OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT

4723/01

MATHEMATICS

Core Mathematics 3

THURSDAY 18 JANUARY 2007

Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

[Turn over

1 Find the equation of the tangent to the curve $y = \frac{2x+1}{3x-1}$ at the point $\left(1, \frac{3}{2}\right)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [5]

2 It is given that θ is the acute angle such that $\sin\theta = \frac{12}{13}$. Find the exact value of

(i) $\cot\theta$, [2]

(ii) $\cos 2\theta$. [3]

3 **(a)** It is given that a and b are positive constants. By sketching graphs of

$$y = x^5 \quad \text{and} \quad y = a - bx$$

on the same diagram, show that the equation

$$x^5 + bx - a = 0$$

has exactly one real root. [3]

(b) Use the iterative formula $x_{n+1} = \sqrt[5]{53 - 2x_n}$, with a suitable starting value, to find the real root of the equation $x^5 + 2x - 53 = 0$. Show the result of each iteration, and give the root correct to 3 decimal places. [4]

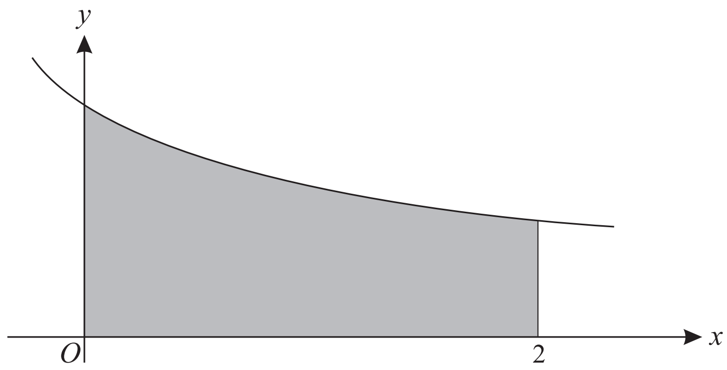
4 **(i)** Given that $x = (4t+9)^{\frac{1}{2}}$ and $y = 6e^{\frac{1}{2}x+1}$, find expressions for $\frac{dx}{dt}$ and $\frac{dy}{dx}$. [4]

(ii) Hence find the value of $\frac{dy}{dt}$ when $t = 4$, giving your answer correct to 3 significant figures. [3]

5 **(i)** Express $4\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(ii) Hence solve the equation $4\cos\theta - \sin\theta = 2$, giving all solutions for which $-180° < \theta < 180°$. [5]

6



The diagram shows the curve with equation $y = \dfrac{1}{\sqrt{3x+2}}$. The shaded region is bounded by the curve and the lines $x = 0$, $x = 2$ and $y = 0$.

(i) Find the exact area of the shaded region. [4]

(ii) The shaded region is rotated completely about the x-axis. Find the exact volume of the solid formed, simplifying your answer. [5]

7 The curve $y = \ln x$ is transformed to the curve $y = \ln\left(\frac{1}{2}x - a\right)$ by means of a translation followed by a stretch. It is given that a is a positive constant.

(i) Give full details of the translation and stretch involved. [2]

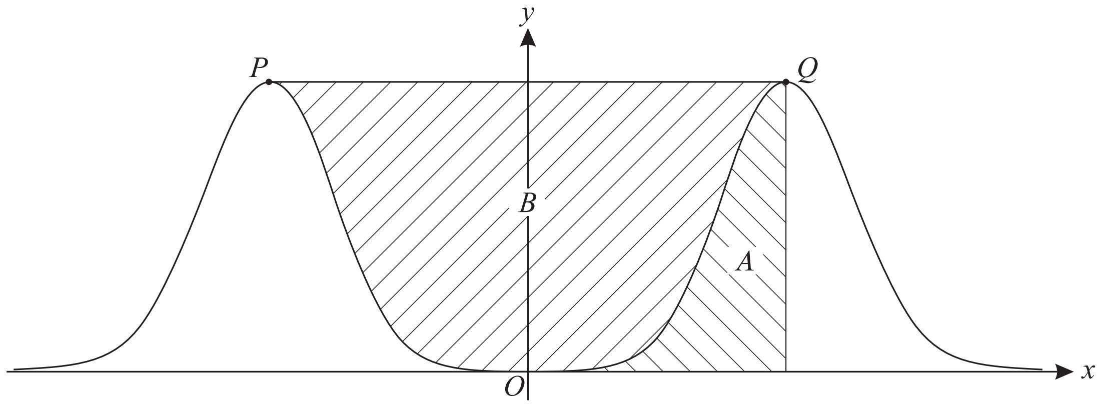
(ii) Sketch the graph of $y = \ln\left(\frac{1}{2}x - a\right)$. [2]

(iii) Sketch, on another diagram, the graph of $y = \left|\ln\left(\frac{1}{2}x - a\right)\right|$. [2]

(iv) State, in terms of a, the set of values of x for which $\left|\ln\left(\frac{1}{2}x - a\right)\right| = -\ln\left(\frac{1}{2}x - a\right)$. [2]

[Questions 8 and 9 are printed overleaf.]

8



The diagram shows the curve with equation $y = x^8 e^{-x^2}$. The curve has maximum points at P and Q. The shaded region A is bounded by the curve, the line $y = 0$ and the line through Q parallel to the y-axis. The shaded region B is bounded by the curve and the line PQ.

(i) Show by differentiation that the x-coordinate of Q is 2. [5]

(ii) Use Simpson's rule with 4 strips to find an approximation to the area of region A. Give your answer correct to 3 decimal places. [4]

(iii) Deduce an approximation to the area of region B. [2]

9 Functions f and g are defined by

$$\begin{aligned} \mathrm{f}(x) &= 2\sin x \quad \text{for } -\tfrac{1}{2}\pi \leqslant x \leqslant \tfrac{1}{2}\pi, \\ \mathrm{g}(x) &= 4 - 2x^2 \quad \text{for } x \in \mathbb{R}. \end{aligned}$$

(i) State the range of f and the range of g. [2]

(ii) Show that $\mathrm{gf}(0.5) = 2.16$, correct to 3 significant figures, and explain why $\mathrm{fg}(0.5)$ is not defined. [4]

(iii) Find the set of values of x for which $\mathrm{f}^{-1}\mathrm{g}(x)$ is not defined. [6]