct equation
Normal to *BCD* is	M1	For finding vector product of any two of $\pm[1, -4, -1], \pm[2, 1, 1], \pm[1, 5, 2]$
$\mathbf{n} = k[1, 1, -3]$	A1	For correct **n**
Equation of *BCD* is $\mathbf{r}.[1, 1, -3] = -6$	A1	For correct equation (or in cartesian form)
Intersect at $(6+t) + 4 + (-3)(8+t) = -6$	M1	For substituting point on *AG* into plane
$t = -4$ ($t = -1$ using $[3, 4, 5]$) $\Rightarrow$ $\mathbf{OM} = [2, 4, 4]$	A1	For correct position vector of *M* **AG**
OR: (**AG** is **r** =) $[6, 4, 8] + tk[1, 0, 1]$ *or* $[3, 4, 5] + tk[1, 0, 1]$	B1	For a correct equation
$\mathbf{r} = \mathbf{u} + \lambda\mathbf{v} + \mu\mathbf{w}$, where $\mathbf{u} = [2, 1, 3]$ *or* $[1, 5, 4]$ *or* $[3, 6, 5]$; $\mathbf{v}, \mathbf{w} =$ two of $[1, -4, -1], [1, 5, 2], [2, 1, 1]$	M1 A1	For a correct parametric equation of *BCD*
e.g. $(x =)\ 6+t = 2 + \lambda + \mu$; $(y =)\ 4 = 1 - 4\lambda + 5\mu$; $(z =)\ 8+t = 3 - \lambda + 2\mu$	M1	For forming 3 equations in t, λ, μ from line and plane, and attempting to solve them
$t = -4$ *or* $\lambda = -\frac{1}{3}, \mu = \frac{1}{3}$	A1	For correct value of t *or* λ, μ
$\Rightarrow$ $\mathbf{OM} = [2, 4, 4]$	A1 **6**	For correct position vector of *M* **AG**
(ii) A, G, M have $t = 0, -3, -4$ *OR* $AG = 3\sqrt{2}$, $AM = 4\sqrt{2}$ *OR* $\mathbf{AG} = [-3, 0, -3]$, $\mathbf{AM} = [-4, 0, -4]$ $\Rightarrow AG : AM = 3 : 4$	B1 **1**	For correct ratio **AEF**
(iii) $\mathbf{OP} = \mathbf{OC} + \frac{4}{3}\mathbf{CG}$	M1	For using given ratio to find position vector of *P*
$= \left[\frac{11}{3}, \frac{11}{3}, \frac{16}{3}\right]$	A1 **2**	For correct vector
(iv) *EITHER*: Normal to *ABD* is	M1	For finding vector product of any two of $\pm[4, 3, 5], \pm[1, 5, 2], \pm[3, -2, 3]$
$\mathbf{n} = k[19, 3, -17]$	A1	For correct **n**
Equation of *ABD* is $\mathbf{r}.[19, 3, -17] = -10$	M1	For finding equation (or in cartesian form)
$19.\frac{11}{3} + 3.\frac{11}{3} - 17.\frac{16}{3} = -10$	A1	For verifying that *P* satisfies equation
OR: Equation of *ABD* is $\mathbf{r} = [6, 4, 8] + \lambda[4, 3, 5] + \mu[1, 5, 2]$ (etc.)	M1	For finding equation in parametric form
$\left[\frac{11}{3}, \frac{11}{3}, \frac{16}{3}\right] = [6, 4, 8] + \lambda[4, 3, 5] + \mu[1, 5, 2]$	M1	For substituting *P* and solving 2 equations for λ, μ
$\lambda = -\frac{2}{3}$, $\mu = \frac{1}{3}$	A1	For correct λ, μ
	A1	For verifying 3rd equation is satisfied
OR: $\mathbf{AP} = \left[-\frac{7}{3}, -\frac{1}{3}, -\frac{8}{3}\right]$	M1 A1	For finding 3 relevant vectors in plane *ABDP*; For correct **AP** *or* **BP** *or* **DP**
$\mathbf{AB} = [-4, -3, -5]$, $\mathbf{AD} = [-3, 2, -3]$	M1	For finding **AB, AD** *or* **BA, BD** *or* **DB, DA**
$\Rightarrow \mathbf{AB} + \mathbf{AD} = [-7, -1, -8]$		
$\Rightarrow \mathbf{AP} = \frac{1}{3}(\mathbf{AB} + \mathbf{AD})$	A1 **4**	For verifying linear relationship
	$\boxed{13}$	

8 (i) $\cos 4\theta + \mathrm{i}\sin 4\theta =$ $c^4 + 4\mathrm{i}c^3s - 6c^2s^2 - 4\mathrm{i}cs^3 + s^4$	M1	For using de Moivre with $n = 4$
$\Rightarrow \sin 4\theta = 4c^3s - 4cs^3$ and $\cos 4\theta = c^4 - 6c^2s^2 + s^4$	A1	For both expressions
$\Rightarrow \tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$	M1	For expressing $\dfrac{\sin 4\theta}{\cos 4\theta}$ in terms of c and s
	A1 **4**	For simplifying to correct expression
(ii) $\cot 4\theta = \dfrac{\cot^4\theta - 6\cot^2\theta + 1}{4\cot^3\theta - 4\cot\theta}$	B1 **1**	For inverting **(i)** and using $\cot\theta = \dfrac{1}{\tan\theta}$ or $\tan\theta = \dfrac{1}{\cot\theta}$. **AG**
(iii) $\cot 4\theta = 0$	B1	For putting $\cot 4\theta = 0$ (can be awarded in **(iv)** if not earned here)
Put $x = \cot^2\theta$	B1	For putting $x = \cot^2\theta$ in the numerator of **(ii)**
$\theta = \frac{1}{8}\pi \Rightarrow x^2 - 6x + 1 = 0$ *OR* $x^2 - 6x + 1 = 0 \Rightarrow \theta = \frac{1}{8}\pi$	B1 **3**	For deducing quadratic from **(ii)** and $\theta = \frac{1}{8}\pi$ *OR* For deducing $\theta = \frac{1}{8}\pi$ from **(ii)** and quadratic
(iv) $4\theta = \frac{3}{2}\pi$ *OR* $\frac{1}{2}(2n+1)\pi$	M1	For attempting to find another value of θ
2nd root is $x = \cot^2\left(\frac{3}{8}\pi\right)$	A1	For the other root of the quadratic
$\Rightarrow \cot^2\left(\frac{1}{8}\pi\right) + \cot^2\left(\frac{3}{8}\pi\right) = 6$	M1	For using sum of roots of quadratic
$\Rightarrow \operatorname{cosec}^2\left(\frac{1}{8}\pi\right) + \operatorname{cosec}^2\left(\frac{3}{8}\pi\right) = 8$	M1 A1 **5** $\boxed{13}$	For using $\cot^2\theta + 1 = \operatorname{cosec}^2\theta$ For correct value

Mark Scheme 4728
January 2007

1	**(i)**	Net force on trailer is +/-(700 - R_T)	B1		
			M1		For applying Newton's second law to the trailer with 2 terms on LHS (no vertical forces)
		700 - R_T = 600 x 0.8	A1ft		ft cv (+/-(700 - R_T))
		Resistance is 220N	A1	4	
	(ii)		M1		For applying Newton's second law to the car or to the whole, with a =+/- 0.8 (no vertical forces)
		2100 – 700 – R_C = 1100 x 0.8	A1ft		
		or 2100 – (R_C + 220) = (1100 + 600)x 0.8			ft cv(220)
		Resistance is 520N	A1	3	

2	**(i)**		M1		For resolving forces vertically
		15 x 0.28 and 11x 0.8	A1		Allow use of □ = 16.3 and □ =53.1
		Y= 15x0.28 + 11x0.8 - 13	A1ft		Ft cv(15 x 0.28 and 11x 0.8)
		Component is zero AG	A1	4	**SR** 15sin□ + 11sin□ -13 = 0 gets M1A0A1ftA0
	(ii)		M1		For resolving forces horizontally
		X = 15 x 0.96 – 11 x 0.6	A1		Allow use of □ = 16.3 and □ =53.1
		Magnitude is 7.8N	A1	3	Accept 7.79, -7.8
	(iii)	Direction is that of the (+ve) x -axis	B1	1	Do not allow horizontal, 90^o from vertical. Do not award if □ = 16.3 and □ =53.1 have been used.

