OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE **A2 7890 - 2**

Advanced Subsidiary GCE **AS 3890 - 2**

Combined Mark Schemes And Report on the Units

January 2006

3890-2/7890-2/MS/R/06J

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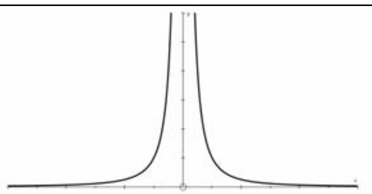
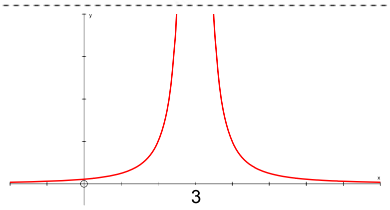
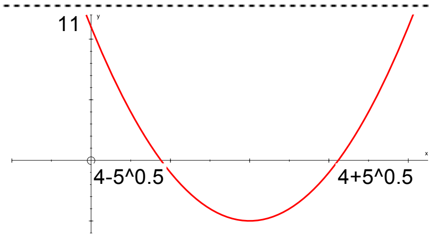
MARK SCHEME ON THE UNITS

REPORTS FOR THE UNITS

Mark Scheme 4721
January 2006

1	(i)	$x^{\frac{1}{3}} = 2$ $x = 8$	B1	**1**	(allow embedded values throughout question 1) 8
	(ii)	$10^t = 1$ $t = 0$	B1	**1**	0
	(iii)	$\left(y^{-2}\right)^2 = \frac{1}{81}$ $y^{-4} = \frac{1}{81}$ $y = \pm 3$	B1 B1	**2**	$y = 3$ $y = -3$
2	(i)	$(3x+1)^2 - 2(2x-3)^2$ $= (9x^2+6x+1) - 2(4x^2-12x+9)$ $= x^2 + 30x - 17$	M1 A1 A1	**3**	Square to get at least one 3 or 4 term quadratic $9x^2+6x+1$ or $4x^2-12x+9$ soi $x^2+30x-17$
	(ii)	$2x^3 + 6x^3 + 4x^3 = 12x^3$ 12	B1 B1	**2**	2 of $2x^3$, $6x^3$, $4x^3$ soi **N.B. www for these terms,** must be positive 12 or $12x^3$
3	(i)	$\frac{dy}{dx} = 15x^4 - \frac{1}{2}x^{-\frac{1}{2}}$	B1 B1 B1	**3**	$15x^4$ $kx^{-\frac{1}{2}}$ $cx^4 - \frac{1}{2}x^{-\frac{1}{2}}$ only
	(ii)	$\frac{d^2y}{dx^2} = 60x^3 + \frac{1}{4}x^{-\frac{3}{2}}$	M1 A1	**2**	Attempt to differentiate their 2 term $\frac{dy}{dx}$ and get one correctly differentiated term $60x^3 + \frac{1}{4}x^{-\frac{3}{2}}$
4	(i)		B1 B1	**2**	Correct curve in one quadrant Completely correct
	(ii)		M1 A1√	**2**	Translate (i) horizontally Translates all of their (i) $\begin{pmatrix}3\\0\end{pmatrix}$ 3 must be labelled or stated
	(iii)	(One-way) stretch, sf 2, parallel to the y-axis	B1 B1 B1	**3**	Stretch (Scale) factor 2 Parallel to y-axis o.e. **SR** Stretch B1 Sf $\sqrt{2}$ parallel to x-axis B2

5	(i)	$x^2+3x=\left(x+\frac{3}{2}\right)^2-\frac{9}{4}$	B1 B1	2	$a=\frac{3}{2}$ $b=-\frac{9}{4}$ o.e.
	(ii)	$y^2-4y-\frac{11}{4}=(y-2)^2-\frac{27}{4}$	B1 B1	2	$p=-2$ $q=-\frac{27}{4}$ o.e.
	(iii)	Centre $\left(-\frac{3}{2},2\right)$	B1√	1	$\left(-\frac{3}{2},2\right)$ N.B. If question is restarted in this part, ft from part (iii) working only
	(iv)	Radius $=\sqrt{\frac{27}{4}+\frac{9}{4}}$ $=\sqrt{9}$ $=3$	M1 A1	2	$\sqrt{-their'b'-their'q'}$ or use $\sqrt{(f^2+g^2-c)}$ 3 (±3 scores A0)
6	(i)	$y=x^3-3x^2+4$ $\frac{dy}{dx}=3x^2-6x$ $3x^2-6x=0$ $3x(x-2)=0$ $x=0 \quad x=2$ $y=4 \quad y=0$	B1 B1 M1 M1 A1 A1√	6	$3x^2-6x$ 1 term correct Completely correct $\frac{dy}{dx}=0$ Correct method to solve quadratic $x=0, 2$ $y=4, 0$ **SR** one correct (x,y) pair **www** **B1**
	(ii)	$\frac{d^2y}{dx^2}=6x-6$ $x=0 \quad y''=-6 \quad -\text{ve max}$ $x=2 \quad y''=6 \quad +\text{ve min}$	M1 B1 B1	3	Correct method to find nature of stationary points (can be a sketch) $x=0$ max $x=2$ min (N.B. If no method shown but both min and max correctly stated, award all 3 marks)
	(iii)	Increasing $x<0 \quad x>2$	M1 A1	2	Any inequality (or inequalities) involving both their x values from part (i) Allow $x\leq 0 \quad x\geq 2$

7	(i)	$x=\dfrac{8\pm\sqrt{64-44}}{2}$ $=\dfrac{8\pm\sqrt{20}}{2}$ $=4\pm\sqrt{5}$	M1 A1 B1 A1	4	Correct use of formula $\dfrac{8\pm\sqrt{20}}{2}$ aef $\sqrt{20}=2\sqrt{5}$ soi $4\pm\sqrt{5}$ <u>Alternative method</u> $(x-4)^2-16+11=0$ M1 $(x-4)^2=5$ A1 $x=4+\sqrt{5}$ A1 or $4-\sqrt{5}$ A1
	(ii)		B1 B1√ B1	3	+ve parabola Root(s) in correct places Completely correct curve with roots and (0, 11) labelled or referenced
	(iii)	$y=x^2=\left(4\pm\sqrt{5}\right)^2$ $=16+5\pm8\sqrt{5}$ $=21\pm8\sqrt{5}$	M1 M1 A1√ A1	4	$y=x^2$ soi Attempt to square at least one answer from part (i) Correct evaluation of $(a+b\sqrt{c})^2$ $(a,b,c\neq0)$ $21\pm8\sqrt{5}$

8	(i)	$y = x^2 - 5x + 15$ $y = 5x - 10$ $x^2 - 5x + 15 = 5x - 10$ $x^2 - 10x + 25 = 0$	M1 A1	 **2**	Attempt to eliminate y $x^2 - 10x + 25 = 0$ **AG** Obtained with no wrong working seen
	(ii)	$b^2 - 4ac = 100 - 100$ $= 0$	B1	**1**	0 Do not allow $\sqrt{(b^2 - 4ac)}$
	(iii)	Line is a tangent to the curve	B1√	**1**	Tangent or 'touches' N.B. Strict ft from their discriminant
	(iv)	$x^2 - 10x + 25 = 0$ $(x-5)^2 = 0$ $x = 5 \quad y = 15$	M1 A1 A1	 **3**	Correct method to solve 3 term quadratic $x = 5$ $y = 15$
	(v)	Gradient of tangent = 5 Gradient of normal = $-\frac{1}{5}$ $y - 15 = -\frac{1}{5}(x - 5)$ $x + 5y = 80$	B1 B1√ M1 A1	 **4**	Gradient of tangent = 5 Gradient of normal = $-\frac{1}{5}$ Correct equation of straight line, any gradient, passing through (5, 15) $x + 5y = 80$

9	(i)	Length AC = $\sqrt{(8-5)^2+(2-1)^2}$ $=\sqrt{3^2+1^2}$ $=\sqrt{10}$	M1 A1		Uses $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\sqrt{10}$ (± $\sqrt{10}$ scores A0)
		Length AB = $\sqrt{(p-5)^2+(7-1)^2}$ $=\sqrt{(p-5)^2+36}$	A1		$\sqrt{(p-5)^2+(7-1)^2}$
		$\sqrt{(p-5)^2+36}=2\sqrt{10}$ $p^2-10p+25+36=40$	M1		AB = 2AC (with algebraic expression) used
		$p^2-10p+21=0$ $(p-7)(p-3)=0$	M1		Obtains 3 term quadratic = 0 suitable for solving <u>or</u> $(p-5)^2=4$
		$p=7,3$	A1 A1	7	$p=7$ $p=3$ **SR <u>If no working seen</u>, and one correct value found, award B2 in place of the final 4 marks in part (i)**
	(ii)	$7 = 3x - 14$ $x=7$	M1 A1		Correct method to find x $x = 7$
		(5, 1) (7, 7)	M1		Use $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$
		Mid-point (6, 4)	A1√	4	(6, 4) or correct midpoint for their AB <u>Alternative method</u> y coordinate of midpoint = 4 M1 A1 sub 4 into equation of line M1 obtains $x = 6$ A1

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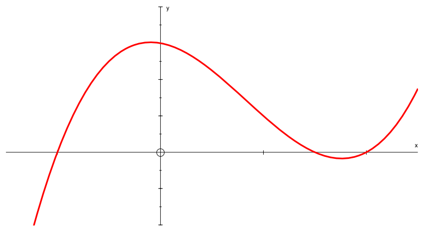
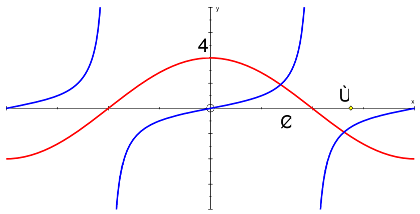
1	(i)	$a+19d=10, \quad a+49d=70$	M1		Attempt to find d from simultaneous equations involving $a+(n-1)d$ or equiv method
		Hence $30d=60 \Rightarrow d=2$	A1		Obtain $d = 2$
		a + (19 x 2) = 10 or a + (49 x 2) = 70	M1		Attempt to find a from $a+(n-1)d$ or equiv
		Hence $a = -28$	A1	**4**	Obtain $a = -28$
	(ii)	$S=\frac{29}{2}\left(2\times -28+(29-1)\times 2\right)=0$	M1		For relevant use of $\frac{1}{2}n\left(2a+(n-1)d\right)$
			A1	**2**	For showing the given result correctly **AG**
				6	
2	(i)	$\Delta=\frac{1}{2}\times 10\times 7\times \sin 80=34.5\text{cm}^2$	M1		For use of $\frac{1}{2}ca\sin B$ or complete equiv.
			A1	**2**	For correct value 34.5
	(ii)	$b^2=10^2+7^2-2\times 10\times 7\times \cos 80$	M1		For attempted use of the correct cosine formula
		Hence length of *CA* is 11.2 cm	A1	**2**	For correct value 11.2
	(iii)	$\sin C=\frac{10\sin 80}{11.166...}=0.8819...$	M1		For use of the sine rule to find *C*, or equivalent
		Hence angle *C* is 61.9°	A1	**2**	For correct value 61.9
				6	
3	(i)	$(1-2x)^{12}=1-24x+264x^2$	B1		Obtain 1 and $-24x$ …
			M1		Attempt x^2 term, including attempt at binomial coeff.
			A1	**3**	Obtain …$264x^2$
	(ii)	$(1\times 264)+(3\times -24)=192$	M1		Attempt coefficient of x^2 from two pairs of terms
			A1√		Obtain correct unsimplified expression
			A1	**3**	Obtain 192
				6	
4	(i)	$\text{perimeter}=(15\times 1.8)+(20\times 1.8)+5+5$	M1		Use $r\theta$ at least once
			A1		Obtain at least one of 27cm or 36cm
		$=73\text{cm}$	A1	**3**	Obtain 73
	(ii)	$\text{area}=\left(\frac{1}{2}\times 20^2\times 1.8\right)-\left(\frac{1}{2}\times 15^2\times 1.8\right)$	M1		Attempt area of sector using $kr^2\theta$
			M1		Find difference between attempts at two sectors
		$=157.5\text{cm}^2$	A1	**3**	Obtain 157.5 / 158
				6	

