OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT

4737/01

MATHEMATICS

Decision Mathematics 2

WEDNESDAY 10 JANUARY 2007

Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
Graph paper
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- There is an **insert** for use in Questions **5** and **6**.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **8** printed pages and an insert.

1 Four friends have rented a house and need to decide who will have which bedroom. The table below shows how each friend rated each room, so the higher the rating the more the room was liked.

	Attic room	Back room	Downstairs room	Front room
Phil	5	1	0	4
Rob	1	6	1	2
Sam	4	2	2	3
Tim	3	5	0	0

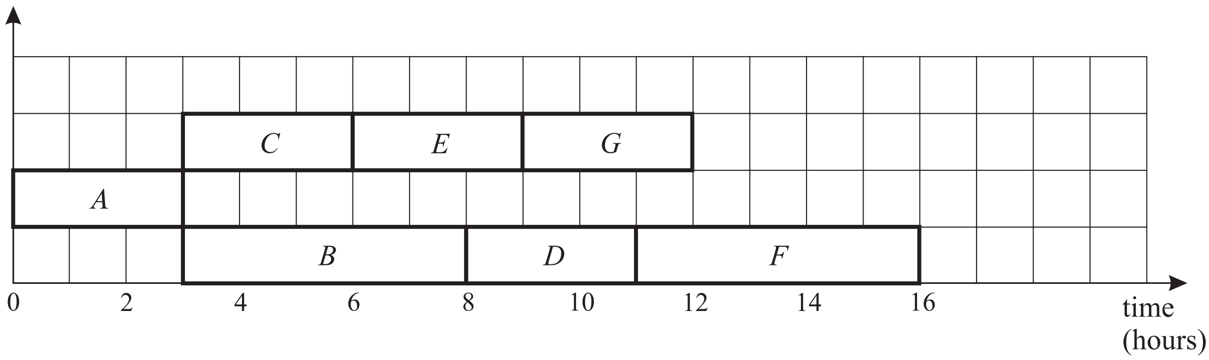
The Hungarian algorithm is to be used to find the matching with the greatest total. Before the Hungarian algorithm can be used, each rating is subtracted from 6.

(i) Explain why the ratings could not be used as given in the table. [1]

(ii) Apply the Hungarian algorithm, **reducing rows first**, to match the friends to the rooms. You must show your working and say how each matrix was formed. [7]

2 The table shows the activities involved in a project, their durations, precedences and the number of workers needed for each activity. The graph gives a schedule with each activity starting at its earliest possible time.

Activity	Duration (hours)	Immediate predecessors	Number of workers
A	3	–	3
B	5	A	2
C	3	A	2
D	3	B	1
E	3	C	3
F	5	D, E	2
G	3	B, E	3



(i) Using the graph, find the minimum completion time for the project and state which activities are critical. [2]

(ii) Draw a resource histogram, using graph paper, assuming that there are no delays and that every activity starts at its earliest possible time. [2]

Assume that only four workers are available but that they are equally skilled at all tasks. Assume also that once an activity has been started it continues until it is finished.

(iii) The critical activities are to start at their earliest possible times. List the start times for the non-critical activities for completion of the project in the minimum possible time. What is this minimum completion time? [4]

3 Rebecca and Claire repeatedly play a zero-sum game in which they each have a choice of three strategies, X, Y and Z.

The table shows the number of points Rebecca scores for each pair of strategies.

Claire

Rebecca		X	Y	Z
	X	5	−3	1
	Y	3	2	−2
	Z	−1	1	3

(i) If both players choose strategy X, how many points will Claire score? [1]

(ii) Show that row X does not dominate row Y and that column Y does not dominate column Z. [2]

(iii) Find the play-safe strategies. State which strategy is best for Claire if she knows that Rebecca will play safe. [3]

(iv) Explain why decreasing the value '5' when both players choose strategy X cannot alter the play-safe strategies. [2]

4 The table gives the pay-off matrix for a zero-sum game between two players, Rowan and Colin.

Colin

Rowan		Strategy X	Strategy Y	Strategy Z
	Strategy P	5	−3	−2
	Strategy Q	−4	3	1

Rowan makes a random choice between strategies P and Q, choosing strategy P with probability p and strategy Q with probability $1 - p$.