3	**(i)**	T = 0.3g	B1		At particle (or 0.3g -T= 0.3a)
		F = T	B1		Or F = cv(T at particle) (or T - F = 0.4a)
		R = 0.4g	B1		
			M1		For using F = μR
		Coefficient is 0.75	A1	5	
	(ii)		M1		For resolving 3 relevant forces on B horizontally, a=0
		X = 0.3g + 0.3g	A1ft		Ft X = 0.3g + cv(μ) cv(R)
		X = 5.88N	A1	3	

4	**(i)**	Momentum before collision $= +/-(0.8 \times 4 - 0.6 \times 2)$	B1		Or momentum change L $0.8x4 +/- 0.8v_L$ Accept inclusion of g in both terms
		Momentum after collision $= +/-0.8v_L + 0.6 \times 2$	B1		Momentum change N $0.6x2 + 0.6x2$ Accept inclusion of g in both terms
			M1		For using the principle of conservation of momentum even if g is included throughout
		Speed is 1 ms^{-1}	A1	4	Accept -1 from correct work (g not used).
	(ii)(a)	$0.6x2 - 0.7x0.5$	M1		Must be a difference. **SR** $0.6x1 - 0.7x0.5$ M1
		Total is $0.85kgms^{-1}$	A1		Must be positive
		Total momentum +ve after the collision. If N continues in its original direction, both particles have a negative momentum.	DM 1		Or $0.6v + 0.7w$ is positive, confirming that the momentum is shared between two particles. No reference need be made to the physically impossible scenario where M and N both might continue in their original directions.
		N must reverse its direction.	A1	4	
	(ii)(b)	$0.6x2 - 0.7x0.5$ (= 0.85) = 0.7v	A1ft		ft cv (0.85). Award M1 if not given in ii(a).
		Speed is $1.21ms^{-1}$	A1	2	Positive. Accept (a.r.t) 1.2 from correct work

5	**(i)**	$1.8t^2/2$ (+C)	M*1		For using $v = \int a dt$
		(t = 0,v = 0) C = 0	B1		May be awarded in (ii). Accept c written and deleted.
		Expression is $1.8t^2/2$	A1	3	also for $1.8t^2$ +c
	(ii)		M1		For using $s = \int v dt$
		$0.9t^3/3$ (+K)	A1		**SR** Award B1 for (s = 0, t = 0) K = 0 if not already given in (i), or +K included and limits used.
		0.3 x 64	M1		For using limits 0 to 4 (or equivalent)
		19.2m AG	A1	4	
	(iii)	$u = 0.9 \times 4^2$	D* M1		For using 'u' = v(4)
			M1		For using $s = ut + ½ x7.2t^2$ with non-zero u
		$s = 14.4 \times 3 + ½\ 7.2 \times 3^2$	A1		(s = 75.6)
		19.2 + 75.6	M1		For adding distances for the two distinct stages
		Displacement is 94.8m	A1	5	
		OR			
		$v = \int 7.2 dt$	D* M1		For finding v(4) Integration and finding non-zero integration constant
		t = 0, v = 14.4, c = 14.4			Nb Using t=4, v=14.4 gives c = -14.4
		$s = \int 7.2t + 14.4 dt$			$s = \int 7.2t - 14.4 dt$
		t = 0, s = 0, k = 0			Integration and finding integration constant. Nb t=4 with s=19.2 and v=7.2t-14.4 gives k=19.2
		$s=3.6x3^2+14.4x3$	M1 A1		Substituting t = 3 (OR 7 into $s = 3.6t^2 - 14.4t + 19.2$) (s=75.6) (OR $s = 3.6 x7^2 - 14.4x7 + 19.2$)
		19.2 + 75.6 = 94.8	M1		Adding two distinct stages OR
		Displacement is 94.8m	A1		$s = 3.6 x7^2 - 14.4x7+19.2 =94.8$ final M1A1

6	**(i)**	$½\ 25v_m = 8$ or $½Tv_m+½(25 – T)v_m =$	B*1		Do not accept solution based on isosceles or right angled triangle

	8			
	Greatest speed is 0.64 ms^{-1}	D*B 1	2	
(ii)		M1		For using v = u + at or the idea that gradient represents acceleration
	V = 0.02 x 40	A1		
	V = 0.8	A1	3	
(iii)		M1		For using the idea that the area represents displacement. nb trapezium area is 16+8+8
		M1		For A = ½ (L_1 + L_2)h or other appropriate breakdown
	½ (70 + T) x 0.8 = 40 - 8	A1ft		½ (30 + T) x 0.8 = 40 - 8- ½ x 40 x 0.8 ft cv(0.8)
	Duration is 10s	A1	4	
(iv)		M1		For using v = u + at or the idea that gradient represents acceleration
	0=0.8+a(30–10)	A1ft		ft cv(10) and cv(0.8)
	Deceleration is 0.04ms^{-2}	A1	3	Accept -0.04 from correct work
	Or	M1		Using the idea that the area represents displacement.
	40-8-½ x 40 x 0.8-10x0.8 =0.8(30-10)-a(30–10)^2/2	A1ft		Ft cv(0.8 and 10)
	Deceleration is 0.04ms^{-2}	A1		Accept -0.04 from correct work. d=-0.04 A0

7	**(i)**	$R = 0.5g\cos40^o$	B1		$R = 3.7536$
		$F = 0.6 \times 0.5g\cos40^o$	M1		For using $F = \mu R$
		Magnitude is 2.25N AG	A1	3	
	(ii)		M1		For applying Newton's second law (either case) //slope, two forces
		$-/+0.5g\sin40^o - F = 0.5a$	A1		Either case
		(a) Acceleration is -10.8ms^{-2}	A1		Accept 10.8 from correct working (both forces have the same sign)
		(b) Acceleration is 1.79ms^{-2}	A1	4	Accept -1.79 from correct working (the forces have opposite sign) Accept ! 1.8(0)
	(iii)a)	$0 = 4 + (-10.8)T_1$	M1		Requires appropriate sign
		$T_1 = 0.370(3)$	A1		Accept 0.37
	b)		M1		For complete method of finding distance from A to highest point using a(up) with appropriate sign
		$0 = 4^2 + 2(-10.8)s$ or $s = (0 + 4) \times 0.37/2$or $s = 4(0.370) + \frac{1}{2}(-10.8)(0.370)^2$	A1 ft		ft a(up) and/or T_1 ($s = 0.7405$)
			M1		For method of finding time taken from highest point to A and not using a(up)
		$0.7405 = \frac{1}{2}(1.79)T_2^2$	A1ft		ft a(down) and cv(0.7405) ($T_2 = 0.908$ approx)
		$0.370 + 0.908$	M1		Using $T = T_1 + T_2$ with different values for T_1 ,T_2
		= 1.28s	A1	8	3 significant figures cao

Mark Scheme 4729
January 2007

1		com directly above lowest point	B1			
		tan α = 6/10	M1			
		α = 31.0	A1	3	or 0.540 rads	**3**

2		e = 1 = $(y–x)/4$	B1		or ½x0.2x^2 + ½x0.1y^2 =	
		0.8 = 0.2x + 0.1y	B1		½x0.2x4^2(B1/B1 for any 2)	
		solving sim. equ.	M1		not if poor quad. soln.	
		x = 4/3 only	A1	4		**4**

3	(i)	$x^2 = 21^2$ + 2x40x9.8	M1			
		x = 35	A1			
		0 = y^2 – 2x40x9.8	M1			
		y = 28	A1		may be implied	
		e = 28/35	M1			
		e = 0.8	A1	6	aef	
	(ii)	0.2x28 – – 0.2x35	M1		must be double negative	
		I = 12.6	A1	2		**8**

4	(i)	½x80x5^2 or ½x80x2^2 either KE	B1		1000/160	
		70 x 25	B1		1750	
		80x9.8x25sin20°	B1		6703.6	
		WD=½x80x5^2-½x80x2^2+70x25+80x9.8x25sin20°	M1		4 parts	
		9290	A1	5		
	(ii)	Pcos30°x25	B1		or a=0.42	
		Pcos30°.25=9290 / Pcos30°-70-80x9.8sin20°=80a	M1			
		P = 429 /if P found 1st then Pcos30°x25 =9290 ok	A1	3		**8**

5	(i)	D = 3000/5^2 = 120	M1			
			A1	2	**AG**	
	(ii)	120 – 75 = 100a	M1			
		a = 0.45 ms^{-2}	A1	2		
	(iii)	100x9.8x1/98	B1		weight component	
		3000/v^2=3v^2+100x9.8x1/98	M1			
		3000 = 3v^4 + 10v^2	A1		aef	
		solving quad in v^2	M1		(v^2 = 30)	
		v = 5.48 ms^{-1}	A1	5	accept √30	**9**

6	(i)	com of Δ 4 cm right of C	B1			
		1.5 x 10 + 7 x 20 = $\bar{x}$ x 30	M1			
			A1			
		$\bar{x}$ = 5.17	A1		5 1/6 31/6	
		com of Δ 6 cm above E	B1		or 3 cm below C	
		4.5 x 10 + 6 x 20 = $\bar{y}$ x 30	M1			
			A1			
		$\bar{y}$ = 5.5	A1	8		
	(ii)	tanθ = 5.17/3.5	M1		right way up and (9 – $\bar{y}$)	
		55.9° or 124°	A1√	2	√ their $\bar{x}$ /(9 – $\bar{y}$)	
	(iii)	d = 15sin45° (10.61)	B1		dist to line of action of T	
		Td = 30 x 5.17	M1		allow Tx15 i.e. T vertical	
		T = 14.6	A1	3		**13**

7	(i)	Tsin30°	B1			
		Tsin30° = 0.3x0.4x2^2	M1		resolving horizontally	
			A1			
		T = 0.96	A1	4		
	(ii)	R + Tcos30° = 0.3x9.8	M1		resolving vertically	
			A1			
		R = 2.11	A1√	3	√ their T (2.94─Tcos30°)	
	(iii)	T_1sin30° = 0.3 x v^2/0.4	M1		or 0.3x0.4xω^2	
			A1		(T_1 = 1.5v^2)	
		T_1cos30° = 0.3 x 9.8	B1		(T_1 = 1.96√3 = 3.3948)	
		R = 0	B1		may be implied or stated	
		tan30°=v^2 /(0.4 x 9.8)for elim of T_1	M1		and v=0.4ω (ω = 3.76)	
		v = 1.50	A1	6		**13**

