

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**Advanced Subsidiary General Certificate of Education  
Advanced General Certificate of Education**

**MATHEMATICS**

**4724**

Core Mathematics 4

Monday            **12 JUNE 2006**            Afternoon            1 hour 30 minutes

Additional materials:  
8 page answer booklet  
Graph paper  
List of Formulae (MF1)

**TIME**    1 hour 30 minutes

**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

---

**This question paper consists of 3 printed pages and 1 blank page.**

- 1 Find the gradient of the curve  $4x^2 + 2xy + y^2 = 12$  at the point  $(1, 2)$ . [4]
- 2 (i) Expand  $(1 - 3x)^{-2}$  in ascending powers of  $x$ , up to and including the term in  $x^2$ . [3]
- (ii) Find the coefficient of  $x^2$  in the expansion of  $\frac{(1 + 2x)^2}{(1 - 3x)^2}$  in ascending powers of  $x$ . [4]
- 3 (i) Express  $\frac{3 - 2x}{x(3 - x)}$  in partial fractions. [3]
- (ii) Show that  $\int_1^2 \frac{3 - 2x}{x(3 - x)} dx = 0$ . [4]
- (iii) What does the result of part (ii) indicate about the graph of  $y = \frac{3 - 2x}{x(3 - x)}$  between  $x = 1$  and  $x = 2$ ? [1]
- 4 The position vectors of three points  $A$ ,  $B$  and  $C$  relative to an origin  $O$  are given respectively by
- $$\begin{aligned}\overrightarrow{OA} &= 7\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}, \\ \overrightarrow{OB} &= 4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k} \\ \text{and } \overrightarrow{OC} &= 5\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}.\end{aligned}$$
- (i) Find the angle between  $AB$  and  $AC$ . [6]
- (ii) Find the area of triangle  $ABC$ . [2]
- 5 A forest is burning so that,  $t$  hours after the start of the fire, the area burnt is  $A$  hectares. It is given that, at any instant, the rate at which this area is increasing is proportional to  $A^2$ .
- (i) Write down a differential equation which models this situation. [2]
- (ii) After 1 hour, 1000 hectares have been burnt; after 2 hours, 2000 hectares have been burnt. Find after how many hours 3000 hectares have been burnt. [6]
- 6 (i) Show that the substitution  $u = e^x + 1$  transforms  $\int \frac{e^{2x}}{e^x + 1} dx$  to  $\int \frac{u - 1}{u} du$ . [3]
- (ii) Hence show that  $\int_0^1 \frac{e^{2x}}{e^x + 1} dx = e - 1 - \ln\left(\frac{e + 1}{2}\right)$ . [5]

7 Two lines have vector equations

$$\mathbf{r} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k} + \lambda(3\mathbf{i} + \mathbf{j} + a\mathbf{k}) \quad \text{and} \quad \mathbf{r} = -8\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(\mathbf{i} - 2\mathbf{j} - \mathbf{k}),$$

where  $a$  is a constant.

(i) Given that the lines are skew, find the value that  $a$  cannot take. [6]

(ii) Given instead that the lines intersect, find the point of intersection. [2]

8 (i) Show that  $\int \cos^2 6x \, dx = \frac{1}{2}x + \frac{1}{24} \sin 12x + c$ . [3]

(ii) Hence find the exact value of  $\int_0^{\frac{1}{12}\pi} x \cos^2 6x \, dx$ . [6]

9 A curve is given parametrically by the equations

$$x = 4 \cos t, \quad y = 3 \sin t,$$

where  $0 \leq t \leq \frac{1}{2}\pi$ .

(i) Find  $\frac{dy}{dx}$  in terms of  $t$ . [3]

(ii) Show that the equation of the tangent at the point  $P$ , where  $t = p$ , is

$$3x \cos p + 4y \sin p = 12. \quad [3]$$

(iii) The tangent at  $P$  meets the  $x$ -axis at  $R$  and the  $y$ -axis at  $S$ .  $O$  is the origin. Show that the area of triangle  $ORS$  is  $\frac{12}{\sin 2p}$ . [3]

(iv) Write down the least possible value of the area of triangle  $ORS$ , and give the corresponding value of  $p$ . [3]

BLANK PAGE

---

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (OCR) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

OCR is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.