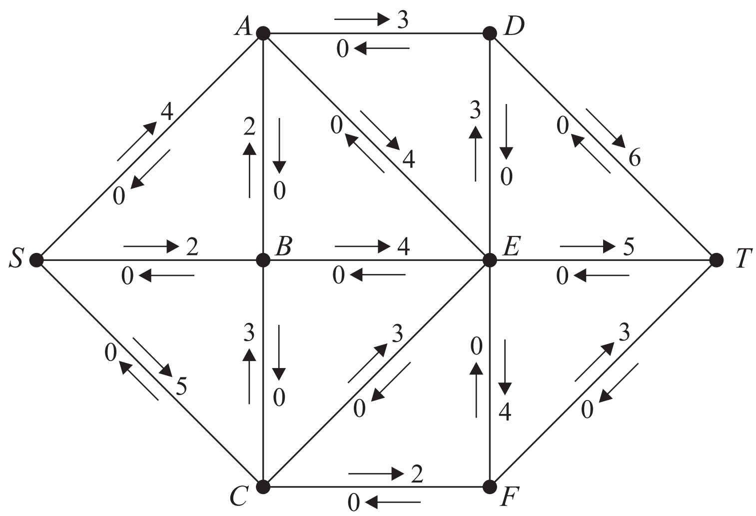
(i) Write down and simplify an expression for the expected pay-off for Rowan when Colin chooses strategy X. [2]

(ii) Using graph paper, draw a graph to show Rowan's expected pay-off against p for each of Colin's choices of strategy. [4]

(iii) Using your graph, find the optimal value of p for Rowan. [2]

(iv) Rowan plays using the optimal value of p. Explain why, in the long run, Colin cannot expect to win more than 0.25 per game. [2]

5 Answer this question on the insert provided.



The diagram represents a system of pipes through which fluid can flow from a source, S, to a sink, T. The arrows are labelled to show excess capacities and potential backflows (how much more and how much less could flow in each pipe). The excess capacities and potential backflows are measured in litres per second. Currently no fluid is flowing through the system.

(i) Calculate the capacity of the cut X = $\{S, B\}$, Y = $\{A, C, D, E, F, G, T\}$. [2]

(ii) Update the first diagram in the insert to show the **changes** to the labelling when 3 litres per second flow along $SADT$, 2 litres per second flow along $SBET$ and 2 litres per second flow along $SCFT$. [3]

(iii) Write down one more flow augmenting route, but do not change the labelling on the diagram. How much can flow through your route? [2]

(iv) What is the maximum flow through the network? Write down a cut that has capacity equal to the maximum flow. [2]

(v) Complete the second diagram in the insert to show a maximum feasible flow through the network. You will need to mark the direction and amount of flow in each arc. [3]

6 Answer this question on the insert provided.

The table shows a partially completed dynamic programming tabulation for solving a maximin problem.

Stage	State	Action	Working	Maximin
1	0	0	4	4
	1	0	3	3
2	0	0	$\min(6, 4) = 4$	
		1	$\min(2, 3) = 2$	
	1	0	$\min(2, 4) =$	
		1	$\min(4, 3) =$	
	2	0	min(2,	
		1	min(3,	
3	0	0	min(5,	
		1	min(5,	
		2	min(2,	

(i) Complete the last two columns of the table in the insert. [8]

(ii) State the maximin value and write down the maximin route. [4]

7 Annie (A), Brigid (B), Carla (C) and Diane (D) are hanging wallpaper in a stairwell. They have broken the job down into four tasks: measuring and cutting the paper (M), pasting the paper (P), hanging and then trimming the top end of the paper (H) and smoothing out the air bubbles and then trimming the lower end of the paper (S). They will each do one of these tasks.

- Annie does not like climbing ladders but she is prepared to do tasks M, P or S
- Brigid gets into a mess with paste so she is only able to do tasks M or S
- Carla enjoys hanging the paper so she wants to do task H or task S
- Diane wants to do task H

Initially Annie chooses task M, Brigid task S and Carla task H.

(i) Draw a bipartite graph to show the available pairings between the people and the tasks. Write down an alternating path to improve the initial matching and write down the complete matching from your alternating path. [3]

Hanging the wallpaper is part of a bigger decorating project. The table lists the activities involved, their durations and precedences.