8	(i)	v_v = 42sin30° (=21)	B1			
		0 = 21^2 – 2x9.8xh	M1			
		h = 22.5	A1	3		
	(ii)	v_h = 42cos30° (=36.4)	B1			
		v_v = ±v_h x tan10°	M1			
		v_v = ±6.41 or 21√3 tan10°	A1		or 42cos30°.tan10°	
		–6.41 = 42sin30° – 9.8t	M1	**	must be –6.41(also see “or” x 2)	
		t = 2.80	A1	**		
		y=42sin30°x2.8 – 4.9x2.8^2	M1	**		
		y = 20.4	A1√	**	√ their t	
		x = 42cos30° x 2.80	M1			
		x = 102	A1√		√ their t	
		√(x^2 + y^2)	M1			
		d = 104	A1	11		
	or	6.41^2 = 21^2 + 2 x -9.8s	M1	**	vert dist first then time	
		s = 20.4	A1	**		
		20.4 = 21t + ½. -9.8t^2	M1	**		
		t = 2.80	A1	**		
	or	22.5 ─ s and 6.41^2=2x9.8s	M1	**	dist from top (s = 2.096)	
		y = 20.4	A1	**		
		22.5 & 2.1 = ½.9.8t^2	M1	**	2 separate times (2.143, 0.654)	
		t = 2.80	A1	**	2.143 + 0.654	**14**
		alternatively				
	(ii)	*y* = *x*/√3 – x^2/270 aef	B1		y=xtan30°– 9.8x^2/2.42^2.$\cos^2$30°	
		d*y*/d*x* = 1/√3 – *x*/135	M1		for differentiating	
			A1		aef	
		d*y*/d*x* = – tan10°	M1		must be –tan10°	
		1/√3 – *x*/135 = – tan10°	A1			
		solve for *x*	M1			
		x = 102	A1√		√ on their d*y*/d*x*	
		y = *x*/√3 – x^2/270	M1			
		y = 20.4	A1√		√ their *x*	
		√(x^2 + y^2)	M1			
		d = 104	A1	(11)		

Mark Scheme 4730
January 2007

1		M1		For using the principle of conservation of energy
	½ 0.6x5^2 – ½ 0.6v^2 = 0.6g(2x0.4) [v^2 = 9.32]	A1		
	[T + 0.6g = 0.6a]	M1		For using Newton's second law
	[a = 9.32/0.4]	M1		For using a = v^2/r
	T + 0.6g = 0.6x9.32/0.4	A1ft		ft incorrect energy equation
	Tension is 8.1N	A1	6	

2	28cos30^o – 10cos30^o [= |Δ v$_H$| = (I/m)cosθ]	B1		
	10sin30^o + 28sin30^o [= |Δ v$_V$| = (I/m)sinθ]	B1		
	[X = - Icosθ = -0.8885, Y = Isinθ = 1.083]	M1		For using mv change for component or resultant
		M1		For using I^2 = X^2 + Y^2
	I = 1.40	A1		
	[tanθ = 1.083/0.8885 or 19/15.588..]	M1		For using θ=tan^{-1}(Y/-X) or tan^{-1}(|Δ v$_V$| / |Δ v$_H$|)
	θ = 50.6	A1	7	

	ALTERNATIVELY			
2		M1		For using cosine rule in correct triangle
	(I/m)^2 = 28^2 + 10^2 –2x28x10cos60^o [=604]	A1		
	[I = 0.057$\sqrt{604}$]	M1		For using I = mv change
	I = 1.40	A1		
		M1		For using sine rule in correct triangle
	(I/m)/sin60^o = 10/sin(θ-30^o) or 28/sin(150^o-θ)	A1		
	θ = 50.6	A1	7	

3	(i) $160a = 2aY$	M1		For taking moments for AB about B
	Component at B is 80N	A1		
	Component at C is 240N	B1ft	3	ft 160 + Y
	(ii)	M1		For taking moments for BC about B or C (and using X = F) or for whole about A
	$160a \cos60^o + 2aF\sin60^o = 240x2a \cos 60^o$ or $80x2a \cos60^o + 160a \cos60^o = 2aX\sin60^o$ or $240(2 + 2\cos60^o)a = 160a + 160(2 + \cos60^o)a + 2aF\sin60^o$	A1ft		
	Frictional force is 92.4N	A1		
	Direction is to the left	B1	4	
	(iii) [92.4/240]	M1		For using $F = \mu R$
	Coefficient is 0.385	A1ft	2	

4	(i)	M1		For using T = mg and $T = \lambda e/L$
	$3.5e/0/7 = 0.2g$ [e = 0.392]	A1		
	Position is 1.092m below O.	A1	3	AG
	(ii)	M1		For using Newton's second law
	$0.2g – 3.5(0.392 + x)/0.7 = 0.2a$	A1ft		ft incorrect e
	$a = -25x$	A1ft		ft incorrect e
	[$25A^2 = 1.6^2$ or $½ (0.2)1.6^2 + 3.5x0.392^2/(2x0.7) + 0.2gA = 3.5x(0.392 + A)^2/(2x0.7)$	M1		For using $A^2n^2 = v_{max}^2$ or Energy at lowest point = energy at equilibrium point (4 terms needed including 2 EE terms)
	Amplitude is 0.32m	A1ft	5	
	(iii) [$x = 0.32\sin2^c$]	M1		For using x = Asin nt or Acos(π/2 - nt)
	x = 0.291	A1		
	[$v = 0.32x5\cos2^c$ or $v^2 = 25(0.32^2 – 0.291^2)$ or $0.256 + 0.38416 + 0.2g(0.291) = ½ 0.2v^2 + 2.5(0.683)^2$	M1		For using v = Ancos nt or $v^2 = n^2(A^2 – x^2)$ or Energy at equilibrium point = energy at x = 0.291
	$v^2 = 0.443$	A1		May be implied
	v = -0.666 (or 0.666 upwards)	A1	5	

5	(i) $[mg - mkv^2 = ma]$	M1		For using Newton's second law
	$(v\, dv/dx)/(g - kv^2) = 1$	A1	2	AG
	(ii) $[-\tfrac{1}{2}[\ln(g - kv^2)]/k = x \quad (+C)]$	M1		For separating variables and attempting to integrate
	$[-(\ln g)/2k = C]$	M1		For using v(0) = 0 to find C
	$x = [-\tfrac{1}{2}[\ln\{(g - kv^2)/g\}]/k$	A1		Any equivalent expression for x
	$[\ln\{(g - kv^2)/g\} = \ln(e^{-2kx})]$	M1		For expressing in the form $\ln f(v^2) = \ln g(x)$ or equivalent
	$v^2 = (1 - e^{-2kx})g/k$	A1		
		M1		For using $e^{-Ax} \rightarrow 0$ for +ve A
	Limiting value is $\sqrt{g/k}$	A1ft	7	AG
	(iii) $[1 - e^{-600k} = 0.81]$	M1		For using $v^2(300) = 0.9^2 g/k$
	$[-600k = \ln(0.19)]$	M1		For using logarithms to solve for k
	k = 0.00277	A1	3	

6	(i) $[u \sin 30^\circ = 3]$	M1		For momentum equation for B, normal to line of centres
	u = 6	A1	2	
	(ii) $[4\sin 88.1^\circ = v \sin 45^\circ]$	M1		For momentum equation for A, normal to line of centres
	v = 5.65	A1		
		M1		For momentum equation along line of centres
	$0.4(4\cos 88.1^\circ) - mu \cos 30^\circ = -0.4v \cos 45^\circ$	A1		
	m = 0.318	A1	5	
	(iii)	M1		For using NEL
	$0.75(4\cos\theta + u \cos 30^\circ) = v \cos 45^\circ$	A1		
	$4\sin\theta = v \sin 45^\circ$	B1		
	$[3\cos\theta + 4.5\cos 30^\circ = 4\sin\theta]$	M1		For eliminating v
	$8\sin\theta - 6\cos\theta = 9\cos 30^\circ$	A1	5	AG
7	(i)(a) Extension = $1.2\alpha - 0.6$	B1		
	$[T = mg\sin\alpha]$	M1		For resolving forces tangentially
	$0.5 \times 9.8\sin\alpha = 6.86(1.2\alpha - 0.6)/0./6$	A1ft		
	$\sin\alpha = 2.8\alpha - 1.4$	A1	4	AG
	(i)(b) [0.8, 0.756.., 0.745.., 0.742.., 0.741.., 0.741..,]	M1		For attempting to find α_2 and α_3
	$\alpha = 0.74$	A1	2	
	(ii) $\Delta h = 1.2(\cos 0.5 - \cos 0.8)$ [0.217…]	B1		
	$[0.5 \times 9.8 \times 0.217.. = 1.06355..]$	M1		For using $\Delta(PE) = mg\Delta h$
	$[6.86(1.2 \times 0.8 - 0.6)^2/(2 \times 0.6) = 0.74088]$	M1		For using EE = $\lambda x^2/2L$
		M1		For using the principle of conservation of energy
	$\tfrac{1}{2}\, 0.5v^2 = 1.06355.. - 0.74088$	A1		Any correct equation for v^2
	Speed is 1.14ms^{-1}	A1		
	Speed is decreasing	B1ft	7	

Mark Scheme 4732
January 2007

Note: "3 sfs" means an answer which is equal to, or rounds to, the given answer. If such an answer is seen and then later rounded, apply ISW. Penalize over-rounding only once in paper, except qu 8(ii).