5	(i)	$r = \frac{4.8}{5} = 0.96 \Rightarrow S_\infty = \frac{5}{0.04} = 125$	B1* B1 dep*	2	For correct value of r used For correct use of $\frac{a}{1-r}$ to show given answer **AG**
	(ii)	$S_n = \frac{5(1-0.96^n)}{1-0.96}$ Hence $1-0.96^n > 0.992 \Rightarrow 0.96^n < 0.008$ $n\log 0.96 < \log 0.008$ Hence $n > \frac{\log 0.008}{\log 0.96} \approx 118.3$ Least value of n is 119	B1 M1 A1 B1 M1 A1	6 **8**	For correct, unsimplified, S_n For linking S_n to 124 (> or =) and multiplying through by 0.04, or equiv. For showing the given result correctly, with correct inequality throughout **AG** For correct log statement seen or implied (ignore sign) For dividing both sides by log 0.96 For correct (integer) value 119
6	(a)	$\frac{2}{3}x^{\frac{3}{2}} + 4x + c$	M1 A1 B1 B1	4	For $kx^{\frac{3}{2}}$ For correct first term $\frac{2}{3}x^{\frac{3}{2}}$, or equiv For correct second term $4x$ For $+c$
	(b)(i)	$\int_1^a 4x^{-2}\,dx = \left[-4x^{-1}\right]_1^a$ $= 4 - \frac{4}{a}$	M1 M1 A1	3	Obtain integral of the form kx^{-1} Use limits $x = a$ and $x = 1$ Obtain $= 4 - \frac{4}{a}$, or equivalent
	(ii)	4	B1√	1 **8**	State 4, or legitimate conclusion from their (b)(i)
7	(i)(a)	$\log_{10}x - \log_{10}y$	B1	1	For the correct answer
	(b)	$1 + 2\log_{10}x + \log_{10}y$	M1 A1 A1	3	Sum of three log terms involving 10, x^2, y For correct term $2\log_{10}x$ For both correct terms 1 and $\log_{10}y$
	(ii)	$2\log_{10}x - 2\log_{10}y = 2 + 2\log_{10}x + \log_{10}y$ Hence $3\log_{10}y = -2$ So $y = 10^{-\frac{2}{3}} \approx 0.215$	M1 A1 M1 A1	4 **8**	For relevant use of results from (i) For a correct, unsimplified, equation in $\log_{10}y$ only For correct use of $a = \log_{10} c \Leftrightarrow c = 10^a$ For the correct value 0.215

8	(i)	$-2+k+1+6=0 \Rightarrow k=-5$	M1 A1		For attempting f(–1) For equating f(-1) to 0 and deducing the correct value of *k* **AG**
		OR	M1 A1		Match coefficients and attempt *k* Show *k* = -5
		OR	B2		Following division, state remainder is 0, hence (*x* + 1) **is** a factor, hence *k* = -5
		EITHER: $(x+1)(2x^2-7x+6)$	B1 M1		For correct leading term $2x^2$ For attempt at complete division by f(*x*) by (*x* + 1) or equiv.
		$=(x+1)(x-2)(2x-3)$	A1 A1		For completely correct quadratic factor For all three factors correct
		OR: f(2) = 16 – 20 – 2 + 6 = 0 Hence (*x* – 2) is a factor Third factor is (2*x* – 3) Hence f(*x*) $=(x+1)(x-2)(2x-3)$	M1 A1 M1 A1	6	For further relevant use of the factor theorem For correct identification of factor (*x* – 2) For any method for the remaining factor For all three factors correct
	(ii)	$\int_{-1}^{2} f(x)dx = \left[\frac{1}{2}x^4 - \frac{5}{3}x^3 - \frac{1}{2}x^2 + 6x\right]_{-1}^{2}$	B1√ B1√		For any two terms integrated correctly For all four terms integrated correctly
		$=\left(8-\frac{40}{3}-2+12\right)-\left(\frac{1}{2}+\frac{5}{3}-\frac{1}{2}-6\right)$	M1		For evaluation of F(2) – F(–1)
		= 9	A1	4	For correct value 9
	(iii)		B1 B1	2	For sketch of positive cubic, with three distinct, non-zero, roots For correct explanation that some of the area is below the axis
				12	
9	(i)		B1 B1 B1	3	For correct sketch of one curve For correct shape and location of second curve, on same diagram For intercept 4 on *y*-axis
	(ii)	(See diagram above) $\beta = 180 - \alpha$	B1 M1 A1	3	For correct identification of intersections – in correct order For attempt to use symmetry of the graphs For the correct (explicit) answer for β
	(iii)	$\sin x = 4\cos^2 x = 4(1-\sin^2 x)$	M1 M1		For use of $\tan x = \frac{\sin x}{\cos x}$ For use of $\cos^2 x = 1 - \sin^2 x$
		Hence $4\sin^2 x + \sin x - 4 = 0$	A1		For showing the given equation correctly
		$\sin x = \frac{-1 \pm \sqrt{65}}{8}$	B1		For correct solution of quadratic
		Hence $\beta - \alpha = 118.02... - 61.97... \approx 56^{\circ}$	M1 A1	6	Attempt value for *x* from their solutions For the correct value 56
				1	

				2	

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1	Obtain integral of form $k \ln x$	**M1**		[any non-zero constant k; or equiv such as $k \ln 3x$]
	Obtain $3 \ln 8 - 3 \ln 2$	**A1**		[or exact equiv]
	Attempt use of at least one relevant log property	**M1**		[would be earned by initial $\ln x^3$]
	Obtain $3 \ln 4$ or $\ln 8^3 - \ln 2^3$ and hence ln 64	**A1**	**4**	[**AG**; with no errors]
2	Attempt use of identity linking $\sec^2\theta$, $\tan^2\theta$ and 1	**M1**		[to write eqn in terms of tan θ]
	Obtain $\tan^2\theta - 4\tan\theta + 3 = 0$	**A1**		[or correct unsimplified equiv]
	Attempt solution of quadratic eqn to find two values of tan θ	**M1**		[any 3 term quadratic eqn in tan θ]
	Obtain at least two correct answers	**A1**		[after correct solution of eqn]
	Obtain all four of 45, 225, 71.6, 251.6	**A1**	**5**	[allow greater accuracy or angles to nearest degree – and no other answers between 0 and 360]
3 (a)	Attempt use of product rule	**M1**		[involving … + …]
	Obtain $2x(x+1)^6$ …	**A1**		
	Obtain … + $6x^2(x+1)^5$	**A1**	**3**	[or equivs; ignore subsequent attempt at simplification]
(b)	Attempt use of quotient rule	**M1**		[or, with adjustment, product rule; allow u / v confusion]
	Obtain $\dfrac{(x^2-3)\,2x-(x^2+3)\,2x}{(x^2-3)^2}$	**A1**		[or equiv]
	Obtain –3	**A1**	**3**	[from correct derivative only]
4 (i)	State $y \le 2$	**B1**	**1**	[or equiv; allow <; allow any letter or none]
(ii)	Show correct process for composition of functions	**M1**		[numerical or algebraic]
	Obtain 0 and hence 2	**A1**	**2**	[and no other value]
(iii)	State a range of values with 2 as one end-point	**M1**		[continuous set, not just integers]
	State $0 < k \le 2$	**A1**	**2**	[with correct < and ≤ now]
5	Obtain integral of form $k(1-2x)^6$	**M1**		[any non-zero constant k]
	Obtain correct $-\frac{1}{12}(1-2x)^6$	**A1**		[or unsimplified equiv; allow + c]
	Use limits to obtain $\frac{1}{12}$	**A1**		[or exact (unsimplified) equiv]
	Obtain integral of form $k\,e^{2x-1}$	**M1**		[or equiv; any non-zero constant k]
	Obtain correct $\frac{1}{2}e^{2x-1} - x$	**A1**		[or equiv; allow + c]
	Use limits to obtain $-\frac{1}{2}e^{-1}$	**A1**		[or exact (unsimplified) equiv]
	Show correct process for finding required area	**M1**		[at any stage of solution; if process involves two definite integrals, second must be negative]
	Obtain $\frac{1}{12} + \frac{1}{2}e^{-1}$	**A1**	**8**	[or exact equiv; no + c]

6 (a)	Either: State proportion $\frac{440}{275}$	**B1**		
	Attempt calculation involving proportion	**M1**		[involving multn and *X* value]
	Obtain 704	**A1**	**3**	
	Or: Use formula of form $275\mathrm{e}^{kt}$ or $275a^t$	**M1**		[or equiv]
	Obtain $k = 0.047$ or $a = \sqrt[10]{1.6}$	**A1**		[or equiv]
	Obtain 704	**A1**	**(3)**	[allow ±0.5]
(b)(i)	Attempt correct process involving logarithm	**M1**		[or equiv including systematic trial and improvement attempt]
	Obtain $\ln\frac{20}{80} = -0.02t$	**A1**		[or equiv]
	Obtain 69	**A1**	**3**	[or greater accuracy; scheme for T&I: M1A2]
(ii)	Differentiate to obtain $k\,\mathrm{e}^{-0.02t}$	**M1**		[any constant *k* different from 80]
	Obtain $-1.6\mathrm{e}^{-0.02t}$ (or $1.6\mathrm{e}^{-0.02t}$)	**A1**		[or unsimplified equiv]
	Obtain 0.88	**A1**	**3**	[or greater accuracy; allow –0.88]

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7 (i)	Sketch curve showing (at least) translation in x direction	**M1**		[either positive or negative]
	Show correct sketch with one of 2 and 3π indicated	**A1**		
	… and with other one of 2 and 3π indicated	**A1**	**3**	
(ii)	Draw straight line through *O* with positive gradient	**B1**	**1**	[label and explanation not required]
(iii)	Attempt calculations using 1.8 and 1.9	**M1**		[allow here if degrees used]
	Obtain correct values and indicate change of sign	**A1**	**2**	[or equiv; $x = 1.8$: LHS = 1.93, diff = 0.13; $x = 1.9$: LHS = 1.35, diff = -0.55; radians needed now]
(iv)	Obtain correct first iterate 1.79 or 1.78	**B1**		[or greater accuracy]
	Attempt correct process to produce at least 3 iterates	**M1**		
	Obtain 1.82	**A1**		[answer required to exactly 2 d.p.; $2 \to 1.7859 \to 1.8280 \to 1.8200$; SR: answer 1.82 only - B2]
	Attempt rearrangement of $3\cos^{-1}(x-1) = x$ or of $x = 1 + \cos(\frac{1}{3}x)$	**M1**		[involving at least two steps]
	Obtain required formula or equation respectively	**A1**	**5**	

8	(i)	Differentiate to obtain $kx(5-x^2)^{-1}$	M1		[any non-zero constant]
		Obtain correct $-2x(5-x^2)^{-1}$	A1		[or equiv]
		Obtain –4 for value of derivative	A1		
		Attempt equation of straight line through (2, 0) with numerical value of gradient obtained from attempt at derivative	M1		[not for attempt at eqn of normal]
		Obtain $y = -4x + 8$	A1	5	[or equiv]
	(ii)	State or imply $h = \frac{1}{2}$	B1		
		Attempt calculation involving attempts at y values	M1		[addition with each of coefficients 1, 2, 4 occurring at least once]
		Obtain $k(\ln 5 + 4\ln 4.75 + 2\ln 4 + 4\ln 2.75 + \ln 1)$	A1		[or equiv perhaps with decimals; any constant k]
		Obtain 2.44	A1	4	[allow ±0.01]
	(iii)	Attempt difference of two areas	M1		[allow if area of their triangle < area A]
		Obtain 8 – 2.44 and hence 5.56	A1√	2	[following their tangent and area of A providing answer positive]

9	(i)	State $\sin 2\theta \cos\theta + \cos 2\theta \sin\theta$	B1		
		Use at least one of $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = 1 - 2\sin^2\theta$	B1		
		Attempt complete process to express in terms of $\sin\theta$	M1		[using correct identities]
		Obtain $3\sin\theta - 4\sin^3\theta$	A1	4	[**AG**; all correctly obtained]
	(ii)	State 3	B1		
		Obtain expression involving $\sin 10\alpha$	M1		[allow θ/α confusion]
		Obtain 9	A1	3	[and no other value]
	(iii)	Recognise cosec 2β as $\frac{1}{\sin 2\beta}$	B1		[allow θ/β confusion]
		Attempt to express equation in terms of $\sin 2\beta$ only	M1		[or equiv involving $\cos 2\beta$]
		Attempt to find non-zero value of $\sin 2\beta$	M1		[or of $\cos 2\beta$]
		Obtain at least $\sin 2\beta = \sqrt{\frac{5}{12}}$	A1		[or equiv, exact or approx]
		Attempt correct process to find two values of β	M1		[provided equation is $\sin 2\beta = k$; or equiv with $\cos 2\beta$]
		Obtain 20.1, 69.9	A1	6	[and no others between 0 and 90]