Activity	Duration (hours)	Immediate predecessors
A Strip the old wallpaper	10	–
B Fill in plaster	2	A
C Undercoat woodwork	1	B
D Paint woodwork with gloss	2	C
E Paint ceiling	2	A
F Hang new wallpaper	4	B
G Prepare floor	3	A
H Lay new flooring	2	C, E, G

(ii) Draw an activity network, using activity on arc, to represent this information. You will need to label the activities and use some dummy activities. Calculate the early event time and late event time for each event and show these clearly at the vertices of your network. [6]

(iii) State the minimum completion time for the whole decorating project, assuming that there are enough workers and there are no delays. Write down the critical activities. [2]

(iv) Construct a cascade chart, showing each activity starting at its earliest possible time. [3]

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MATHEMATICS

Decision Mathematics 2 INSERT for Question 5 and 6

WEDNESDAY 10 JANUARY 2007 Afternoon

Time: 1 hour 30 minutes

Candidate Name	

Centre Number						Candidate Number					

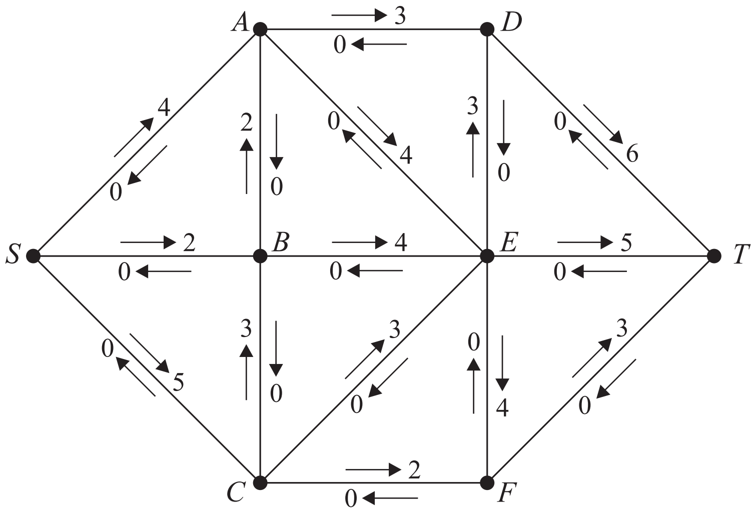
INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the boxes above.
- This insert should be used to answer Questions **5** and **6**.
- Write your answers to Questions **5** and **6** in the spaces provided in this insert, and attach it to your answer booklet.

This document consists of **3** printed pages and **1** blank page.

5 **(i)** Capacity = ...

(ii)

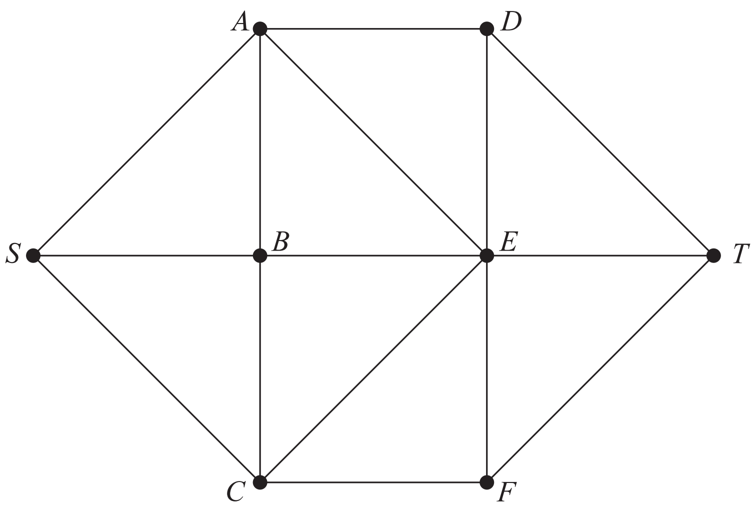


(iii) Route: .... Flow = ...

(iv) Maximum flow = ...

Cut: X = { } Y = { }

(v)



6 **(i)**

Stage	State	Action	Working	Maximin
1	0	0	4	4
	1	0	3	3
2	0	0	$\min(6, 4) = 4$	
		1	$\min(2, 3) = 2$	
	1	0	$\min(2, 4) =$	
		1	$\min(4, 3) =$	
	2	0	$\min(2,$	
		1	$\min(3,$	
3	0	0	$\min(5,$	
		1	$\min(5,$	
		2	$\min(2,$	

(ii) Maximin value = ...

Route = ...

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