1i	$1 - (^3/_{10} + ^1/_5 + ^2/_5)$ $^1/_{10}$	M1 A1 2	or $(^3/_{10} + ^1/_5 + ^2/_5) + p = 1$
ii	$^3/_{10} + 2 \times ^1/_5 + 3 \times ^2/_5$ $^{19}/_{10}$ oe	M1 A1 2	$\div$ 4or6 $\Rightarrow$ M0A0
Total		**4**	
2i	. □x =20;□y =11;□x^2 =96; □y^2 = 31; □xy =52) $S_{xx} = 16$ or 3.2 $S_{yy} = 6.8$ or 1.36 $S_{xy} = 8$ or 1.6 $r = \frac{8}{\sqrt{(16 \times 6.8)}}$ or $\frac{1.6}{\sqrt{(3.2 \times 1.36)}}$ = 0.767 (3 sfs)	B1 B1 B1 M1 A1 5	dep $-1 \le r \le 1$ ft their S's (S_{xx} & S_{yy} +ve) for M1 only
ii	Small sample oe	B1f 1	
Total		**6**	
3i	120	B1 1	not just 5!
iia	3 x 4! or 72 ($\div$ 5!) $^3/_5$ oe	M1 A1 2	oe, eg $^{72}/_{120}$
b	Starts 1 or 21 (both) $^1/_5 + ^1/_5 \times ^1/_4$ $= ^1/_4$ oe	M1 M1 A1 3	12,13,14,15, ($\ge$2 of these incl 21, or allow 1 extra) can be implied by wking or 5x 3! or 4! + 3! ($\div$5!) complement: full equiv steps for Ms
Total		**6**	
4ia	***W & Y oe***	B1 1	
b	***X oe***	B1 1	
ii	Geo probs always decrease or Geo has no upper limit to x or $x \neq 0$	B1 1	Geo not fixed no. of values diags have fixed no of trials not Geo has +ve skew
iii	**W** Bin probs cannot fall then rise or bimodal	B1 B1dep 2	indep allow Bin probs rise then fall
Total		**5**	
5i	$\dfrac{2685 - \frac{140 \times 106.8}{8}}{3500 - \frac{140^2}{8}}$ or $\dfrac{2685 - 8 \times 17.5 \times 13.35}{3500 - 8 \times 17.5^2}$ $= ^{136}/_{175}$ or 0.777 (3 sfs) $y - ^{106.8}/_8 = 0.777(x - ^{140}/_8)$ y=0.78x –0.25 or better or $y = ^{136}/_{175}x - ^1/_4$	M1 A1 M1 A1 4	Correct sub in any correct formula for b (incl. (x - $\bar{x}$) etc) or $a = ^{106.8}/_8 - 0.777 \times ^{140}/_8$ ft b for M1 $\ge$ 2 sfs sufficient for coeffs
ii	0.78 x 12 – 0.25 = 9.1 (2 sfs)	M1 A1f 2	M1: ft their equn A1: dep const term in equn
iiia b	Reliable Unreliable because extrapolating oe	B1 B1 2	Just "reliable" for both: B1
Total		**8**	

6i	Geo($^2/_3$) stated $(^1/_3)^3 \times ^2/_3$ $= ^2/_{81}$ or 0.0247 (3 sfs)	M1 M1 A1 3	or implied by $(^1/_3)^n \times ^2/_3$

ii	$(^1/_3)^3$ $1 - (^1/_3)^3$ $^{26}/_{27}$ or 0.963 (3 sfs)	M1 M1 A1 3	or $^2/_3+^1/_3\text{x}^2/_3+(^1/_3)^2\text{x}^2/_3$: M2 one term omitted or extra or wrong: M1 $1 - (^1/_3)^4$ or $1- (^2/_3+^1/_3\text{x}^2/_3+(^1/_3)^2\text{x}^2/_3)$:M1
iii	1 / 2/3 = 3/2 oe	M1 A1 2	
Total		**8**	
7i	$^2/_9$ or $^7/_9$ oe seen $^3/_9$ or $^6/_9$ oe seen $^1/_8$ or $^7/_8$ oe seen Correct structure All correct	B1 B1 B1 B1 B1 5	 ie 8 correct branches only, ignore probs & values including probs and values, but headings not req'd
ii	$^3/_{10}\text{ x }^7/_9 + {}^7/_{10}\text{ x }^3/_9 + {}^7/_{10}\text{ x }^6/_9$ $^{14}/_{15}$ or 0.933 oe	M2 A1 3	or $^3/_{10}\text{x}^7/_9 + {}^7/_{10}$ or $1 - {}^3/_{10}\text{ x }^2/_9$ M1: one correct prod or any prod + $^7/_{10}$ or $^3/_{10}\text{ x }^2/_9$
iii	$^3/_{10}\text{ x }^2/_9\text{ x }^7/_8 + {}^7/_{10}\text{ x }^6/_9$ $^{21}/_{40}$ or 0.525 oe	M2 A1 3	M1: one correct prod cao
	No ft from diag except: with replacement: (i) structure: B1 (ii) $^{91}/_{100}$: B2 (iii) 0.553: B2		
Total		**11**	
8i	Med = 2 LQ = 1 or UQ = 4 IQR = 3	B1 M1 A1 3	cao or if treat as cont data: read cf curve or interp at 25 & 75 cao
ii	Assume last value = 7 (or eg 7.5 or 8 or 8.5) Σxf attempted ≥ 5 terms 2.6 or 3 sf ans that rounds to 2.6 Σx^2f or $\Sigma(x-m)^2f$ ≥ 5 terms $\sqrt{(\Sigma x^2f/100 - m^2)}$ or $\sqrt{(\Sigma(x-m)^2f)/100}$ fully correct but ft m 1.6 or 1.7 or 3 sf ans that rounds to 1.6 or 1.7	B1 M1 A1 M1 M1 A1 6	stated, & not contradicted in wking eg 7-9 or 7,8, 9 Not just in wking allow "midpts" in Σxf or Σx^2f dep M3 penalize > 3 sfs only once
iii	Median less affected by extremes or outliers etc (NOT anomalies)	B1 1	or median is an integer or mean not int. or not affected by open-ended interval general comment acceptable
iv	Small change in var'n leads to lge change in IQR UQ for W only just 4, hence IQR exaggerated orig data shows variations are similar	 B1 1	 for Old Moat LQ only just 1 & UQ only just 3 oe specific comment essential
v	OM % (or y) decr (as x incr) oe Old Moat	B1 B1 2	ranks reversed in OM or not rev in W NIS
Total		**13**	

9i	${}^{11}C_5 \times ({}^1/_4)^6 \times ({}^3/_4)^5$ 0.0268 (3 sfs)	M1 A1 2	or $462 \times ({}^1/_4)^6 \times ({}^3/_4)^5$
ii	$q^{11} = 0.05$ or $(1 - p)^{11} = 0.05$ $\sqrt[11]{0.05}$ $q = 0.762$ or 0.7616 . . . $p = 0.238$ (3 sfs)	M1 M1 A1 A1f 4	(any letter except p)$^{11} = 0.05$ oe oe or invlog($\frac{\log 0.05}{11}$) ft dep M2
iii	$11 \times p \times (1 - p) = 1.76$ oe $11p - 11p^2 = 1.76$ or $p - p^2 = 0.16$ $11p^2 - 11p + 1.76 = 0$ or $p^2 - p + 0.16 = 0$ $(25p^2 - 25p + 4 = 0)$ $(5p - 1)(5p - 4) = 0$ or $p = \frac{11 - \sqrt{(11^2 - 4 \times 11 \times 1.76)}}{2 \times 11}$ $p = 0.2$ or 0.8	M1 A1 A1 M1 A1 5	not $11pq = 1.76$ any correct equn after mult out or equiv with = 0 or correct fact'n or subst'n for their quad equ'n eg $p = \frac{1 \pm \sqrt{(1-4 \times 0.16)}}{2}$
Total		**11**	
	Total 72 marks		

Mark Scheme 4733
January 2007

For over-specified answers (> 6SF where inappropriate) deduct 1 mark, no more than once in paper.