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1		Attempt to factorise numerator and denominator	M1		
		num = $xx(x-3)$ <u>or</u> denom = $(x-3)(x+3)$	A1		Not num = $x(x^2-3x)$
		<u>Final</u> answer = $\dfrac{x^2}{x+3}$ [Not $\dfrac{xx}{x+3}$]	A1	3	Do not ignore further cancellation.
2		$\dfrac{d}{dx}(\sin y)=\cos y.\dfrac{dy}{dx}$	B1		
		$\dfrac{d}{dx}(xy)=x\dfrac{dy}{dx}+y$ s.o.i.	B1		[SR: If xy taken to LHS, accept $-x\dfrac{dy}{dx}+y$ as s.o.i.]
		$\cos y.\dfrac{dy}{dx}=x\dfrac{dy}{dx}+y+2x$ AEF	B1		
		[If written as $\frac{dy}{dx}=\cos y\frac{dy}{dx}=x\frac{dy}{dx}+y+2x$, accept for prev B1 but not for following marks if the $\frac{dy}{dx}$ is used]			
		$f(x,y)\dfrac{dy}{dx}=g(x,y)$	M1		Regrouping provided > one $\dfrac{dy}{dx}$ term
		$\dfrac{y+2x}{\cos y-x}$ or $-\dfrac{y+2x}{x-\cos y}$ or $\dfrac{-2x-y}{x-\cos y}$	A1	5	ISW Answer could imply M1
3	(i)	Quotient = $3x+...$	B1		For correct leading term in quotient
		For evidence of correct division process	M1		Or for cubic $\equiv(x^2-2x+5)(gx+h)\ (+...)$
		$3x+4$	A1		For correct quotient
		$-6x-13$	A1	4	For correct remainder ISW
	(ii)	$a=7$	B1√		<u>Follow through</u> If rem in (i) is $Px+Q$,
		$b=20$	B1√	2	then B1√ for $a=1-P$ and B1√ for $b=7-Q$
		[SR: If B0+B0, award B1√ for $a=1+P$ AND $b=7+Q$; also SR B1 for $a=20, b=7$]			
4	(i)	Parts using correct split of $u=x, \dfrac{dv}{dx}=\sec^2 x$	M1		1st stage result of form $f(x)+/-\int g(x)dx$
		$x\tan x-\int \tan x\,dx$	A1		Correct 1st stage
		$\int \tan x\,dx=-\ln\cos x$ or $\ln\sec x$	B1		
		$x\tan x+\ln\cos x+c$ or $x\tan x-\ln\sec x+c$	A1	4	
	(ii)	$\tan^2 x=+/-\sec^2 x+/-1$	M1		or $\sec^2 x=+/-1+/-\tan^2 x$
		$\int x\sec^2 x\,dx-\int x\,dx$ s.o.i.	A1		Correct 1st stage
		$x\tan x+\ln\cos x-\frac{1}{2}x^2+c$	A1√	3	f.t. their answer to part (i) $-\frac{1}{2}x^2$

5	(i)	$\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}$	M1		Used, not just quoted
		$\frac{1}{t}$ or t^{-1}	A1	2	Not $\frac{2}{2t}$ as final answer
		SR: M1 for Cart conv, finding $\frac{dy}{dx}$ & ans involv t + A1			M1 is attempt only, accuracy not involved
	(ii)	Finding equation of tangent (using p or t)	M1		
		$py = x + p^2$	A1	2	**AG**; p essential; at least 1 line inter working
	(iii)	$(25,-10) \Rightarrow p = -5$ or $-5y = x + 25$ seen	B1		$5y = x + 25$ seen $\Rightarrow$ B0
		Substitution of their values of p into given tgt eqn	M1		Producing 2 equations
		Solving the 2 equations simultaneously	M1		
		$(-15,-2)$ $x = -15, y = -2$	A1	4	Common wrong ans $(15,8) \Rightarrow$ B0, M2, A0
6	(i)	Attempt to connect $dx, d\theta$	M1		But not $dx = d\theta$
		$dx = 2\sin\theta\cos\theta\, d\theta$	A1		AEF
		$\sqrt{\frac{x}{1-x}} = \frac{\sin\theta}{\cos\theta}$	B1		Ignore any references to $\pm$.
		Reduction to $\int 2\sin^2\theta\, d\theta$	A1	4	**AG** WWW
	(ii)	$\sin^2\theta = k(+/-1+/-\cos 2\theta)$	M1		Attempt to change $(2)\sin^2\theta$ into $f(\cos 2\theta)$
		$2\sin^2\theta = 1 - \cos 2\theta$	A1		Correct attempt
		$\int \cos 2\theta\, d\theta = \frac{1}{2}\sin 2\theta$	B1		Seen anywhere in this part
		Attempting to change limits	M1		<u>Or</u> Attempting to resubstitute; Accept degrees
		$\frac{1}{2}\pi$	A1	5	
		<u>Alternatively</u> Parts once & use			
		$\cos^2\theta = 1 - \sin^2\theta$	(M2)		Instead of the M1 A1 B1
		$\frac{1}{2}(\theta - \sin\theta\cos\theta)$	(A1)		Then the final M1 A1 for use of limits
7	(i)	$A = 3$	B1		For correct value stated
		$C = 1$	B1		For correct value stated
		$11 + 8x \equiv A(1+x)^2 + B(2-x)(1+x) + C(2-x)$	M1		AEF; any suitable identity
		e.g. $A - B = 0, 2A + B - C = 8, A + 2B + 2C = 11$	A1		For any correct (f.t.) equation involving B
		$B = 3$	A1	5	
	(ii)	$\left(1 - \frac{x}{2}\right)^{-1} = 1 + \frac{x}{2} + \frac{x^2}{4} + \ldots$	B1		s.o.i.
		$(1+x)^{-1} = 1 - x + x^2 - \ldots$	B1		s.o.i.
		$(1+x)^{-2} = 1 - 2x + 3x^2 - \ldots$	B1,B1		s.o.i.
		Expansion = $\frac{11}{2} - \frac{17}{4}x + \frac{51}{8}x^2 + \ldots$	B1	5	CAO. No f.t. for wrong A and/or B and/or C

SR(1) If partial fractions not used but product of $(11+8x)(2-x)^{-1}(1+x)^{-2}$ attempted, then

B1 for $\left(1-\frac{x}{2}\right)^{-1} = 1+\frac{x}{2}+\frac{x^2}{4}+\ldots$

B1,B1 for $(1+x)^{-2} = 1-2x+\ldots+3x^2+\ldots$

B1,B1 for $\frac{11}{2}-\frac{17}{4}x+\ldots+\frac{51}{8}x^2+\ldots$

SR(2) If partial fractions not used but denominator multiplied out, then

B1 for denom = $2+3x(+0x^2)+\ldots$

B1 for $\left(1+\frac{3x}{2}\right)^{-1} = 1-\frac{3x}{2}+\frac{9x^2}{4}+\ldots$

B1,B1,B1 for $\frac{11}{2}\ldots-\frac{17}{4}x\ldots+\frac{51}{8}x^2+\ldots$

N.B. In both SR, if final expansion given B0, -------------------*allow SR B1 for* $22-17x+51/2\,x^2$

8	**(i)**	$\int (y-3)\,dy = \int (2-x)\,dx$ or equiv	M1		For separation & integration of both sides
		$\frac{1}{2}y^2-3y = 2x-\frac{1}{2}x^2$	A1		or $\frac{1}{2}(y-3)^2 = -\frac{1}{2}(x-2)^2$
		For an arbitrary const on one/both sides	*B1		} (or + M2 for equiv statement using limits)
		Substituting $(x,y)=(5,4)$ or (4,5) & finding 'c'	dep*M1		}
		$\frac{1}{2}y^2-3y = -\frac{1}{2}x^2+2x-\frac{3}{2}$ AEF ISW	A1	**5**	or $\frac{1}{2}(y-3)^2 = -\frac{1}{2}(x-2)^2+5$ AEF
	(ii)	Attempt to clear fracts (if nec) & compl square	M1		
		$a=2, b=3, k=10$	A2	**3**	For all 3; SR: A1 for 1 or 2 correct
	(iii)	Circle clearly indicated in a sketch	B1		
		Centre $(2,3)$ or their (a,b)	B1√		
		Radius $\sqrt{10}$ or their $\sqrt{k}$	B1√	**3**	√ provided $k>0$

9	**(i)**	Using $\begin{pmatrix}-8\\1\\-2\end{pmatrix}$ and $\begin{pmatrix}-9\\2\\-5\end{pmatrix}$ as the relevant vectors	M1		i.e. correct direction vectors
		Using $\cos\theta = \frac{\underline{a}.\underline{b}}{\lvert\underline{a}\rvert\lvert\underline{b}\rvert}$ AEF for any 2 vectors	M1		Accept $\cos\theta = \left\lvert\frac{\underline{a}.\underline{b}}{\lvert\underline{a}\rvert\lvert\underline{b}\rvert}\right\rvert$
		Method for scalar product of any 2 vectors	M1		
		Method for finding magnitude of any vector	M1		
		15° (15.38…), 0.268 rad	A1	**5**	
	(ii)	Produce (at least) 2 of the 3 eqns in t and s	M1		e.g. $4-8t=-2-9s$, $-6-2t=-2-5s$
		Solve the (x) and (z) equations	M1		
		$t=3$ or $s=2$	A1		for first value found
		$s=2$ or $t=3$ f.t.	A1√		for second value found
		Substituting their (t,s) into (y) equation	M1		
		$a=1$	A1		
		Substituting their t into l_1 or their (s,a)			

into l_2	M1		
$\begin{pmatrix}-20\\5\\-12\end{pmatrix}$	A1	**8**	Any format but not $\begin{pmatrix} \\ \\ \end{pmatrix}+\begin{pmatrix} \\ \\ \end{pmatrix}$

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		Mark	Total	
1.	(i) $2 + 16i - i - 8i^2$ $10 + 15i$ (ii) $\frac{1}{5}(10 + 15i)$ or $2 + 3i$	M1 A1 M1 A1 A1ft	 2 3 **5**	Attempt to multiply correctly Obtain correct answer Multiply numerator & denominator by conjugate Obtain denominator 5 Their part (i) or 10 + 15i derived again / 5
2.	$1^2 = \frac{1}{6} \times 1 \times 2 \times 3$ $\frac{1}{6}n(n+1)(2n+1) + (n+1)^2$ $\frac{1}{6}(n+1)(n+2)\{2(n+1)+1\}$	B1 M1 DM1 A1 A1	 5 **5**	Show result true for n = 1or 2 Add next term to given sum formula, any letter OK Attempt to factorise or expand and simplify Correct expression obtained Specific statement of induction conclusion, with no errors seen
3.	(i) $2\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} - 1\begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} + 3\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ 2 x 5 – 1 x 2 +3 x -1 5 (ii)	M1 A1 A1 B1ft	 3 1 **4**	Show correct expansion process, allow sign slips Obtain correct (unsimplified) expression Obtain correct answer State that **M** is non-singular as det **M** non-zero, ft their determinant
4.	$u^2 + 4u + 4$ $u^3 + 6u^2 + 12u + 8$ $u = \sqrt[3]{5}$ $x = 2 + \sqrt[3]{5}$	B1 M1 A1 A1ft A1ft	 5 **5**	$u + 2$ squared and cubed correctly Substitute these and attempt to simplify Obtain $u^3 - 5 = 0$ or equivalent Correct solution to their equation Obtain 2 + their answer [Decimals score 0/2 of final A marks]

5.	$8\Sigma r^3 - 6\Sigma r^2 + 2\Sigma r$	M1		Consider the sum of three separate terms
	$8\Sigma r^3 = 2n^2(n+1)^2$	A1		Correct formula stated or used a.e.f.
	$6\Sigma r^2 = n(n+1)(2n+1)$	A1		Correct formula stated or used a.e.f.
	$2\Sigma r = n(n+1)$	A1		Correct term seen
	$2n^3(n+1)$ **AG**	M1 A1	6 **6**	Attempt to factorise or expand and simplify Obtain given answer correctly

