OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4731

Mechanics 4

Wednesday **21 JUNE 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

[Turn over

1 A straight rod AB of length a has variable density. At a distance x from A its mass per unit length is $k(a + 2x)$, where k is a positive constant. Find the distance from A of the centre of mass of the rod. [5]

2 A flywheel takes the form of a uniform disc of mass 8 kg and radius 0.15 m. It rotates freely about an axis passing through its centre and perpendicular to the disc. A couple of constant moment is applied to the flywheel. The flywheel turns through an angle of 75 radians while its angular speed increases from $10\,\text{rad s}^{-1}$ to $25\,\text{rad s}^{-1}$.

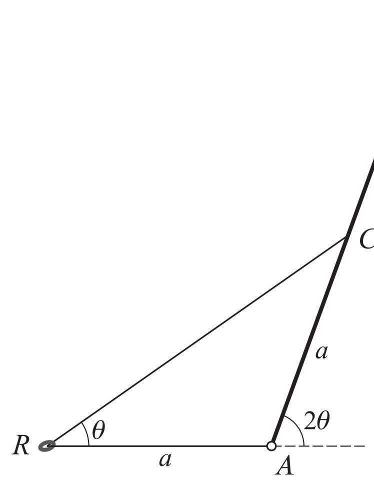
(i) Find the moment of the couple about the axis. [5]

When the flywheel is rotating with angular speed $25\,\text{rad s}^{-1}$, it locks together with a second flywheel which is mounted on the same axis and is at rest. Immediately afterwards, both flywheels rotate together with the same angular speed $9\,\text{rad s}^{-1}$.

(ii) Find the moment of inertia of the second flywheel about the axis. [3]

3 The region bounded by the x-axis, the lines $x = 1$ and $x = 2$ and the curve $y = \dfrac{1}{x^2}$ for $1 \leqslant x \leqslant 2$, is occupied by a uniform lamina of mass 24 kg. The unit of length is the metre. Find the moment of inertia of this lamina about the x-axis. [8]

4



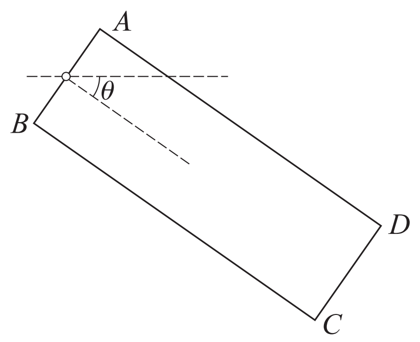
A uniform rod AB, of mass m and length $2a$, is freely hinged to a fixed point at A. A particle of mass $2m$ is attached to the rod at B. A light elastic string, with natural length a and modulus of elasticity $5mg$, passes through a fixed smooth ring R. One end of the string is fixed to A and the other end is fixed to the mid-point C of AB. The ring R is at the same horizontal level as A, and is at a distance a from A. The rod AB and the ring R are in a vertical plane, and RC is at an angle θ above the horizontal, where $0 < \theta < \frac{1}{4}\pi$, so that the acute angle between AB and the horizontal is 2θ (see diagram).

(i) By considering the energy of the system, find the value of θ for which the system is in equilibrium. [7]

(ii) Determine whether this position of equilibrium is stable or unstable. [3]

5 A uniform rectangular lamina $ABCD$ has mass 20 kg and sides of lengths $AB = 0.6$ m and $BC = 1.8$ m. It rotates in its own vertical plane about a fixed horizontal axis which is perpendicular to the lamina and passes through the mid-point of AB.