1	$\frac{22-\mu}{5} = -\Phi^{-1}(0.242)$ $= -0.7$ $\mu = $ **25.5**	M1 A1 B1 A1	 **4**	Standardise with Φ^{-1}, allow +, "1 –" errors, cc, √5 or 5^2 Correct equation including signs, no cc, can be wrong Φ^{-1} 0.7 correct to 3 SF, can be + Answer 25.5 correct to 3 SF
2	(i) 900 ÷ 12 = **75**	B1	**1**	75 only
	(ii) (a) True, first choice is random	B1	**1**	True stated with reason based on first choice
	(b) False, chosen by pattern	B1	**1**	False stated, with any non-invalidating reason
	(iii) Not equally likely e.g. P(1) = 0, or triangular	M1 A1	 **2**	"Not equally likely", or "Biased" stated Non-invalidating reason
3	Let R be the number of 1s $R \sim B(90, 1/6)$ $\approx N(15, 12.5)$ $\frac{13.5-15}{\sqrt{12.5}}$ [= –0.424] **0.6643**	B1 B1 B1 M1 A1 A1	 **6**	B(90, 1/6) stated or implied, e.g. Po(15) Normal, μ = 15 stated or implied 12.5 or √12.5 or 12.5^2 seen Standardise, np and npq, allow errors in √ or cc or both √ and cc both right Final answer, a.r.t. 0.664. [Po(15): 1/6]
4	(i) $\overline{w} = 100.8 \div 14 = 7.2$ $\frac{938.70}{14} - \overline{w}^2$ [= 15.21] × 14/13 = **16.38**	B1 M1 M1 A1	 **4**	7.2 seen or implied Use Σw^2 – their $\overline{w}^2$ Multiply by $n/(n-1)$ Answer, a.r.t. 16.4
	(ii) N(7.2, 16.38 ÷ 70) [= N(7.2, 0.234)]	B1 B1√ B1√	 **3**	Normal stated Mean their $\overline{w}$ √ Variance [their (i)√ ÷ 70], allow arithmetic slip
5	(i) λ = 1.2 Tables or formula used **0.6626**	B1 M1 A1	 **3**	Mean 1.2 stated or implied Tables or formula [allow ± 1 term, or "1 –"] correctly used Answer in range [0.662, 0.663] [.3012, .6990, .6268 or .8795: B1M1A0]
	(ii) B(20, 0.6626√) ${}^{20}C_{13}\ 0.6626^{13} \times 0.3374^7$ **0.183**	M1 M1 A1	 **3**	B(20, p), p from (i), stated or implied Correct formula for their p Answer, a.r.t. 0.183
	(iii) Let S be the number of stars $S \sim Po(24)$ $\approx N(24, 24)$ $\frac{29.5-24}{\sqrt{24}}$ [= 1.1227] **0.8692**	B1 B1 B1√ M1 A1 A1	 **6**	Po(24) stated or implied Normal, mean 24 Variance 24 or 24^2 or √24, √ if 24 wrong Standardise with λ, λ, allow errors in cc or √ or both √λ and cc both correct Answer, in range [0.868, 0.8694]

Question	Answer	Marks	Guidance
6 (i)	$\left[ax+\frac{bx^2}{2}\right]_0^2=1$	M1	Use total area = 1
		B1	Correct indefinite integral, or convincing area method
	$2a + 2b = 1$ **AG**	A1 **3**	Given answer correctly obtained, "1" appearing before last line [if + c, must see it eliminated]
(ii)	$\left[\frac{ax^2}{2}+\frac{bx^3}{3}\right]_0^2=\frac{11}{9}$	M1	Use $\int xf(x)dx = 11/9$, limits 0, 2
		B1	Correct indefinite integral
	$2a+\frac{8b}{3}=\frac{11}{9}$	A1	Correct equation obtained, a.e.f.
	Solve simultaneously	M1	Obtain one unknown by correct simultaneous method
		A1	a correct, 1/6 or a.r.t 0.167
	$a = \frac{1}{6}$, $b = \frac{1}{3}$	A1 **6**	b correct, 1/3 or a.r.t. 0.333
(iii)	e.g. P(< 11/9) = 0.453, *or*	M1	Use $P(x < 11/9)$, *or* integrate to find median m
	$\left[ax+\frac{bx^2}{2}\right]_0^m=0.5$, $m=1.303$ or $\frac{\sqrt{13}-1}{2}$	M1	Substitute into $\int f(x)dx$, √ on a, b, limits 0 and 11/9 or m [if finding m, need to solve 3-term quadratic]
		A1	Correct numerical answer for probability or m
	Hence median > mean	A1√ **4**	Correct conclusion, cwo
			["Negative skew", M2; median > mean, A2]
7 (i)	$H_0 : p = 0.35$ [*or* $p \geq 0.35$]	B1	Each hypothesis correct, B1+B1, allow $p \geq .35$ if .35 used
	$H_1 : p < 0.35$	B1	[Wrong or no symbol, B1, but r or x or $\bar{x}$: B0]
	B(14, 0.35)	M1	Correct distribution stated or implied, can be implied by N(4.9, …), but *not* Po(4.9)
α:	P(≤ 2) = 0.0839 > 0.025		
β:	CR ≤ 1, probability 0.0205	A1	0.0839 seen, *or* P(≤ 1) = 0.0205 if clearly using CR
	Do not reject H_0. Insufficient	B1	Compare binomial tail with 0.025, *or* R = 2 binomial CR
	evidence that proportion that can	M1	Do not reject H_0, √ on their probability, *not* from N or Po
	receive Channel C is less than 35%	A1√ **7**	or P(< 2); Contextualised conclusion √
(ii)	B(8, 0.35): P(0) = 0.0319	M1	Attempt to find P(0) from B(n, 0.35)
	B(9, 0.35): P(0) = 0.0207	A1	One correct probability [P(≤ 2) = .0236, n = 18: M1A1]
		A1	Both probabilities correct
	Hence largest value of n is **8**	A1 **4**	Answer 8 or ≤ 8 only, needs minimum M1A1
or	$0.65^n > 0.025$; $n \ln 0.65 > \ln 0.025$	M1M1	$p^n > 0.025$, any relevant p; take ln, *or* T&I to get 1 SF
	8.56; largest value of n = 8	A1A1	In range [8.5, 8.6]; answer 8 or ≤ 8 only
8 (i) α:	$\frac{100.7-102}{5.6/\sqrt{80}}=-2.076$	M1	Standardise 100.7 with √80 or 80
		A1	a.r.t. –2.08 obtained, must be –, *not* from μ = 100.7
	Compare with –2.576	B1 **3**	–2.576 or –2.58 seen and compare z, allow both +
or β:	Φ(–2.076) = 0.0189	M1	Standardise 100.7 with √80 or 80
	[or Φ(2.076) = 0.981]	A1	a.r.t. 0.019, allow 0.981 only if compared with 0.995
	and compare with 0.005 [or 0.995]	B1 (3)	Compare correct tail with 0.005 or 0.995
or γ:	$102-\frac{k\times5.6}{\sqrt{80}}$	M1	This formula, allow +, 80, wrong SD, any k from Φ^{-1}
	k = 2.576, compare 100.7	B1	k = 2.576/2.58, – sign, and compare 100.7 with CV
	100.39	A1 (3)	CV a.r.t. 100.4
	Do not reject H_0	M1	Reject/Do not reject, √, needs normal, 80 or √80, Φ^{-1} or equivalent, correct comparison, *not* if clearly μ = 100.7
	Insufficient evidence that quantity of SiO_2 is less than 102	A1 **2**	Correct contextualised conclusion
(ii) (a)	$\frac{c-102}{5.6/\sqrt{n}}=-2.326$	M1	One equation for c and n, equated to Φ^{-1}, allow cc,
		B1	wrong sign, σ^2; 2.326 or 2.33
	$102-c=\frac{13.0256}{\sqrt{n}}$ **AG**	A1 **3**	Correctly obtain given equation, needs in principle to have started from c – 102, –2.326
(b)	$\frac{c-100}{5.6/\sqrt{n}}=1.645$ or $c-100=\frac{9.212}{\sqrt{n}}$	M1	Second equation, as before
		A1 **2**	Completely correct, aef
(c)	Solve simultaneous equations	M1	Correct method for simultaneous equations, find c or √n
	√n =11.12	A1	√n correct to 3 SF
	n_{min} = **124**	A1	n_{min} = 124 only
	c = **100.83**	A1 **4**	Critical value correct, 100.8 or better

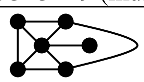
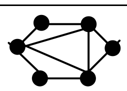
Mark Scheme 4734
January 2007

1	(i)	$E(T) = E(X) + \lambda E(Y)$ $\Rightarrow$ $100 = 45 + 33\lambda$ $\Rightarrow$ $\lambda = {}^5/_3$ AG	M1 A1	 2	Use $E(X+\lambda Y)$ aef
	(ii)	$\text{Var}(T) = \text{Var}(X) + ({}^5/_3)^2\text{Var}(Y)$ $= 256$ $T \sim N(100, 256)$	M1 A1 B1√	 3	 ft variance
	(iii)	Same student for X and Y so independence unlikely.	B1	1	Sensible reason
2	(i)	Use $3a/2 = 1$	B1	1	Or similar
	(ii)	$y = \frac{2}{3}x$ $y = 1 - \frac{1}{3}x$	B1 M1A1	 3	 M1 for correct gradient B1M1A0 if not y=…
	(iii)	$\text{f}(x) = \begin{cases} \frac{2}{3}x & 0 \le x \le 1 \\ 1 - \frac{1}{3}x & 1 < x \le 3. \end{cases}$	B1√	1	ft (ii)
	(iv)	$\int_0^1 \frac{2}{3}x^2 dx + \int_1^3 (x - \frac{1}{3}x^2)dx$ $\left[\frac{2}{9}x^3\right]_0^1 + \left[\frac{1}{2}x^2 - \frac{1}{9}x^3\right]_1^3$ $= 4/3$	M1 A1√A1√ A1	 4	One correct, with limits ft from similar f aef
3	(i)	Assumes breaking strengths have normal normal distributions Equal variances	B1 B1	 2	
	(ii)	H_0: $\mu_T = \mu_U$, H_1: $\mu_T > \mu_U$ where μ_T, μ_U are means for treated and untreated thread.	B1		For both hypotheses
		$\bar{x}_T = 18.05$, $\bar{x}_U = 17.26$	B1		May be implied below by 0.79
		$s_T^2 = 0.715$, $s_U^2 = 0.738$	B1		Allow biased, 0.596, 0.590 if s^2
		$s^2 = (5\times0.715 + 4\times0.738)/9$	M1		$=(6\times0.596 + 5\times0.590)/9$
		EITHER: $(18.05 - 17.26)/[s\sqrt{(1/5+1/6)}]$ $= 1.532$	M1 A1		With pooled variance est.
		Compare correctly with 1.383	M1		
		Reject H_0 and accept there is sufficient evidence that mean has increased so that the treatment has been successful.	A1√		Conclusion in context. Ft 1.532
		OR: $\bar{X}_T - \bar{X}_U \ge ks\sqrt{1/5+1/6}; = 0.713$	M1A1		Allow > or =
		$0.79 > 0.713$, reject H_0 etc	M1A1√	8	Or equivalent. Ft 0.713