6.	(i) $\frac{1}{2}\begin{pmatrix} 8 & -2 \\ -3 & 1 \end{pmatrix}$	B1 B1	2	Transpose leading diagonal and negate other diagonal Divide by determinant
	(ii) Either	B1 M1A1		State or imply $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ Use this result and obtain $\mathbf{B}^{-1} = \mathbf{C}^{-1}\mathbf{A}$, or equivalent matrix algebra
	$\frac{1}{2}\begin{pmatrix} 14 & 2 \\ -5 & 0 \end{pmatrix}$	M1 A1ft	5	Matrix multn., two elements correct, for any pair All elements correct ft their (i)
	Or			
	$\frac{1}{5}\begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}$	B1		Find $\mathbf{A}^{-1}$
	$\mathbf{B} = \mathbf{A}^{-1}\mathbf{C}$	M1		Premultiply by $\mathbf{A}^{-1}$ stated or implied
	$\mathbf{B} = \frac{1}{5}\begin{pmatrix} 0 & -2 \\ 5 & 14 \end{pmatrix}$	M1 A1ft		Matrix multn. Two elements correct All elements correct
	$\frac{1}{2}\begin{pmatrix} 14 & 2 \\ -5 & 0 \end{pmatrix}$	A1		Correct $\mathbf{B}^{-1}$
	Or			
	$\mathbf{AB} = \begin{pmatrix} 2a+c & 2b+d \\ a+3c & b+3d \end{pmatrix}$	B1		Find $\mathbf{AB}$
	$a=0, c=1, b=-0.4, d=2.8$	M1 A1A1		Solve one pair of simultaneous equations Each pair of answers
	$\frac{1}{2}\begin{pmatrix} 14 & 2 \\ -5 & 0 \end{pmatrix}$	A1	**7**	Correct $\mathbf{B}^{-1}$

7.	(a) (i) $\sqrt{13}$	B1	1	Obtain correct answer, decimals OK
	(ii)			
		M1		Using $\tan^{-1} {}^{b}/_{a}$, or equivalent trig allow + or -
	- 0.59	A1		Obtain 0.59
		A1	3	Obtain correct answer
	(b)	M1		Express LHS in Cartesian form & equate real and imaginary parts
	1 – 2i	A1A1		Obtain x = 1 and y = -2
		A1	4	Correct answer written as a complex number
	(c)	B1		Sketch of vertical straight line
		B1	2	Through (- 0.5, 0)
			10	
8.	(i)	B1		For correct vertex (2, -2)
	$\begin{pmatrix}0\\0\end{pmatrix}$ $\begin{pmatrix}2\\0\end{pmatrix}$ $\begin{pmatrix}2\\-2\end{pmatrix}$ $\begin{pmatrix}0\\-2\end{pmatrix}$	B1		For all vertices correct
		B1	3	For correct diagram
	(ii) Either $\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$	B1,B1		Reflection, in x-axis
		B1		Correct matrix
	$\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}$	B1,B1		Enlargement, centre O s.f.2
		B1	6	Correct matrix
	Or $\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}$	B1,B1		Reflection, in the y-axis
		B1		Correct matrix
	$\begin{pmatrix}-2 & 0\\0 & -2\end{pmatrix}$	B1,B1		Enlargement, centre O s.f. –2
		B1		Correct matrix
	Or $\begin{pmatrix}2 & 0\\0 & 1\end{pmatrix}$	B1,B1		Stretch, in x-direction s.f. 2
		B1		Correct matrix
	$\begin{pmatrix}1 & 0\\0 & -2\end{pmatrix}$	B1,B1		Stretch, in y-direction s.f. -2
		B1		Correct matrix
			9	

9.	(i) $\frac{r+2-r}{r(r+2)}$	M1		Show correct process for subtracting fractions
	$\frac{2}{r(r+2)}$ **AG**	A1	2	Obtain given answer correctly
	(ii)	M1		Express terms as differences using (i)
		M1		Express 1st 3 (or last 3) terms so that cancelling occurs
		A1		Obtain $1+\frac{1}{2}$
		A1		Obtain $-\frac{1}{n+2},\ -\frac{1}{n+1}$
	$\frac{3}{2}-\frac{1}{n+1}-\frac{1}{n+2}$	A1	5	Obtain correct answer in any form
	(iii) (a) $\frac{3}{2}$	B1ft	1	Obtain value from their sum to *n* terms
	(b) $\frac{1}{n+1}+\frac{1}{n+2}$	M1		Using (iii) (a) – (ii) or method of differences again [$n\to\infty$ is a method error]
		A1 ft	2	Obtain answer in any form
			10	
10.	(i) $\alpha+\beta+\gamma=9$	B1	1	
	(ii)	B1		State or use other root is *p* - i*q*
		M1		Substitute into (i)
		A1		Obtain $2p+\alpha=9$
	$p=\frac{9-\alpha}{2}$	A1	4	Obtain correct answer a.e.f.
	(iii) $\alpha\beta\gamma=29$	B1	1	
	(iv)	M1		Substitute into (iii)
	$\alpha(p^2+q^2)=29$	A1ft		Obtain unsimplified expression with no i's
		M1		Rearrange to obtain *q* or q^2
		M1		Substitute their expression for *p* a.e.f.
	$q=\sqrt{\frac{29}{\alpha}-\frac{(9-\alpha)^2}{4}}$	A1	5	Obtain correct answer a.e.f.
			11	
	(iv) Alternative method	M1		Substitute into $\alpha\beta+\beta\gamma+\gamma\alpha=27$
	$2p\alpha+p^2+q^2=27$	A1		Obtain unsimplified expression with no i's
		M1		Rearrange to obtain *q* or q^2
		M1		Substitute their expression for *p* a.e.f.
	$q=\sqrt{27-\frac{(9-\alpha)^2}{4}-\alpha(9-\alpha)}$	A1		Obtain correct answer a.e.f.

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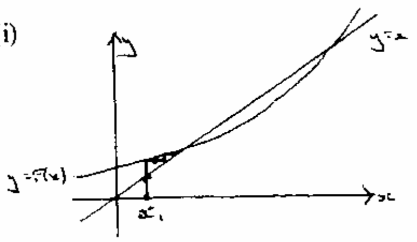
1(i) Use standard $\ln(1+3x) = 3x - \frac{(3x)^2}{2} + \frac{(3x)^3}{3}$	M1	Allow e.g. $3x^2$, 2! etc.
	M1	Attempt to simplify $(3x)^2$ etc.
$= 3x - 9x^2/2 + 9x^3$	A1	cao
(ii) Produce $(1 + x + x^2/2)$	B1	
	M1	Mult. 2 reasonable attempts, each of 3 terms (non-zero)
Get $3x - 3x^2/2 + 6x^3$	A1√	From their series
	SC M1	Reasonable attempt at diff. and replace $x = 0$ (2 correct)
	M1√	Put <u>their</u> values into correct Maclaurin expansion
	A1	cao
		(Applies to either/both parts)
2 Write as $f(x) = \pm(x - e^{-x})$	B1	Or equivalent
So $f'(x) = \pm(1 + e^{-x})$	B1	Correct from their f(x)
Use $x_{n+1} = x_n - f(x_n)/f'(x_n)$ with $x_0 = 0.5$	M1	Clear evidence of N-R on their f, f′
Get $x_1 = 0.56631$, $x_2 = 0.56714$	A1√	At least one to 4d.p.
Get $x_3 = 0.567(1)$	A1	cao to 3 d.p.
3 Use $A/x + (Bx + C)/(x^2 + 2)$	B1	
Equate $x+6$ to $A(x^2 + 2) + (Bx+C)x$ (or equiv.)	M1√	Equate to their P.F. (e.g. if B = 0 or C = 0 used)
Use $x = 0$ or equiv. for A (or equate coeff.etc.)	M1√	Include cover-up
Correctly find one of B,C	A1	
Get A=3, B=-3, C=1	A1	
4(i) 	B1	Line from x_1 to curve
	B1	Then to line
	B1	Clear explanation; allow use of step/staircase
(ii)(a) Converges to $x=\alpha$	B1, B1	
(b) Diverges (does not give either root)	B1	
5 (i) Give $x = -2$	B1	
Attempt to divide out	M1	Giving $y = x+k$; allow $k = 0$ here
Get $y = x + 1$	A1	Must be =
(ii) Write as quad. $x^2 + x(3 - y) + (3-2y) = 0$	M1	SC Differentiate M1
Use for real x, $b^2 - 4ac \geq 0$	M1	Solve $dy/dx=0$ M1
Produce quad. inequality in y	M1	Get 2 x,y values correct A1
Attempt to solve quad. inequality	M1	Attempt at max/min M1
Get A.G. clearly e.g. graph	A1	Justify, e.g. graph, constraints on y A1

6 (i) Use parts to $(-e^{-x}.x^n - \int -e^{-x}.nx^{n-1}\,dx)$	M1 Reasonable attempt e.g. $+e^{-x}$ A1 cao
Use limits to get e^{-1}	B1 Allow ±
Tidy correctly to A.G.	A1
(ii) Use $I_3 = 3I_2 - e^{-1}$ $I_2 = 2I_1 - e^{-1}$ $I_1 = I_0 - e^{-1}$	B1 One such seen
Work out $I_0 = 1 - e^{-1}$ or $I_1 = 1 - 2e^{-1}$	M1,A1
Get $6 - 16e^{-1}$	A1
7 (i) Area under graph = $\int \sqrt{x}\,dx$	B1 Explain RHS (limits need not be specified)
> Sum of areas of rectangles from 1 to $N+1$	B1
Area of each rect. = Width x Height = 1 x $\sqrt{x}$	B1
(ii) Similarly, area under curve from 0 to N	B1
< sum of areas of rect. from 0 to N	B1
Clear explanation of A.G.	B1
(iii) Integrate $x^{0.5}$ and use 2 different sets of limits	M1,M1
Get area between ${}^2/_3((N+1)^{1.5}-1)$ and ${}^2/_3N^{1.5}$	A1
8 (i) Max. $r = 2$ at $\theta = 0$ and π	B1,B1 Two θ needed (rads only); ignore θ out of range
(ii) Solve $r = 0$ for θ, giving $\theta = ½\pi$ and ${}^3/_2\pi$	M1,A1 Two θ needed (rads only); ignore θ out of range
(iii) Use correct formula with correct r	M1
Expand r	M1
Get $\int A + B\cos 2\theta + C\cos 4\theta\,d\theta$	M1 $C \neq 0$
Integrate their expression correctly	M1√
Get $3\pi/8$	A1 cao
(iv) Express $\cos 2\theta = \cos^2\theta - \sin^2\theta$ or similar	M1
Use $\cos\theta = x/r$ and/or $\sin\theta = y/r$	M1
Simplify to $(x^2 + y^2)^{1.5} = 2x^2$ or similar	M1,A1
9 (i) Correct def^n of cosh x and sinh x	B1,B1
Expand $2.½(e^x - e^{-x}).½(e^x + e^{-x})$	M1 Reasonable attempt
Clearly get $½(e^{2x} - e^{-2x})$ to A.G.	A1
(ii) Attempt to diff. and solve $dy/dx = 0$	M1 Reasonable attempt
Use (ii) to get A cosh x (B sinh x+ C)=0	M1
Clearly see cosh $x > 0$ or similar for one useable factor only	B1
Attempt to solve sinh x = - C/B	M1 Quote or via e^{-x} correctly
Get $x = \ln((3+\sqrt{13})/2)$	A1
Justify one answer only for sinh x = -C/B	B1
Accurate test for MINIMUM	B1 First or second diff^l test with numeric evidence B1 Correct value(s) for min.

