RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4727

Further Pure Mathematics 3

Thursday **15 JUNE 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

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1 **(a)** For the infinite group of non-zero complex numbers under multiplication, state the identity element and the inverse of $1 + 2\mathrm{i}$, giving your answers in the form $a + \mathrm{i}b$. [3]

(b) For the group of matrices of the form $\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}$ under matrix addition, where $a \in \mathbb{R}$, state the identity element and the inverse of $\begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix}$. [2]

2 **(a)** Given that $z_1 = 2\mathrm{e}^{\frac{1}{6}\pi\mathrm{i}}$ and $z_2 = 3\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}}$, express $z_1 z_2$ and $\dfrac{z_1}{z_2}$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $0 \leqslant \theta < 2\pi$. [4]

(b) Given that $w = 2(\cos\frac{1}{8}\pi + \mathrm{i}\sin\frac{1}{8}\pi)$, express w^{-5} in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $r > 0$ and $0 \leqslant \theta < 2\pi$. [3]

3 Find the perpendicular distance from the point with position vector $12\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$ to the line with equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + t(8\mathbf{i} + 3\mathbf{j} - 6\mathbf{k})$. [6]

4 Find the solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{x^2 y}{1 + x^3} = x^2$$

for which $y = 1$ when $x = 0$, expressing your answer in the form $y = \mathrm{f}(x)$. [8]

5 A line l_1 has equation $\dfrac{x}{2} = \dfrac{y+4}{3} = \dfrac{z+9}{5}$.

(i) Find the cartesian equation of the plane which is parallel to l_1 and which contains the points $(2, 1, 5)$ and $(0, -1, 5)$. [5]

(ii) Write down the position vector of a point on l_1 with parameter t. [1]

(iii) Hence, or otherwise, find an equation of the line l_2 which intersects l_1 at right angles and which passes through the point $(-5, 3, 4)$. Give your answer in the form $\dfrac{x-a}{p} = \dfrac{y-b}{q} = \dfrac{z-c}{r}$. [4]

6 **(i)** Find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4y = \sin x. \qquad [6]$$

(ii) Find the solution of the differential equation for which $y = 0$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{4}{3}$ when $x = 0$. [4]

7 The series C and S are defined for $0 < \theta < \pi$ by

$$C = 1 + \cos\theta + \cos 2\theta + \cos 3\theta + \cos 4\theta + \cos 5\theta,$$
$$S = \sin\theta + \sin 2\theta + \sin 3\theta + \sin 4\theta + \sin 5\theta.$$

(i) Show that $C + \mathrm{i}S = \dfrac{\mathrm{e}^{3\mathrm{i}\theta} - \mathrm{e}^{-3\mathrm{i}\theta}}{\mathrm{e}^{\frac{1}{2}\mathrm{i}\theta} - \mathrm{e}^{-\frac{1}{2}\mathrm{i}\theta}}\,\mathrm{e}^{\frac{5}{2}\mathrm{i}\theta}$. [4]

(ii) Deduce that $C = \sin 3\theta \cos \frac{5}{2}\theta \operatorname{cosec} \frac{1}{2}\theta$ and write down the corresponding expression for S. [4]

(iii) Hence find the values of θ, in the range $0 < \theta < \pi$, for which $C = S$. [4]

8 A group D of order 10 is generated by the elements a and r, with the properties $a^2 = e$, $r^5 = e$ and $r^4 a = ar$, where e is the identity. Part of the operation table is shown below.

	e	a	r	r^2	r^3	r^4	ar	ar^2	ar^3	ar^4
e	e	a	r	r^2	r^3	r^4	ar	ar^2	ar^3	ar^4
a	a	e	ar	ar^2	ar^3	ar^4				
r	r		r^2	r^3	r^4	e				
r^2	r^2		r^3	r^4	e	r				
r^3	r^3		r^4	e	r	r^2				
r^4	r^4	ar	e	r	r^2	r^3				
ar	ar		ar^2	ar^3	ar^4	a				
ar^2	ar^2		ar^3	ar^4	a	ar			**E**	
ar^3	ar^3		ar^4	a	ar	ar^2				
ar^4	ar^4		a	ar	ar^2	ar^3				

(i) Give a reason why D is not commutative. [1]

(ii) Write down the orders of any possible proper subgroups of D. [2]

(iii) List the elements of a proper subgroup which contains

(a) the element a, [1]

(b) the element r. [1]

(iv) Determine the order of each of the elements r^3, ar and ar^2. [4]

(v) Copy and complete the section of the table marked **E**, showing the products of the elements ar, ar^2, ar^3 and ar^4. [5]

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