4	(i) $s^2 = {}^1/_{11}(2604.4 - 177.6^2/12)$	M1		aef
	$= 1.0836\ldots$	A1		
	Use $\bar{x} \pm t\sqrt{\frac{s^2}{12}}$	M1		
	$t = 2.201$	B1		
	$\bar{x} = 177.6/12 = 14.8$	A1		
	(14.14,15.46) , (14.1, 15.5)	A1	**6**	
	(ii) EITHER:$(14.8 - 15.4)/(\sqrt{(s^2/12)})$	M1		With their variance
	= -1.997	A1		
	Compare correctly with –1.796	M1		
	Reject H_0 and accept that there is evidence that the mean is less than 15.4	A1√		In context. Ft – 1.997
	OR: $\bar{X} - 15.4 \le -k\sqrt{\frac{s^2}{12}}$; $\bar{X} \le 14.86$	M1A1		Allow < or =
	14.8 < 14.86, reject H_0 etc	M1A1√	**4**	Or equivalent. Ft 14.86
5	(i) 978/1200 = 0.815	B1	**1**	
	(ii) Use $\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{1200}}$	M1		Reasonable variance
	$z = 1.645$	B1		
	$\sqrt{(0.815\times0.185/1200)}$	A1√		ft $\hat{p}$ Allow 1199
	(0.797,0.833)	A1	**4**	Interval
	(iii) If a large number of such samples were taken, p would be contained in about 90% of the confidence intervals.	B2	**2**	B1 if idea correct but badly expressed.
	(iv) $1.645\sqrt{(0.815\times0.185/n)} = 0.01$	M1		Allow one error; > or <
		A1		All correct
	$n = 1.645^2(0.815\times0.185)/0.01^2$	M1		Correct procedure for sim equ
	= 4080	A1	**4**	Integer rounding to 4100

6	(i) $\int_1^t \frac{3}{x^4}\,dx$	M1		Any variable
	$F(t) = \begin{cases} 1-\frac{1}{t^3} & t \ge 1, \\ (0 & \text{otherwise.}) \end{cases}$	A1	**2**	
	(ii) $G(y) = P(Y \le y)$	M1		
	$= P(T \le y^{1/3})$	A1		
	$= F(y^{1/3})$	M1		
	$= 1 - 1/y$	A1 √		ft $F(t)$
	$g(y) = G'(y)$	M1		
	$= 1/y^2,\ y \ge 1$ AG	A1	**6**	
	(iii) EITHER $\int_1^\infty \frac{\sqrt{y}}{y^2}\,dy$ OR $\int_1^\infty \frac{3t^{\frac{3}{2}}}{t^4}\,dt$	M1		
	$\left[-2y^{-1/2}\right]_1^\infty$ $\left[-2t^{-3/2}\right]_1^\infty$	B1		
	$= 2$	A1	**3**	
7	(i)(a) H_0: Eye colour and reaction are not associated.	B1		Or equivalent (independent, or unrelated)
	H_1: Eye colour and reaction are associated	B1	**2**	
	(b) 65×39/140	B1	**1**	
	(c) $6.11^2/18.11 + 5.3^2/11.7 + 0.81^2/9.19$	M1		Or equivalent ; one correct
	2.061 + 2.401+ 0.071	A1		At least 3 dp here
	4.533, 4.53 AG	A1	**3**	But accept from 2 dp
	(d) $\nu = 4$	B1		Stated or implied
	Use tables to obtain α = 2½	B1	**2**	
	(ii) H_0: $p_{BL} = p_{BR} = 0.4$, $p_O = 0.2$	B1		Or in words, in terms of probs or proportions
	(H_1: At least two prob. not as above)			
	E values 56 56 28	M1A1		
	$\chi^2 = 9^2/56 + 14^2/56 + 8^2/28$	M1		
	= 5.839	A1		Accept 5.84
	Compare correctly with 5.991	M1		M1A0 if 5.991 seen and the consistent conclusion but no explicit comparison
	Accept that sample is consistent with the hypothesis.	A1√	**7**	
	SR: If three tests for p then count only $p_{BR} = 0.4$.			
	$(42/140 - 0.4)/\sigma$	M1		
	$\sigma = \sqrt{(0.4\times0.6/140)}$; -2.415	A1A1		
	Compare with –1..96; conclusion in context	M1A1		Max 6/7 (with H_0)

Mark Scheme 4736
January 2007

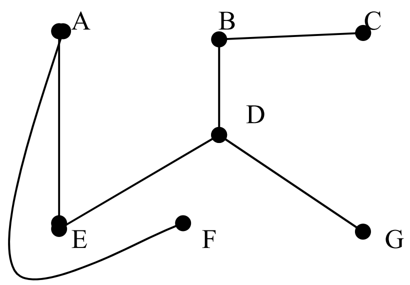
1	(i)	10 4 2 3 5 13 7 2 2 4 5 8 5 3 10 5 5 3	M1 M1 A1	First bundle starting with 10 4 2 and has at least one more bag in it Second bundle correct All bundles correct	[3]
	(ii)	Decreasing order: 13 10 10 8 7 5 5 5 5 5 4 4 3 3 3 2 2 2 13 10 2 10 8 7 5 5 5 5 5 4 4 3 3 3 2 2	 M1 M1 A1	A value missing from written out list may be treated as a misread and lose the A mark only Sorting into decreasing order (may be implied from first bundle starting with 13) If each row sorted, award first M1 only Second and third bundles correct All bundles correct	[3]
	(iii)	Each person has roughly the same number of bags <u>or</u> the total weights are more evenly spread	B1	Saying that (i) gives a more even/equal allocation Five bundles in either part ϕ B0	[1]
				Total =	**7**
2	(i)	a = number of apple cakes b = number of banana cakes c = number of cherry cakes	B1 B1	Identifying variables as 'number of cakes' Indicating a as apple, b as banana and c as cherry.	[2]
	(ii)	$4\times30 = 3\times40 = 4\times30 = 120$ $\frac{a}{30}+\frac{b}{40}+\frac{c}{30}=30\times40\times30$ $4a + 3b + 4c \leq 120$ or $X = 4$, $Y = 3$, $Z = 4$	M1 A1	Any reasonable attempt 4, 3 and 4	[2]
	(iii)	$a + b + c \geq 30$ (or $a + b + c = 30$) $0 \leq a \leq 20$, $0 \leq b \leq 25$, $0 \leq c \leq 10$ (no need to say 'all integer')	B1 M1 A1	Constraint from total number of cakes correct All three upper constraints correct All three lower constraints correct also	[3]
	(iv)	$4a + 3b + 2c$	B1	Any multiple of this expression	[1]
				Total =	**8**
3	(i) a	$9\times2 = 18$	B1	18	[1]
	b	Since the graph is simple, the two nodes of order 5 are each connected to every other node and hence every node has order at least 2 (exactly 2)	B1 B1	Explicitly using the fact that the graph is simple Deducing that each node has order at least 2 or that all other nodes have order 2 A diagram on its own is not enough.	[2]
	c	$3\times5 = 15$ and $18 - 15 = 3$ but the orders of the other nodes must sum to at least $3\times3 = 9$ (must sum to more than 3)	B1 B1	Or, the nodes of order 5 contribute 5+4+3 = 12 arcs But there are only 9 arcs available	[2]
	(ii)	 or equivalent	M1 A1	A simply connected graph with 6 nodes and 9 arcs, with at least one odd node For such a graph with node orders 1, 3, 3, 3, 3, 5	[2]
	(iii)	 or equivalent	M1 A1	A simply connected graph with 6 nodes and 9 arcs, with at least one even node For such a graph with node orders 2, 2, 2, 4, 4, 4	[2]
				Total =	**9**

4 (i)

	1	4	5	3	2	7	6
	A	*B*	*C*	*D*	*E*	*F*	*G*
A	0	4	5	3	2	5	6
B	4	0	1	**2**	4	7	6
C	5	**1**	0	3	4	6	7
D	3	2	3	0	**2**	6	4
E	**2**	4	4	2	0	6	6
F	**5**	7	6	6	6	0	10
G	6	6	7	**4**	6	10	0