Mark Scheme 4727
January 2006

1 Directions $[1, 1, -1]$ and $[2, -3, 1]$	B1	For identifying both directions (may be implied by working)
$\theta = \cos^{-1}\dfrac{\lvert[1, 1, -1].[2, -3, 1]\rvert}{\sqrt{3}\quad\sqrt{14}}$	M1	For using scalar product of their direction vectors
$= \cos^{-1}\dfrac{\lvert -2\rvert}{\sqrt{42}}$	M1	For completely correct process for their angle
$= 72.0°, 72°$ or 1.26 rad	A1 **4** [4]	For correct answer
2 **(i)** Identities b, 6 Subgroups $\{b, d\}$, $\{6, 4\}$	B1 B1 B1 B1 **4**	For correct identities For correct subgroups
(ii) $\{a, b, c, d\} \leftrightarrow \{2, 6, 8, 4\}$ or $\{8, 6, 2, 4\}$	B1 B1 B1 **3** [7]	For $b \leftrightarrow 6$, $d \leftrightarrow 4$ For $a, c \leftrightarrow 2, 8$ in either order **SR** If B0 B0 B0 then M1 A1 may be awarded for stating the orders of all elements in G and H
3 **(i)** $3y^2\dfrac{dy}{dx} = \dfrac{dz}{dx}$	M1	For differentiating substitution
$\Rightarrow \dfrac{dz}{dx} + 2xz = e^{-x^2}$	A1	For resulting equation in z and x
Integrating factor $\left(e^{\int 2x\,dx} =\right) e^{x^2}$	B1 √	For correct IF f.t. for an equation in suitable form
$\Rightarrow \dfrac{d}{dx}\left(ze^{x^2}\right)$ *OR* $\dfrac{d}{dx}\left(y^3e^{x^2}\right) = 1$	M1	For using IF correctly
$\Rightarrow ze^{x^2}$ *OR* $y^3e^{x^2} = x\,(+c)$	A1	For correct integration (+c not required here)
$\Rightarrow y = (x+c)^{\frac{1}{3}}\,e^{-\frac{1}{3}x^2}$	A1 **6**	For correct answer **AEF**
(ii) As $x \to \infty$, $y \to 0$	B1 **1** [7]	For correct statement
4 **(i)** $\cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right)$, $\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)$	B1	For either expression, seen or implied z may be used for $e^{i\theta}$ throughout
$\Rightarrow \cos^2\theta\sin^4\theta = \frac{1}{4}\left(e^{i\theta} + e^{-i\theta}\right)^2 \frac{1}{16}\left(e^{i\theta} - e^{-i\theta}\right)^4$		
$= \frac{1}{4}\left(e^{2i\theta} + 2 + e^{-2i\theta}\right).\frac{1}{16}\left(e^{4i\theta} - 4e^{2i\theta} + 6 - 4e^{-2i\theta} + e^{-4i\theta}\right)$	M1 A1 A1	For expanding terms For the 2 correct expansions **SR** Allow A1 A0 for $k\left(e^{2i\theta} + 2 + e^{-2i\theta}\right)\left(e^{4i\theta} - 4e^{2i\theta} + 6 - 4e^{-2i\theta} + e^{-4i\theta}\right)$, $k \neq \frac{1}{64}$
$= \frac{1}{64}\left(\left(e^{6i\theta} + e^{-6i\theta}\right) - 2\left(e^{4i\theta} + e^{-4i\theta}\right) - \left(e^{2i\theta} + e^{-2i\theta}\right) + 4\right)$	M1	For grouping terms and using multiple angles
$= \frac{1}{32}\left(\cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2\right)$ **AG**	A1 **6**	For answer obtained correctly

(ii) $\int_0^{\frac{1}{3}\pi} \cos^2\theta \sin^4\theta \, d\theta =$		
$= \frac{1}{32}\left[\frac{1}{6}\sin 6\theta - \frac{1}{2}\sin 4\theta - \frac{1}{2}\sin 2\theta + 2\theta\right]_0^{\frac{1}{3}\pi}$	M1 A1	For integrating answer to **(i)** For all terms correct
$= \frac{1}{32}\left[0 + \frac{1}{4}\sqrt{3} - \frac{1}{4}\sqrt{3} + \frac{2}{3}\pi - 0\right] = \frac{1}{48}\pi$	A1 **3** **9**	For correct answer
5 (i) *EITHER* $z = \sqrt{8}\,\mathrm{cis}(2k+1)\frac{\pi}{4},\ k = 0, 1, 2, 3$	B1	For correct modulus **AEF**
OR $z = \sqrt{8}\,e^{(2k+1)\frac{\pi}{4}i}, k = 0, 1, 2, 3$	B1 **2**	For correct arguments **AEF**
(ii) $z = 2\sqrt{2}\left\{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i, -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i, -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i, \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right\}$	B1	For any of $\pm\frac{1}{\sqrt{2}} \pm \frac{1}{\sqrt{2}}i$
$z = 2 + 2i, -2 + 2i, -2 - 2i, 2 - 2i$	B1	For any one value of z correct
	B1	For all values of z correct **AEFcartesian** (may be implied from symmetry or factors)
$(z - \alpha), (z - \beta), (z - \gamma), (z - \delta)$	B1 √ **4**	f.t., where $\alpha, \beta, \gamma, \delta$ are answers above
(iii) *EITHER* $(z - (2 + 2i))(z - (2 - 2i))$	M1	For combining factors from **(ii)** in pairs
$\times (z - (-2 + 2i))(z - (-2 - 2i))$	M1	Use of complex conjugate pairs
$= (z^2 + 4z + 8)(z^2 - 4z + 8)$	A1	For correct answer
OR $z^4 + 64 = (z^2 + az + b)(z^2 + cz + d)$	M1	For equating coefficients
$\Rightarrow a + c = 0, b + ac + d = 0, ad + bc = 0, bd = 64$	M1	For solving equations
Obtain $(z^2 + 4z + 8)(z^2 - 4z + 8)$	A1 **3** **9**	For correct answer
6 (i) **MB** $= [2, 1, -2]$, **OF** $= [4, 1, 2]$	B1	For either vector correct (allow multiples)
MB × **OF**	M1	For finding vector product of their **MB** and **OF**
$= [4, -12, -2]$ *OR* $k[2, -6, -1]$	A1 **3**	For correct vector
(ii) *EITHER* Find vector product of any two of $\pm[2, -1, 2]$, $\pm[0, 0, 2]$, $\pm[2, -1, 0]$ and any two of $\pm[4, 0, 2]$, $\pm[4, -1, 0]$, $\pm[0, 1, 2]$	M1	For finding two relevant vector products
Obtain $k[1, 2, 0]$	A1	For correct LHS of plane *CMG*
Obtain $k[1, 4, -2]$	A1	For correct LHS of plane *OEG*
	M1	For substituting a point into each equation
$x + 2y = 2$ and $x + 4y - 2z = 0$	A1	For both equations correct **AEF**
OR Use $ax + by + cz = d$ with coordinates of *C*, *M*, *G* *OR* *O*, *E*, *G* substituted	M1	For use of cartesian equation of plane
Obtain $a : b : c = 1 : 2 : 0$ for *CMG*	A1	For correct ratio
Obtain $a : b : c = 1 : 4 : -2$ for *OEG*	A1	For correct ratio
	M1	For substituting a point into each equation
$x + 2y = 2$ and $x + 4y - 2z = 0$	A1 **5**	For both equations correct **AEF**

(iii) *EITHER* Put x, y *OR* $z = t$ in planes *OR* evaluate $k[1, 2, 0] \times k[1, 4, -2]$	M1	For solving plane equations in terms of a parameter *OR* for finding vector product of normals to planes from **(ii)**
Obtain $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ where $\mathbf{a} = [0, 1, 2], [2, 0, 1]$ *OR* $[4, -1, 0]$	A1	Obtain a correct point **AEF**
$\mathbf{b} = k[-2, 1, 1]$	A1 **3** **11**	Obtain correct direction **AEF**
7 (i) $(x^{-1}ax)^m = (x^{-1}ax)(x^{-1}ax)\ldots(x^{-1}ax)$	M1	For considering powers of $x^{-1}ax$
$= x^{-1}aa\ldots ax$, associativity, $xx^{-1} = e$	A1 A1	For using associativity and inverse properties
$= x^{-1}a^m x = x^{-1}ex$ when $m = n$, not $m < n$	B1	For using order of *a* correctly
$= x^{-1}x$	A1	For using property of identity
$= e \Rightarrow$ order n	A1 **6**	For correct conclusion
(ii) ***EITHER*** $(x^{-1}ax)z = e$	M1	For attempt to solve for *z* **AEF**
$\Rightarrow axz = xe = x \Rightarrow xz = a^{-1}x$	A1	For using pre- or post multiplication
$\Rightarrow z = x^{-1}a^{-1}x$	A1	For correct answer
OR Use $(pq)^{-1} = q^{-1}p^{-1}$ *OR* $(pqr)^{-1} = r^{-1}q^{-1}p^{-1}$	M1	For applying inverse of a product of elements
State $(x^{-1})^{-1} = x$	A1	For stating this property
Obtain $x^{-1}a^{-1}x$	A1 **3**	For correct answer with no incorrect working **SR** correct answer with no working scores B1 only
(iii) $ax = xa \Rightarrow x = a^{-1}xa$	M1	Start from commutative property for *ax*
$\Rightarrow xa^{-1} = a^{-1}x$	A1 **2** **11**	Obtain commutative property for $a^{-1}x$
8 (i) $m^2 + 2km + 4 = 0$	M1	For stating and attempting to solve auxiliary eqn
$\Rightarrow m = -k \pm \sqrt{k^2 - 4}$	A1 **2**	For correct solutions, at any stage **AEF**
(a) $x = e^{-kt}\left(Ae^{\sqrt{k^2-4}\,t} + Be^{-\sqrt{k^2-4}\,t}\right)$	M1 A1 **2**	For using $e^{f(t)}$ with distinct real roots of aux eqn For correct answer **AEF**
(b) $x = e^{-kt}\left(Ae^{i\sqrt{4-k^2}\,t} + Be^{-i\sqrt{4-k^2}\,t}\right)$	M1	For using $e^{f(t)}$ with complex roots of aux eqn This form may not be seen explicitly but if stated as final answer earns M1 A0
$x = e^{-kt}\left(A'\cos\sqrt{4-k^2}\,t + B'\sin\sqrt{4-k^2}\,t\right)$ *OR* $x = e^{-kt}\left(C' \begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\left(\sqrt{4-k^2}\,t + \alpha\right)\right)$	A1 **2**	For correct answer
(c) $x = e^{-2t}(A'' + B''t)$	M1 A1 **2**	For using $e^{f(t)}$ with equal roots of aux eqn For correct answer. Allow *k* for 2

(ii)(a) $x = B'e^{-t}\sin\sqrt{3}\,t$	B1 √	For using $t=0, x=0$ correctly. f.t. from **(b)**
$\dot{x} = B'e^{-t}\left(\sqrt{3}\cos\sqrt{3}\,t - \sin\sqrt{3}\,t\right)$	M1 A1 √	For differentiating *x* For correct expression. f.t. from their *x*
$t=0, \dot{x}=6 \Rightarrow B' = 2\sqrt{3}$, $x = 2\sqrt{3}e^{-t}\sin\sqrt{3}\,t$	A1 **4**	For correct solution **AEF** **SR** √ and **AEF** OK for $x = C'e^{-t}\cos\left(\sqrt{3}\,t + \frac{1}{2}\pi\right)$
(b) $x \to 0$	B1	For correct statement
$e^{-t} \to 0$ and sin( ) is bounded	B1 **2** **14**	For both statements

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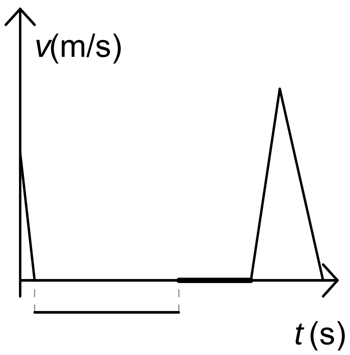
1	**(i)**	$0.3g - T = 0.3a$ and	M1		For using Newton's second law (either particle) condone 0.3ga,0.4ga and !(LHS)
		$T - 0.4g = 0.4a$	A1		Both correct. ***SR*** Accept T -0.3g = 0.3a etc as correct only if consistent with a shown as upwards for P on c's diagram
		$-0.1g = 0.7a$	M1		Eliminating T
		$a = -1.4$	A1	[4]	AG
		<u>See appendix for substituting a = -1.4</u>			
	(ii)	$0 = 2.8t - \frac{1}{2}\, 1.4t^2$	M1		For using $s = ut + \frac{1}{2}at^2$ with s = 0
		$0 = t(2.8 - 0.7t)$	M1		Solving QE
		Time taken is 4 s	A1	[3]	From correct equation only
		OR			
	(i)	$(0.3 + 0.4)a = (0.3 - 0.4)g$	M2		For using $(m_1 + m_2)a = (m_1 - m_2)g$
			A1		No application of *SR* shown above
		$a = -1.4$	A1	[4]	AG
	(ii)	$0 = 2.8 + -1.4t$	M1		For using v = u + at with v = 0
		$t = 2.8/1.4$	M1		Solve for t, and double <u>or any other complete method</u> for return time
		Time taken is 4 s	A1	[3]	

