OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4725

Further Pure Mathematics 1

Thursday **8 JUNE 2006** Morning 1 hour 30 minutes

Additional materials:
- 8 page answer booklet
- Graph paper
- List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

1 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 0 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$.

(i) Find $\mathbf{A} + 3\mathbf{B}$. [2]

(ii) Show that $\mathbf{A} - \mathbf{B} = k\mathbf{I}$, where $\mathbf{I}$ is the identity matrix and k is a constant whose value should be stated. [2]

2 The transformation S is a shear parallel to the x-axis in which the image of the point (1, 1) is the point (0, 1).

(i) Draw a diagram showing the image of the unit square under S. [2]

(ii) Write down the matrix that represents S. [2]

3 One root of the quadratic equation $x^2 + px + q = 0$, where p and q are real, is the complex number $2 - 3\mathrm{i}$.

(i) Write down the other root. [1]

(ii) Find the values of p and q. [4]

4 Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} (r^3 + r^2) = \tfrac{1}{12}n(n+1)(n+2)(3n+1).$$ [5]

5 The complex numbers $3 - 2\mathrm{i}$ and $2 + \mathrm{i}$ are denoted by z and w respectively. Find, giving your answers in the form $x + \mathrm{i}y$ and showing clearly how you obtain these answers,

(i) $2z - 3w$, [2]

(ii) $(\mathrm{i}z)^2$, [3]

(iii) $\dfrac{z}{w}$. [3]

6 In an Argand diagram the loci C_1 and C_2 are given by

$$|z| = 2 \qquad \text{and} \qquad \arg z = \tfrac{1}{3}\pi$$

respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [5]

(ii) Hence find, in the form $x + \mathrm{i}y$, the complex number representing the point of intersection of C_1 and C_2. [2]

7 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$.

(i) Find $\mathbf{A}^2$ and $\mathbf{A}^3$. [3]

(ii) Hence suggest a suitable form for the matrix $\mathbf{A}^n$. [1]

(iii) Use induction to prove that your answer to part **(ii)** is correct. [4]

8 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} a & 4 & 2 \\ 1 & a & 0 \\ 1 & 2 & 1 \end{pmatrix}$.

(i) Find, in terms of a, the determinant of $\mathbf{M}$. [3]

(ii) Hence find the values of a for which $\mathbf{M}$ is singular. [3]

(iii) State, giving a brief reason in each case, whether the simultaneous equations

$$\begin{aligned} ax + 4y + 2z &= 3a, \\ x + ay \qquad &= 1, \\ x + 2y + \ z &= 3, \end{aligned}$$

have any solutions when

(a) $a = 3$,

(b) $a = 2$.

[4]

9 **(i)** Use the method of differences to show that

$$\sum_{r=1}^{n} \left\{ (r+1)^3 - r^3 \right\} = (n+1)^3 - 1.$$ [2]

(ii) Show that $(r+1)^3 - r^3 \equiv 3r^2 + 3r + 1$. [2]

(iii) Use the results in parts **(i)** and **(ii)** and the standard result for $\sum_{r=1}^{n} r$ to show that

$$3 \sum_{r=1}^{n} r^2 = \tfrac{1}{2} n(n+1)(2n+1).$$ [6]

10 The cubic equation $x^3 - 2x^2 + 3x + 4 = 0$ has roots α, β and γ.

(i) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. [3]

The cubic equation $x^3 + px^2 + 10x + q = 0$, where p and q are constants, has roots $\alpha + 1$, $\beta + 1$ and $\gamma + 1$.

(ii) Find the value of p. [3]

(iii) Find the value of q. [5]

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