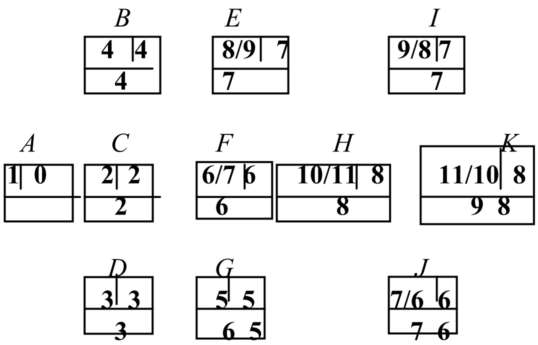
Order: *A E D B C G F*

Minimum spanning tree:



Total weight: 16 (or 1600 m)

Q	Part	Answer	Mark	Guidance	Total
4	(i)			FIRST THREE MARKS ARE FOR WORK ON THE TABLE ONLY	
			M1	(Starting by) choosing row E in column A	
			M1 dep	Choosing more than one entry from column A	
			A1	Correct entries chosen (or all transposed)	
		Order: *A E D B C G F*	B1	Correct order, listed or marked on arrows or table, or arcs listed *AE ED DB BC DG AF*	
		Minimum spanning tree	B1	Tree (correct or follow through from table, provided solution forms a spanning tree)	
		Total weight: 16 (or 1600 m)	B1	16 or 1600m (or follow through from table or diagram, provided solution forms a spanning tree)	[6]
	(ii)	Travelling salesperson (problem)	B1	Identifying TSP by name	[1]
	(iii)	Two shortest arcs from *H*: 12 + 13 = 25	B1	12 + 13 or 25, or implied from final answer	
		25 + 16 = 41	M1	Adding their 25 to their 16 or for 41 (must be using two arcs from *H*)	
		4100 m	A1	4100 m or 4.1 km (correct and with units)	[3]
	(iv)	*H A E D B C F G H*	M1	(*H*) *A E D B C* ...	
			A1	Correct tour	
		12+2+2+2+1+6+10+16 = 51	M1	A substantially correct attempt at sum	
		5100 m	A1	5100m or 5.1 km (correct and with units)	[4]
				Total =	**14**

5	**(i)**		M1 M1 A1 B1 B1	Correct temporary labels at B to G, no extras Correct temporary labels at H to J, no extras All temporary labels correct Order of becoming permanent correct (follow through their permanent labels) All permanent labels correct	
		Note: H may have only a temporary label if left until last Route: *A D G J K* Number of speed cameras on route: 8	 B1 B1	 Correct route 8 (cao)	[7]
	(ii)	Odd nodes: *A I J K* $AI = 7$ $AJ = 6$ $AK = 8$ $JK = \underline{2}$ $IK = \underline{4}$ $IJ = \underline{6}$ 9 10 14 Repeat AI and $JK \Rightarrow AB$ BI and JK Route (example): *K J D A B I K J G K H G F H E F C G D C A B C E B I E K* Number of speed cameras on route: 81	M1 A1 A1 M1 A1 B1	Identifying or using *A I J K* Weight of AI + weight of $JK = 9$ Weight of AJ + weight of $IK = 10$ (follow through weight of AI, AJ from (i) if necessary) A list of 28 nodes that starts and ends with K Such a list that includes each of AB, BI, JK (or reversed) twice 72 + weight of their least pairing	[6]
	(iii)	The only odd nodes are I and J so she only needs to repeat $IJ = 6$ 72 + 6 = 78	B1 M1 A1	Identifying I and J or IJ (not just implied from 6 or 72+6 or 78) Correct calculation (may be implied from 78)	[3]
				Total =	**16**

6	(i)	$\begin{array}{c\|cccccc} P & x & y & z & s & t & \\ \hline 1 & -3 & 5 & -4 & 0 & 0 & 0 \\ \hline 0 & 1 & 2 & -3 & 1 & 0 & 12 \\ 0 & 2 & 5 & -8 & 0 & 1 & 40 \end{array}$	B1 B1 B1	Correct use of two slack variable columns $\pm$ (-3 5 -4) in objective row 1 2 -3 12 and 2 5 -8 40 in constraint rows	[3]
	(ii)	The entries in rows 2 and 3 of the z column are negative Pivot on 1 in x column x and z columns have negative entries in obj. row but no value in z column is positive so choose x $12 \div 1 = 12$, $40 \div 2 = 20$ Least positive ratio is 12 so pivot on the 1	B1 B1 B1	Entries for potential pivots are not positive Correct pivot choice (cao) (stated or entry ringed) Follow through their table 'Negative in top row for x' <u>and</u> a correct explanation of choice of row 'least ratio $12 \div 1$'	[3]
	(iii)	$\begin{array}{c\|cccccc} P & x & y & z & s & t & \\ \hline 1 & 0 & 11 & -13 & 3 & 0 & 36 \\ \hline 0 & 1 & 2 & -3 & 1 & 0 & 12 \\ 0 & 0 & 1 & -2 & -2 & 1 & 16 \end{array}$ $x = 12$, $y = 0$, $z = 0$	M1 A1 B1	Follow through their tableau if possible Correct method evident Correct tableau (ft if reasonable and possible, column representing RHS of equations must contain non-negative entries) Correct non-negative values for their tableau	[3]
	(iv)	z can increase without limit and increasing z will increase P	B1	Discussing the effect of increasing z Not just referring to pivoting in tableau	[1]
	(v)	Initial tableau is unchanged except entry in z col of obj. row becomes +40 First iteration tableau is also unchanged except for this entry which becomes 31 36	B1 B1 B1 B1	Describing change to obj. row of initial tableau or showing tableau that results Identifying 31 instead of -13 (cao) No other changes 36 stated (cao)	[4]
	(vi)	Adding the constraints gives $3x - 5y + 7z \leq 52$ so $Q \leq 52$	B1	52	[1]
	(vii)	$x - 3z = 12$ and $2x + 10z = 40$ (Accept $\leq$) $\Rightarrow 10z + 6z = 40 - 24$ $\Rightarrow x = 15$ and $z = 1$	M1 M1 A1	Eliminating y terms (may be implied) Trying to solve simultaneous equations Correct values (may imply method marks)	[3]
				Total =	**18**

Mark Scheme 4737
January 2007

1 (i)

Answer	Mark	Guidance	
The Hungarian algorithm finds the minimum cost allocation, we need to subtract each score from 6 to convert a maximisation into a minimisation.	B1	A valid reference to maximising/minimising	[1]

(ii)

First subtract each entry from 6

	Attic	Back	Down	Front
Phil	1	5	6	2
Rob	5	0	5	4
Sam	2	4	4	3
Tim	3	1	6	6

B1 Correctly subtracting each entry from 6 (cao)

Reduce rows

0	4	5	1
5	0	5	4
0	2	2	1
2	0	5	5

M1 Reducing rows first

Then reduce columns

0	4	3	0
5	0	3	3
0	2	0	0
2	0	3	4

M1 dep Then reducing columns

A1 A correct reduced cost matrix from rows reduced first (cao)

Cover 0's using 3 lines

0	4	3	0
5	0	3	3
0	2	0	0
2	0	3	4

M1 Covering zeros using minimum number of lines and augmenting by (their) 2

Augment by 2

0	6	3	0
3	0	1	1
0	4	0	0
0	0	1	2

A1 A correct augmented matrix (cao) from rows reduced first

Phil = Front room
Rob = Back room
Sam = Downstairs room
Tim = Attic room

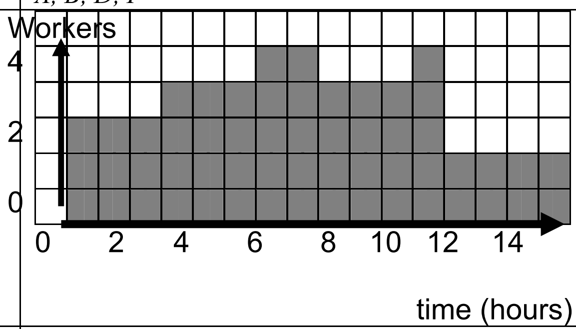
B1 Correct matching [7]

Total = 8

2 (i)

Answer	Mark	Guidance	
16 hours	B1	16 with units	
A, B, D, F	B1	All four critical activities and no others	[2]

(ii)



M1 A reasonable attempt at a resource histogram

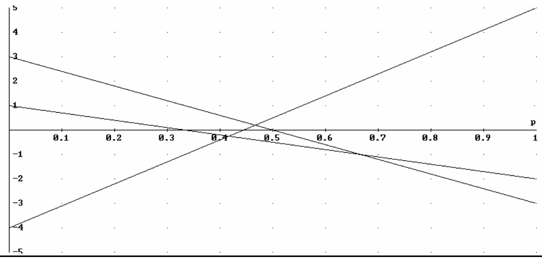
A1 An entirely correct graph with scales and labels [2]

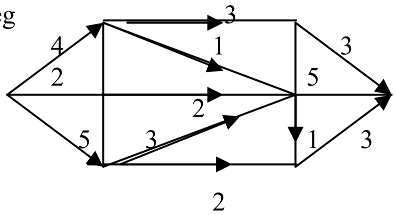
(iii)