2	**(i)**	$T\sin\alpha = 0.08 \times 1.25$	M1		Newton's second law condone cos, and 0.08g for mass but not part of force
		$= 0.1$	A1	[2]	
	(ii)	$T\cos\alpha = 0.08g$	M1		Resolving forces vertically, condone sin
			A1		May be implied by $T^2 = 0.1^2 + 0.784^2$
			M1		For eliminating α or T
		$T^2 = 0.1^2 + 0.784^2$ or $\alpha = 7.3^\circ$	A1		$\alpha = 7.3^\circ$ or better
		$T = 0.79$	A1	[5]	Accept anything rounding to 0.79

3	**(i)**		M1		For using $a = dv/dt$
		$a = 7.2 - 0.9t$	A1		
			M1		For attempting to solve $a(t) = 0$
		$T = 8$	A1	[4]	
		<u>See also special case in appendix.</u>			
	(ii)	$v(T) = 28.8$ <u>See also special case in appendix.</u>	B1	[1]	AG (From $7.2 \times 8 - 0.45 \times 8^2$)
	(iii)		M1		For using $s = \int v\,dt$
		$s = 3.6t^2 - 0.15t^3 \quad (+C)$	A1		
			DM1		For finding s(T or 31) or using limits (0) to T or (0) to 31 (dep on integration)
		$s = 153.6\ (+C)$	A1		Condone +C
		s at constant speed = 662.4	B1ft		For using $(31 - \text{cv } T) \times 28.8$
		Displacement is 816 m	A1ft	[6]	cv 153.6 + cv 662.4 (non-zero numerical)

4	**(i)**	$F = 12\cos 15^{\circ}$ Frictional component is 11.6 N	M1 A1 [2]	Resolve horizontally (condone sin) Accept 12cos 15°
	(ii)	$N + 12\sin 15^{\circ} = 2g$ Normal component is 16.5 N	M1 A1 [2]	Resolve vert 3 forces (accept cos) AG
	(iii)	$11.591\ldots = \mu\, 16.494\ldots$ Coefficient is 0.7(0)	M1 A1ft [2]	For using cv $F = \mu$ cv N Ft cv F to 2 sf. $\mu = 0.7027....$
	(iv)	$N = 2g$ $F = 19.6 \times 0.7027\ldots$ $20 - 13.773\ldots = 2a$ Acceleration is 3.11 ms^{-2} ***MISREAD*** (omits "horizontal") $N = 2g$ - 20sin15 F = 0.7027 x 14.4 20cos15- 10.14 = $2a$ Acceleration is 4.59 ms^{-2}	B1 M1 M1 A1ft A1 [5] MR-1 B1ft M1 M1 A1ft A1ft [4]	 For using Newton's second law cv Tractive - cv Friction (e.g. from (i)) Accept either 3.11 or 3.12 only All A and B marks now ft. Subtract "MR-1" from initial B1 or final A1 (not A1ft in main scheme). Equals 14.42... Equals 10.1.... For using Newton's second law cv Tractive - cv Friction Accept 4.59, 4.6(0)

5	**(i)**		Graph with 5 straight line segments and with v single valued. Line segment for car stage Line segment for walk stage Line segment for wait stage 2 line segments for motor-cycle stage	B1 B1 B1 B1 B1 [5]	'Wait' line segment may not be distinguishable from part of the t axis. Attempt at all lines segments fully straight. Mainly straight, ends on t-axis Horizontal below t-axis. Ignore linking to axis. Can be implied by gap between walk and motor-cycle stages Inverted V not U, mainly straight. Condone vertex below x intercept.
	(ii)	$d = 12/8$ Deceleration is 1.5 ms^{-2}		M1 A1 [2]	Using gradient represents accn Or a = -1.5 ms^{-2}
	(iii)	 $t_{walk} = 420/0.7$ $t_{motorcycle} = 42$ $T = 8 + 600 + 250 + 42 = 900$		M1 B1 B1 A1 [4]	Using area represents displacement. Accept 600 Ignore method

6	**(i)**	$T_A\cos\alpha - T_B\cos\beta = W$	M1	For resolving 3 forces vertically, condone Wg, sin
		$T_A = T_B$ (= T)	B1	May be implied or shown in diagram
		$\cos\alpha > \cos\beta \Rightarrow \alpha < \beta$	A1 [3]	AG
	(ii)(a)	$T\sin\alpha + T\sin\beta = 14$	M1	Resolve 3 forces horiz accept cos
		$\sin\alpha = 0.6$ and $\sin\beta = 0.8$	DM1	
		Tension is 10 N	A1 [3]	
	(ii)(b)	$10\cos\alpha - 10\cos\beta = W$	M1	Must use cv T, and W (not Wg)
		$\alpha = 36.9°$, $\beta = 53.1°$	DM1	Or $\cos\alpha = 0.8$ and $\cos\beta = 0.6$ ***SR*** -1 for assuming $\alpha + \beta = 90°$
		$W = 2$ See appendix for solution based on resolving along *RA* and *RB*.	A1 ft [3]	ft for $T/5$ (accept 1.99)
	(iii)	*R* is below *B*	B1	Accept *R* more than 0.5 m below *A*
		Tension is 1 N	B1 ft [2]	ft for $W/2$ accept $W/2$

7	(i)	Initial momentum $= 0.15\times 8 + 0.5\times 2$	B1	(or loss in A's momentum $= 0.15\times 8$ B1
		Final momentum $= 0.5v$	B1	and gain in B's momentum $= 0.5(v-2)$ B1)
		$0.15\times 8 + 0.5\times 2 = 0.5v$ (or $0.15\times 8 = 0.5\times(v - 2)$)	M1	For using the principle of conservation of momentum
		$v = 4.4$	A1 [4]	condone inclusion of g in all terms
		$(m)g\sin\alpha = (\pm)(m)a$	M1	***SR*** Awarded even if g in all terms
		$a = (\pm)4.9$	A1	Condone cos
		EITHER (see also part (ii)) $0 = 4.4^2 - 2\times 4.9s$	M1	For using $v^2 = u^2 + 2as$ with $v = 0$
		s = 1.97 or 1.98 m	A1ft	Accept $s < 2$ iff $s = 4.4^2/(2\times 4.9)$
		OR $v^2 = 4.4^2 - 2\times 4.9\times 2$	M1	For using $v^2 = u^2 + 2as$ with $s = 2$
		$v^2 = -0.24$	A1ft	Accept $v^2 < 0$
		OR (see also part (ii)) $t = 4.4/4.9$ (=0.898) with either $s = 4.4\times 0.898 - 0.5\times 4.9\times 0.898^2$ or $s = (4.4 + 0)/2\times 0.898$	M1	Both parts of method needed
		s = 1.97 or 1.98 m	A1ft [4]	Accept $s < 2$
	(ii)	$2 = \frac{1}{2}4.9\,t_A^2$	M1	cv for acceleration
		$t_A = 0.904$	A1	Accept 0.903=<time=<0.904
		EITHER $2 = (-4.4)t_B + \frac{1}{2}\,4.9t_B^2$	M1	Appropriate use of $s = ut + \frac{1}{2}at^2$
		$t_B = (4.4 ! \oplus (4.4^2 + 4\times 2.45\times 2))/4.9$	M1	Correct method for solving QE
		$t_B = 2.17$	A1	2.171...
		$t_B - t_A = (2.17 - 0.9) = 1.27$ s	A1	
		OR $t_{up} = 4.4/4.9$ (=0.898)	M1	Or using s_{up} to find t_{up}
		$(2 + 1.98) = 0.5\times 4.9\times t_{down}^2$	M1	$s = ut + \frac{1}{2}at^2$ with cv s in part (i)
		$t_{down} = 1.27$	A1	Not the final answer
		$t_B - t_A = (0.9 + 1.27 - 0.9) = 1.27$s	A1	
		OR $0 = 4.4t - \frac{1}{2}\,4.9t^2$ (i.e. approx 1.8 s to return to start)	M1	$s = ut + \frac{1}{2}at^2$ with $s = 0 = 1.796$
		$2 = 4.4t + 4.9t^2$	M1	
		$t = 0.376$	A1	
		$t_B - t_A = 1.796 + 0.376 - 0.9 = 1.27$s	A1 [5]	

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1		$\tan\theta = ⅓$ ($\theta = 18.4°$ at B)	B1		71.6° at C	
		3 x T $\sin\theta$ = 20 x 1.5 must	M1		M(A) ($d=3/\sqrt{10}$)	
		have two distances and no g	A1			
		T = 31.6 N	A1	4		**4**

2	(i)	$0 = 50 \sin 25° t - 4.9t^2$	M1		or 0=50sin25° - 9.8t &2t :2x2.16	
			A1			
		t = 4.31 s	A1	3		
	(ii)	d = 50cos25° x 4.31	M1		or $u^2\sin(2x25°)/g$	
		195 m	A1√	2	√ 50cos25° x their t	**5**

3	(i)a	100 J	B1	1		
	b	7500 Nm	B1	1		
	(ii)	400cosα x 25 =7500 + 100	M1		sc N II gets M1A1only.This M1	
		√ for = a + b	A1√		for total M (a=0.08)&A1for α	
		α = 40.5	A1	3	or 0.707 rads	**5**

4	(i)	horiz comps in opp direct	B1		at E & F	
		Right at E + Left at F	B1	2		
	(ii)	1.6x9.8x30 = 20X or	M1		or 10X + 1.6gx30 = 30X M(A)	
		0.5x30g + 0.7x30g + 0.2x60g = 20X	A1		or 10X + (. . . = 470.4) = 30X M mark ok without g but 3 parts	
		X = 23.5 N	A1	3		
	(iii)	$1.6\bar{y}$ =	M1		must be moments with vert dists	
		20x0.2 + 20x0.2 +40x0.5	A1		or $1.6\bar{y}$ =20x0.2x2+40x0.7(22.5)	
		$\bar{y}$ = 17.5 cm	A1	3		**8**

5	(i)	6m = 3mx + 2my	M1		- 3mx ok if clear on diagram	
		6 = 3x + 2y	A1		m must have been cancelled	
		e = 1 = (y-x)/2	M1		**or** $½.3m.2^2 = ½.3mx^2 + ½.2my^2$	
			A1		$6 = 3x^2/2 + y^2$ aef	
		x = 0.4 or 2/5	A1		sc A1A0 if x = 2, y = 0 not rejected	
		y = 2.4 or 12/5	A1	6		
	(ii)	4.8m or 24m/5	B1√		√ 2m x their y or 3m(2-their x)	
		same as original dir. of A	B1	2	use their diagram(or dir. of B)	
	(iii)	e = (2.8 – 1.0)/2.4	M1			
		0.75 watch out for ± fiddles	A1√	2	√(1.8/their y) with $0 \leq e \leq 1$	**10**

6	(i)	x = 7t	B1			
		y = - 4.9t^2 or -½gt^2	M1		some attempt at vertical motion	
			A1		sc y=xtanθ −gx^2 /(2V^{2}cos^{2}θ)	
		y = - x^2/10 **AG** (no fiddles)	A1	4	with θ=0 M1 then A1 (max = 2)	
	(ii)	-20 = -x^2/10	M1		or t=√(20/4.9) & x=7t	
		14.1 m	A1	2	sc B1 for 14.1 after wrong work	
	(iii)	½mv^2 = ½m7^2 + mgx20 n.b. v^2=u^2	M1		**OR** v_h = 7 (B1)	
		+2as gets M0	A1		v_v = ±19.8 (B1) 14√2,2√98 etc	
		v = 21 ms^{-1}	A1		v = 21 (B1)	
		dy/dx = -2x/10 & tanθ	M1		**OR** tanθ = 19.8/7 or	
			A1		cosθ=7/21 or sinθ=19.8/21	
		70.5° to horizontal	A1	6	or 19.5° to vertical	**12**