(i) Show that the moment of inertia of the lamina about the axis is 22.2 kg m^2. [3]

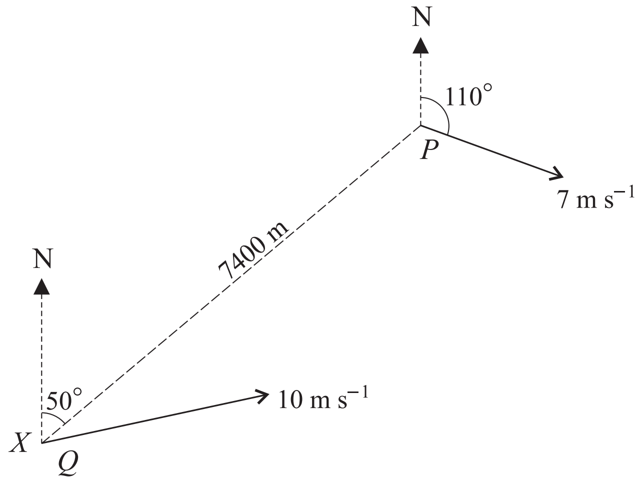


The lamina is released from rest with BC horizontal and below the level of the axis. Air resistance may be neglected, but a frictional couple opposes the motion. The couple has constant moment 44.1 N m about the axis. The angle through which the lamina has turned is denoted by θ (see diagram).

(ii) Show that the angular acceleration is zero when $\cos\theta = 0.25$. [3]

(iii) Hence find the maximum angular speed of the lamina. [5]

6



A ship P is moving with constant velocity 7 m s^{-1} in the direction with bearing 110°. A second ship Q is moving with constant speed 10 m s^{-1} in a straight line. At one instant Q is at the point X, and P is 7400 m from Q on a bearing of 050° (see diagram). In the subsequent motion, the shortest distance between P and Q is 1790 m.

(i) Show that one possible direction for the velocity of Q relative to P has bearing 036°, to the nearest degree, and find the bearing of the other possible direction of this relative velocity. [3]

Given that the velocity of Q relative to P has bearing 036°, find

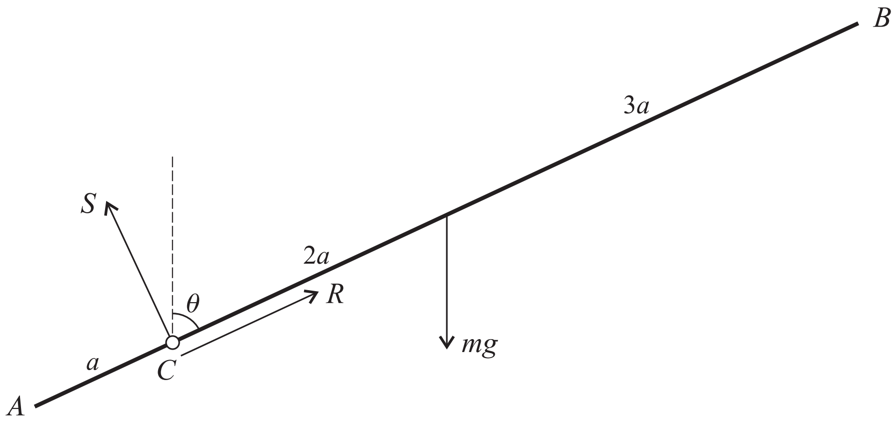
(ii) the bearing of the direction in which Q is moving, [4]

(iii) the magnitude of the velocity of Q relative to P, [2]

(iv) the time taken for Q to travel from X to the position where the two ships are closest together, [3]

(v) the bearing of P from Q when the two ships are closest together. [1]

7



A uniform rod AB has mass m and length $6a$. It is free to rotate in a vertical plane about a smooth fixed horizontal axis passing through the point C on the rod, where $AC = a$. The angle between AB and the upward vertical is θ, and the force acting on the rod at C has components R parallel to AB and S perpendicular to AB (see diagram). The rod is released from rest in the position where $\theta = \frac{1}{3}\pi$. Air resistance may be neglected.

(i) Find the angular acceleration of the rod in terms of a, g and θ. [4]

(ii) Show that the angular speed of the rod is $\sqrt{\dfrac{2g(1 - 2\cos\theta)}{7a}}$. [3]

(iii) Find R and S in terms of m, g and θ. [6]

(iv) When $\cos\theta = \frac{1}{3}$, show that the force acting on the rod at C is vertical, and find its magnitude. [4]