Answer	Mark	Guidance	
Start *C* at time 3	B1	'*C*' and '3' or 'after *A*' or 'with *B*'	
Start *E* at time 8	B1	'*E*' and '8' or 'after *B*' or 'with *D*'	
Start *G* at time 16	B1	'*G*' and '16' or 'after *F*'	
Complete in 19 hours	B1	19	[4]

Total = 8

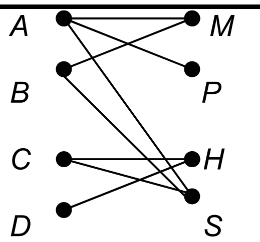
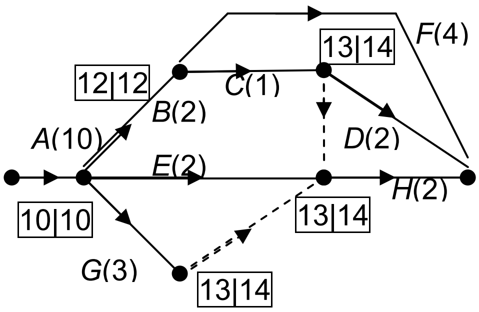
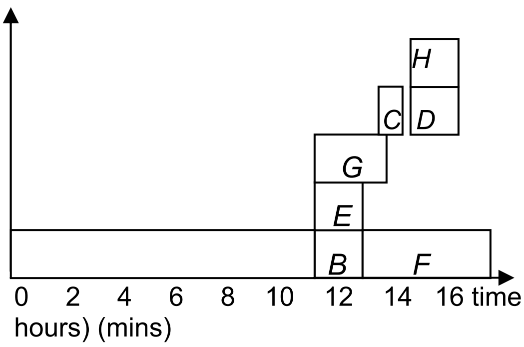
3	**(i)**	-5	B1	-5	[1]
	(ii)	Because -3 < 2 in column *Y* and 2 > -2 in row *Y*	M1 A1	Either of these, possibly with others Both of these comparisons and no others	 [2]
	(iii)	Play-safe for Rebecca is *Z* Play-safe for Claire is *Y* Best choice is *X*	B1 B1 B1 ft	Indicating row *Z* Indicating column *Y* The correct choice with their play-safe	 [3]
	(iv)	For Rebecca, -1 > smaller of {-3, value that 5 becomes} For Claire, 2 < larger of {3, value that 5 becomes}	 B1 B1	 This, or equivalent, or 5 is not in the play-safe row This, or equivalent (but NOT '5 is not in the play-safe column')	 [2]
				Total =	**8**

4	**(i)**	$5p$ -4(1-p) = $9p$ - 4	M1 A1	This, or implied $9p - 4$ or $-4 + 9p$	 [2]
	(ii)		M1 A1 A1 A1	Correct structure to graph Line $E = 9p - 4$ plotted from (0,-4) to (1, 5) Line $E = 3 - 6p$ plotted from (0, 3) to (1,-3) Line $E = 1 - 3p$ plotted from (0, 1) to (1,-2) Withhold an A1 for horizontal scale beyond 0 to 1	[4]
	(iii)	$9p - 4 = 1 - 3p$ $\Rightarrow p = 5/12$ or 0.41 to 0.42 (or better)	M1 A1 ft	Solving the correct pair of lines for their graph Correct value for their lines	 [2]
	(iv)	If Colin plays *X* or *Z*, Rowan's expected winnings are -0.25 so Colin's expected winnings are +0.25 Even if Colin plays optimally he cannot expect, in the long run, to do better on average than to win what Rowan loses.	B1 B1	Showing why it is +0.25 for Colin Realising that Colin need to play his optimal strategy as well as Rowan	 [2]
				Total =	**10**

5	(i)	4+2+4+0+5 = 15	M1 A1	At least four correct terms 15 from correct calculation	[2]
	(ii)	Subtract 3 from *SA*, *AD*, *DT* and add 3 to *TD*, *DA*, *AS* Subtract 2 from *SB*, *BE*, *ET* and add 2 to *TE*, *EB*, *BS*□ Subtract 2 from *SC*, *CF*, *FT* and add 2 to *TF*, *FC*, *CS*	M1 M1 A1	Correctly subtracting along one of the three flow augmenting routes Correctly adding along one of the three flow augmenting routes All changes correct and no other changes made	[3]
	(iii)	eg Route *SCET* Flow = 3	B1 B1 ft	Any valid flow augmenting route (not ft) Maximum extra flow on their route	[2]
	(iv)	Maximum flow = 11 litres per second Cut: X = {*S*}, Y = {*A,B,C,D,E,F,T*}	B1 B1	11 with units This cut described in this way	[2]
	(v)	eg 	M1 M1 A1	At each vertex, flow in = flow out On each arc, flow $\leq$ capacity A valid directed flow of 11	[3]
				Total =	**12**

6	(i)	(table below)	B1 M1 A1 M1 A1 B1 ft M1 ft A1 ft	Maximin value correct for (2;0) Completing working column of (2;1) Maximin value correct for (2;1) Completing working column for (2;2) Maximin value correct for (2;2) Transferring maximin values from stage 2 Completing working column for stage 3 Maximin value correct for stage 3	[8]
	(ii)	4 (3;0) – (2;0) – (1;0) – (0;0) (or in reverse)	B1 ft M1 ft M1 ft A1	4, or ft their table if possible (3;0) – (2;0), or ft their table if possible (2;0) – (1;0), or ft their table if possible For maximin route correct	[4]
				Total =	**12**

Stage	State	Action	Working	Maximin
1	0	0	4	4
	1	0	3	3
2	0	0	min(6, 4) = 4	**4**
		1	min(2, 3) = 2	
	1	0	min(2, 4**) = 2**	
		1	min(4, 3**) = 3**	**3**
	2	0	min(2, **4**) **= 2**	
		1	min(3, **3**) **= 3**	**3**
3	0	0	min(5, **4**) **= 4**	**4**
		1	min(5, **3**) **= 3**	
		2	min(2, **3**) **= 2**	

7	(i)		B1	Correct bipartite graph seen Ignore further working on graph for incomplete matching or alternating path	
		Alternating path: $D – H – C – S – B – M – A – P$	B1	This, or in reverse, listed (not just deduced from labelling of diagram)	
		Matching: A - P B - M C - S D - H	B1	This matching	[3]
	(ii)		M1 A1 M1 A1 ft M1 A1 ft	Precedences correct A correct network (directions may be implied) Forwards pass Early event times correct (need not use boxes) Backwards pass Late event times (need not use boxes)	[6]
	(iii)	Completion time: 16 hours Critical activities: *A B F*	B1 B1	16 with units Correct list	[2]
	(iv)		 M1 A1 ft A1 ft	Accept any variation of cascade chart Structure of chart correct, activities may be collected together or on individual rows Non-critical activities correct, none split across rows (floats not necessary) Critical activities correct	[3]
				Total =	**14**

Advanced GCE Mathematics (3892 – 2, 7890 - 2)
January 2007 Assessment Series

Unit Threshold Marks

Unit		Maximum Mark	a	b	c	d	e	u
4721	Raw	72	63	55	48	41	34	0
	UMS	100	80	70	60	50	40	0
4722	Raw	72	57	49	41	33	26	0
	UMS	100	80	70	60	50	40	0
4723	Raw	72	55	48	41	34	28	0
	UMS	100	80	70	60	50	40	0
4724	Raw	72	57	49	41	33	26	0
	UMS	100	80	70	60	50	40	0
4725	Raw	72	58	50	42	34	26	0
	UMS	100	80	70	60	50	40	0
4726	Raw	72	54	46	39	32	25	0
	UMS	100	80	70	60	50	40	0
4727	Raw	72	55	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
4728	Raw	72	61	53	45	38	31	0
	UMS	100	80	70	60	50	40	0
4729	Raw	72	61	53	45	37	29	0
	UMS	100	80	70	60	50	40	0
4730	Raw	72	54	47	40	33	26	0
	UMS	100	80	70	60	50	40	0
4732	Raw	72	59	52	45	38	32	0
	UMS	100	80	70	60	50	40	0
4733	Raw	72	61	54	47	40	33	0
	UMS	100	80	70	60	50	40	0
4734	Raw	72	56	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4736	Raw	72	53	46	39	32	25	0
	UMS	100	80	70	60	50	40	0
4737	Raw	72	61	53	45	38	31	0
	UMS	100	80	70	60	50	40	0

Specification Aggregation Results

Overall threshold marks in UMS (i.e. after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
3890/3892	300	240	210	180	150	120	0
7897892	600	480	420	360	300	240	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
3890	19.1	36.8	59.5	80.6	94.3	100	299
3892	66.7	77.8	88.9	88.9	100	100	9
7890	40.2	62.5	87.5	95.5	100	100	112
7892	50.0	83.3	83.3	83.3	91.7	100	12

For a description of how UMS marks are calculated see;
http://www.ocr.org.uk/exam_system/understand_ums.html

Statistics are correct at the time of publication

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

(General Qualifications)
Telephone: 01223 553998
Facsimile: 01223 552627
Email: helpdesk@ocr.org.uk

www.ocr.org.uk

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553