7	(i)	F = 300/12	M1			
		R = 25	A1	2		
	(ii)	P=17.5x12 (R_2=17.5 & F_2 = 17.5)	M1		n.b. B1 only for 210 W	
		P = 210 W	A1	2	without working	
	(iii)	500 = Fx12	M1			
		F = 41.67 or 500/12 aef	A1			
		41.67 – 25 – 75x9.8sin1°= 75a	M1			
			A1			
		0.0512 ms^{-2}	A1	5	or 0.051	
	(iv)	PE = 75x9.8x200sin10° (25530)	B1		**OR** 75x9.8sin10° -120= 75a	
		WD = 200x120 (24000)	B1		(M1 + A1)	
		½.75v^2 =	M1		a = 0.102 (A1)	
		½.75.13^2+75x9.8x200sin10°-200.120	A1		v^2 =169+2x0.102x200 (M1)	
		14.5 ms^{-1}	A1	5	v = 14.5	**14**

8	(i)	R cos30° = 0.1 x 9.8	M1		resolving vertically	
			A1			
		R = 1.13 N	A1	3		
	(ii)	r = 0.8cos30° = 0.693 or 2√3/5	B1		may be implied	
		Rcos60° = 0.1 x 0.693 ω^2	M1		or 0.1v^2/r & ω = v/r	
			A1			
		ω = 2.86	A1	4		
	(iii)	T = 1.96 N	B1	1		
	(iv)	Rcos30° = Tcos60° + 0.1x9.8	M1			
			A1			
		R = 2.26 N	A1			
		Rcos60° + Tcos30° = 0.1 x v^2/r	M1		or mrω^2 & use of v = rω	
			A1		with R=1.13 can get M1 only	
		4.43 ms^{-1}	A1	6		**14**
or	(iv)	LHS (or RHS)	M1*		method without finding R	
		T + 0.1x9.8cos60°	A1		i.e. resolving along PA	
		RHS (or LHS)	M1*			
		0.1 x v^2/r x cos30°	A1		r to be 0.8 cos30° for A1	
		solve to find v	M1*		depends on 2* Ms above	
		4.43 ms^{-1}	A1	(6)		

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1	$\pm(5.4\cos 45^{o} - 8.7)$	M1		For attempting to find Δv in **i** dir'n
		M1		For using I = m(Δv) in **i** direction
	$I\cos\theta = \pm 0.4(5.4\cos 45^{o} - 8.7)$	A1		(= $\mp$ 1.953)
	$I\sin\theta$ = 0.4x5.4sin45	B1		(= 1.527)
	$I = \sqrt{(1.527^2 + 1.953^2)}$ or $\theta = \tan^{-1}[1.527/(-1.953)]$	M1		For using Pythagoras or trig.
	Magnitude is 2.48 $kgms^{-1}$	A1		
	Direction is 142^{o} to original dir'n.	A1	[7]	Accept $\theta = 38.0^{o}$ with θ shown appropriately
OR		M1		For using Impulse = mass x Δv
		M1		For appropriate use of cosine rule
	$I = 0.4\,(5.4^2 + 8.7^2 - 2x5.4x8.7\cos 45^{o})^{½}$	A1		
	Magnitude is 2.48 $kgms^{-1}$	A1		
		M1		For appropriate use of sine rule
	$\sin\theta/5.4 = \sin 45^{o}/6.1976$	A1		
	$\theta = 38.0^{o}$	A1		

2	(i)	M1		For correct use of Newton's 2nd law
	$0.5dv/dt = 1 + kt^2$	A1		
	$v = 2t + 2kt^3/3$	A1	[3]	
				SR(max 1/3) for omission of mass but otherwise correct $v = t + kt^3/3$ B1
	(ii) $x = t^2 + kt^4/6$	M1		For integration w.r.t. t
	2 = 1 + k/6	M1		For substitution and attempting to solve for k
	k = 6	A1		
		M1		For attempting to solve quadratic in t^2 for t
	t = 2	A1	[5]	With no extra solutions

3	(i)	M1		For use of EE formula
	$EE = \lambda \times (5-3)^2/(2 \times 3)$	A1		
	$2\lambda/3$ = 1.6 x 9.8 x 5	M1		For equating EE and PE
	λ = 117.6 N	A1	[4]	AG
	(ii)	M1		For use of conservation of energy
	$0.5x1.6v^2 = 1.6x9.8x4.5 - 117.6x1.5^2/(2x3)$	A2,1,0		-1 each error
	v = 5.75 ms^{-1}	A1	[4]	

4	Perp. vel. of A after impact = 4	B1		
		M1		For using cons'n of m'm'tum // l.o.c
	[5x0] - 2x4 = 5a + 2b	A1		
		M1		Using N.E.L. // l.o.c.
	0.75 x 4 = b-a	A1		
		M1		For solving sim. equ.
	Speed of B is 1ms^{-1}; direction //l.o.c. and to the right	A1		
	$v_A = \sqrt{(4^2 + (-2)^2)}$	M1		For method of finding the speed of A
	tan(angle) = 4/2	M1		For method of finding the direction of A
	Speed of A is 4.47 ms^{-1}; direction is 63.4^o to l.o.c. and to the left	A1	[10]	

5	(i)	M1		For any moment equ. that includes F and all other relevant forces
	1.8F = 0.9x40 +1.4x9	A2,1,0		-1 each error
	Magnitude is 27 N	A1	[4]	AG
	(ii) Vertical comp. is 22 N downwards	B1		
		M1		For any moment equ. that includes X and all other relevant forces
	1.2X = (40+9-27)x(3.8-1.8) + 64 x1 (1.2X = 44 + 64)	A2,1,0 ft		-1 each error. ft wrong vert. comp.
	Horizontal comp. is 90 N to the left	A1	[5]	
	(iii) μ = 27/[90]	M1		For use of μ =F/R
	Coefficient of friction is 0.3	A1 ft	[2]	ft wrong answer in (ii)

6	(i)	M1		For use of conservation of energy
	$0.5\text{x}0.3v^2 - 0.5\text{x}0.3\text{x}2^2 = 0.3\text{x}9.8\text{x}0.5\cos60 - 0.3\text{x}9.8\text{x}0.5\cos\theta$	A2,1,0		-1 each error
	$v^2 = 8.9 - 9.8\cos\theta$	A1	[4]	AG
	(ii)	M1		For using Newton's 2nd law radially
	$T + 0.3\text{x}9.8\cos\theta = 0.3v^2/0.5$	A1		
	$T + 2.94\cos\theta = 0.6(8.9 - 9.8\cos\theta)$	M1		For correct substitution for v^2
	Tension is$(5.34 - 8.82\cos\theta)$N	A1	[4]	Accept any correct form
	(iii)	M1		For using T = 0
	Basic value $\theta = 52.7^o$	A1 ft		ft any T of the form a - b$\cos\theta$
	Angle = (360-52.7) - 60	M1		
	Angle turned through is 247^o	A1	[4]	

7	(i)	M1		For using $T = \lambda e/L$ once
	For 180e/1 or 360(0.8-e)/1.2 **or** T_A = 180 x 0.5/1 or T_B = 360 x 0.3/1.2	A1		
	480e = 240 or T_A = 90, T_B = 90	M1		For using $T_A(e) = T_B(e)$ or attempting to show $T_A = T_B$ when BQ = 1.5
	BQ = 1 + 0.5 = 1.5 m or $T_A = T_B$	A1	[4]	AG
	(ii) T_B = 360(0.3 - x)/1.2	B1		
	T_A = 180(0.5 + x)	B1		
	$1.2d^2x/dt^2$ = 300(0.3-x) - 180(0.5+x)	M1		For using Newton's 2nd law
	$d^2x/dt^2 = -400x$	A1		
	Period is $2\pi/\sqrt{[400]}$ = 0.314 s	A1	[5]	AG
	(iii)	M1		For using $T_B = 0$
	Max amplitude = 1.5 - 1.2 = 0.3 m	A1		
	amplitude = $u/\sqrt{400}$ or $180 \times 0.5^2/(2 \times 1) + 360 \times 0.3^2/(2 \times 1.2) + \frac{1}{2} 1.2 u_{max}^2 = 180 \times 0.8^2/(2 \times 1)$	M1		For using Amp. = u/ω or 'energy at equil. pos'n = energy at max. displ.'
	Maximum value of u is 6	A1	[4]	AG
	(iv) -0.2 = 0.3sin20t	M1		For relevant trig. equation
	20t = 0.7297 + 3.142	M1		For method of obtaining relevant solution
	Time taken is 0.194s	A1	[3]	

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1(i)	$^2/_3$+prod of 2 P's or 1– prod of 2 P's $^2/_3 + ^1/_3 \times ^3/_4$ or $1 - ^1/_3 \times ^1/_4$ $= ^{11}/_{12}$ or 0.917 (3 sfs)	M1 M1 A1 **3**	or $^1/_3 \times ^3/_4$ or $^1/_3 \times ^1/_4$
(ii)	$^1/_3 \times p$ $^2/_3 + ^1/_3 \times p = ^5/_6$ oe $p = ^1/_2$	M1 M1 A1 **3**	or $^1/_3(1-p)$ or $^1/_3(1-p) = 1 - ^5/_6$ SW: $½ \times ^1/_3 = ^1/_6$ M2A0, unless clear this is a check
Total		**6**	
2(i)	124.5, 4.8	B1B1 **2**	for 4.8 allow "same"
(ii)	mean smaller or generally smaller or means similar or hts similar oe More widely spread or varied oe	B1f B1f **2**	Assume 2nd referred to unless clear 1st or less consistent or gter dispersion or further from mean, gter variance Not "range" greater Allow opposite if ft (i)
(iii)	("124.5" + 2 x 123)/3 = 123.5	M1 A1 **2**	or (50 x "124.5" + 100 x 123)/150 cao
Total		**6**	
3(i)	$^3/_5 \times ^2/_4 \times ^1/_3$ or $^2/_5 \times ^3/_4 \times ^1/_3$ x 2 or + $^3/_5 \times ^2/_4 \times ^1/_3 + ^2/_5 \times ^3/_4 \times ^1/_3$ $= ^1/_5$ AG	M1 M1 M1 A1 **4**	or $^1/_{10}$ from tree add 2 equal products of 3 probs all correct Must see correct working NB incorrect methods eg $^3/_5 x^2/_4 x^2/_3$
(ii)	Σxp = 4 Σx^2p (= 17) $-\mu^2$ = 1	M1 A1 M1 M1 A1 **5**	$\geq$ 3 terms added. Allow arith errors. $\geq$ 3 terms added. Allow arith errors Indep if +ve result $\Sigma(x-\mu)^2p$ M2; 3 terms: M1 dep +ve result Σxp & Σx^2p, if ÷ eg 4: M0A0 (- μ^2 poss M1)
Total		**9**	

4(i)(a)	Total area = 60 sqs Recog that total area reps 300 8 x 300/60 = 40	M1 M1 M1 A1 **4**	Attempt total area, eg 15000 or 15 cm^2 eg 1 squ = 5 or 15000 ÷ (300 or 50) or 2000/50 cao
(b)	Splitting classes 1.2x4x5 <u>or</u> 0.8x6x5 oe 48	M1 M1 A1 **3**	or 0.3x16x5 <u>or</u> 0.4x12x5 or 24 NB other correct eg 2x4x5 + $^4/_5$x2x5 Alt method: estimate: 46-50 SC B1
(ii)(a) (b)	Box & whisker Cum freq diag	B1 **1** B1 **1**	
Total		**9**	
5(i)(a)	$(^3/_5)^4$ x $^2/_5$ = 0.0518 (3sfs) or $^{162}/_{3125}$ oe	M1 A1 **2**	Allow index 3 or 5
(b)	$(^3/_5)^4$ $1 - (^3/_5)^4$ = 0.870 (3 sfs) or $^{544}/_{625}$ oe	M1 M1 A1 **3**	$^2/_5 + {^3/_5}\text{x}{^2/_5} + (^3/_5)^2\text{x}{^2/_5} + (^3/_5)^3\text{x}{^2/_5}$: M2 (1extra or omit or wrong: M1) Allow $1 - (^3/_5)^3$ or $1 - (^3/_5)^5$
(ii)(a)	B(5, $^2/_5$) stated 5 x $^2/_5$ x $(^3/_5)^4$ or 0.3370 – 0.0778 = 0.259 (3 sfs) or $^{162}/_{625}$ oe	M1 M1 A1 **3**	or $(^5C_a$ or $^5C_b)\text{x}(^2/_5)^a\text{x}(^3/_5)^b$ & $a + b = 5$
(b)	"0.259" x $^2/_5$ = 0.104 (3 sfs) or $^{324}/_{3125}$ oe	M1 A1f **2**	eg ft: (a) 0.0518 → 0.0207 (a) 0.922 → 0.369
Total		**10**	
6(i)	$^4C_3$ x $^7C_4$ = 140	M1M1 A1 **3**	M1 either comb. 140/330: M1M1
(ii)	$^3C_2$ x $^6C_4$ or $\frac{^3C_2}{^4C_3}$ or $\frac{^6C_4}{^7C_4}$ $\frac{^3C_2 \text{ x } ^6C_4}{\text{"140"}}$ or $^3/_4$ x (1 – $^4/_7$) = $^9/_{28}$ oe or 0.321 (3 sfs)	M1 M1 A1 **3**	or $^3C_2$(x..)/"140" or (…x)$^6C_4$/"140" or ($^3C_2$+$^6C_4$)/"140" or (3+15)/"140" or $^3/_4$ or 1–$^4/_7$ seen all correct
(iii)	$^3C_2$x $^6C_4$ (or i x ii) or ($^3C_3$x)$^7C_4$ or 45 or 35 or $^1/_4$x$^4C_3$x$^7C_4$ or $^3/_4$x $^4C_3$x$^6C_4$ $^3C_2$x$^6C_4$+($^3C_3$x)$^7C_4$ or "140" –$^3C_2$x$^6C_3$ = 80	M1 M1 A1ft **3**	1 correct prod or "140"– any prod or $^1/_4$x$^4C_3$x$^7C_4$ + $^3/_4$x $^4C_3$x$^6C_4$ ft only "140"
Total		**9**	

7(i)	Binomial $n = 10$, $p = 0.9$ Each seed equally likely germ or P(germ) same for all seeds oe Seeds independent oe	B1 B1 B1 B1 **4**	 Both requ'd. Ignore $q = 0.1$ or seeds grown in same conditions Context nec'y for each B1
(ii)	0.0702 (3 sfs)	 B2 **2**	0.07 or 0.2639: B1 Σ or 1-Σ: 1 term extra or omit or wrong: M1
(iii)	1 – "0.0702" $0.9298^{20} + {}^{20}C_1 \times 0.0702 \times 0.9298^{19}$ = 0.585 (3 sfs)	 M1 M1M1 A1 **4**	 Or 0.9298 or 0.93(0) seen M1 each term cao eg ft (ii) 0.2639 → (iii) 0.0178 from correct wking: M3A0 $0.0702^{20}+{}^{20}C_1 \times 0.9298 \times 0.0702^{19}$ $(=2.25\times10^{-21})$: SC M1M1 NB ft (ii) for all M mks. But if 0.1, 0.9 used, must be clear using (ii) rounded
Total		**10**	

8(i)(a)	Ranks 1 2 3 4 5 6 7 8 9 9 8 7 6 5 4 3 2 1 3 2 1 5 4 7 8 6 9 7 8 9 5 6 3 2 4 1 Σd^2 (= 16) $r_s = 1 - \frac{6 \text{ x their } 16}{9 \text{ x } (9^2-1)}$ = 0.867 (3 sfs) or $^{13}/_{15}$ oe	M1 A1 M1dep M1dep A1 **5**	Attempt ranks, same dir'n Correct ranks Dep ranks attempted Correct formula with n = 9, dep M1M1
(b)	Countries with larger pops tend to have larger capital pops. oe	B1ft **1**	or ft (a) Must <u>interp</u> & refer to context. Not "Gd corr'n country & cap pops" Not "Gd agree't country & cap pops" Not "Gd rel'nship country & cap pops" Not "proportional"
(ii)	$\frac{1533.76 - (337.5 \text{ x } 28.3)/9}{\sqrt{((18959.11-337.5^2/9)(161.65-28.3^2/9))}}$ = 0.698 (3 sfs)	M1 A1 **2**	(= 472.51/√(6302.86x72.66)) Or correct subst in 2 "S" formulae, any version No wking: 0.7 M0A0; 0.70: M1A0
~~(iii)~~	~~Increase~~	~~B1~~ **1**	or nearer to 1
~~(iv)~~(a)	~~x on y~~ Est country pop from cap or x from y oe	B1ind B1ind **2**	y indep or known or given or x unknown or x dep on y oe
~~(b)~~	~~any indication different context,~~ eg "Africa", "remote areas" unreliable	B1 B1dep **2**	or reliable because r (or r_s) high: B1 or unreliable because r (or r_s) not hi: B1 "accurate": B0
Total		**13**	

Total 72 marks

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1	(i) (a) Po(2): 1 – P(≤ 3) = 0.1429	M1 A1	 2	Po(2) tables, "1 – " used Answer, a.r.t. 0.143
	(b) Po(2/3): $e^{-2/3}\frac{(\frac{2}{3})^2}{2!}$ = 0.114	M1 M1 A1	 3	Parameter 2/3 Poisson formula correct, $r = 2$, any μ Answer, a.r.t. 0.114
	(ii) Foxes may congregate so not independent	B1 B1	 2	Independent/not constant rate/singly used Any valid relevant application in context
2	N(80/7, 400/49) $\frac{13.5 - \frac{80}{7}}{\frac{20}{7}}$ = 0.725 1 – Φ(0.725) = 0.2343	B1 B1 M1 A1 A1 M1 A1	 7	80/7, a.e.f (11.43) 400/49 or 20/7 seen, a.e.f. (8.163 or 2.857) Standardise with *np* & *npq* or √*npq* or *nq*, no √*n* √*npq* correct 13.5 correct Normal tables used, answer < 0.5 Answer, a.r.t. 0.234 [SR: Binomial, complete expression M1, 0.231 A1 Po(80/7) B1, complete expression M1, 0.260 A1 Normal approx to Poisson, B1B0 M1A0A1 M1A0]
3	H_0: $p = 0.3$ H_1: $p \neq 0.3$ B(8, 0.3) P(≤ 4) = 0.9420; P(> 4) = 0.0580 P(≤ 5) = 0.9887; P(> 5) = 0.0113 Compare 0.025 or critical value 6 Do not reject H_0 Insufficient evidence that manufacturer's claim is wrong	B1 B1 M1 A1 M1 M1 A1√	 7	NH stated, must be this form (or π) AH stated, must be this form (or π) [μ: B1 both] B(8, 0.3) stated or implied Any one of these four probabilities seen *Either* compare P(≥ 5) & 0.025 / P(≤ 4) & 0.975 *Or* critical region ≥ 6 with 5 H_0 not rejected, can be implied, needs essentially correct method Correct conclusion in context [SR: Normal, Poisson: can get B2M1A0M0M1A1 P(≤ 5): first 4 marks. P(= 5): first 3 marks only.]
4	(i) B(80, 0.02) approx Po(1.6) 1 – P(≤ 1) = 1 – 0.5249 = 0.4751	M1 M1 M1 A1	 4	B(80, 0.02) seen or implied, e.g. N(1.6, 1.568) Po(*np*) used 1 – P(≤ 1) used Answer, a.r.t. 0.475 [SR: Exact: M1 M0 M0, 0.477 A1]
	(ii) P(≤ 4) = 0.9763, P(≥ 5) = 0.0237 P(≤ 5) = 0.9940, P(≥ 6) = 0.0060 Therefore least value is 6	M1 A1 A1	 3	Evidence for correct method, e.g. answer 6 At least one of these probabilities seen Answer 6 only [SR N(1.6,1.568): 2.326 = $(r - 1.6)/\sqrt{1.568}$ M1 $r = 5$ or (with cc) 6 A1 Exact: M1 A0 A1]

5	(i) $\frac{0-\mu}{\mu/2} = -2$, independent of μ; $1 - \Phi(2) = 1 - 0.9772 = 0.0228$	M1 A1 A1 A1	4	Standardise, allow –, allow $\mu^2/4$ $z = 2$ or -2 z-value independent of μ and any relevant statement Answer, a.r.t. 0.023
	(ii) $\Phi[(9-6)/3]$ $\Phi(1.0) = 0.8413$ $[\Phi(1.0)]^3$ $= 0.59546$	M1 A1 M1 A1	4	Standardise and use Φ *[no √n]* 0.8413 [not 0.1587] Cube previous answer Answer, in range [0.595, 0.596]
	(iii) Annual increases not independent	B1	1	Independence mentioned, in context. Allow "one year affects the next" but not "years not random"
6	H_0: μ = 32; H_1: μ > 32, where μ is population mean waist measurement $\overline{W} = 32.3$ $s^2 = 52214.50/50 - \overline{W}^2$ [= 1] $\hat{\sigma}^2 = 50/49 \times s^2$ [= 50/49 or 1.0204]	B1 B1 B1 M1 M1		One hypothesis correctly stated, *not* x or $\bar{x}$ or $\bar{w}$ Both completely correct, μ used Sample mean 32.3 seen Correct formula for s^2 used Multiply by 50/49 or √
	α: $z = (32.3 - 32)\times\sqrt{49}$ $= 2.1$ Compare 2.1 with 3.09 or 0.0179 with 0.001	M1 A1 B1		Correct formula for z, can use s, aef, need μ = 32 $z = 2.1$ or $1 - \Phi(z) = 0.0179$, *not* –2.1 Explicitly compare their 2.1 with 3.09(0) or their 0.0179 with 0.001
	β: CV $= 32 + 3.09 \div\sqrt{49}$ $= 32.44$ Compare CV with 32.3	M1 B1 A1√		$32 + z \times \sigma/\sqrt{n}$ [allow ±, s, any z] $z = 3.09$ and (later) compare $\bar{x}$ CV in range [32.4, 32.5], √ on k
	Do not reject H_0 Insufficient evidence that waists are actually larger	M1√ A1√ **10**		Correct conclusion, can be implied, needs essentially correct method including $\sqrt{n}$, any reasonable σ, but not from μ = 32.3 Interpreted in context
7	(i) $\frac{80-c}{8/\sqrt{12}} = 2.326$ $c = 74.63$	M1 A1 B1 A1	4	Equate standardised variable to Φ^{-1}, allow – √12, 8 correct 2.326 or a.r.t 2.33 seen, signs must be correct Answer, a.r.t. 74.6, cwo, allow ≤ or ≥
	(ii) (a) Type I error (b) Correct	B1 B1√	1 1	"Type I error" stated, needs evidence "Correct" stated or clearly implied Wrong c: $74 < c < 75$, B1√ B1√ $c < 74$, both "correct", B1. $75 < c < 80$, both "Type I", B1 Also allow if only one is answered
	(iii) $\frac{74.63-\mu}{8/\sqrt{12}} = -1.555$ Solve for μ μ = 78.22	M1*dep A1√ dep*M1 A1 **4**		$\frac{c-\mu}{8/\sqrt{12}} = (\pm)\Phi^{-1}$, allow no √12 but not 80, not 0.8264 Correct including sign, √ on their c Solve to find μ, dep, answer consistent with signs Answer, a.r.t. 78.2

8	(i)	$\int_0^1 x^n\,dx = \left[\frac{x^{n+1}}{n+1}\right]_0^1 = \frac{1}{n+1}$ $k/(n+1) = 1$ so $k = n + 1$	M1 M1 A1 **3**	Integrate x^n, limits 0 and 1 Equate to 1 and solve for k Answer $n + 1$, *not* 1^{n+1}, c.w.o.
	(ii)	$\int_0^1 x^{n+1}\,dx = \left[\frac{x^{n+2}}{n+2}\right]_0^1 = \frac{1}{n+2}$ $\mu = \frac{k}{n+2} = \frac{n+1}{n+2}$ **AG**	M1 A1 A1 **3**	Integrate x^{n+1}, limits 0 and 1, not just $x.x^n$ Answer $\frac{1}{n+2}$ Correctly obtain given answer
	(iii)	$\int_0^1 x^5\,dx = \left[\frac{x^6}{6}\right]_0^1$ $[=\frac{1}{6}]$ $\sigma^2 = \frac{4}{6} - \left(\frac{4}{5}\right)^2 = \frac{2}{75}$	M1 M1 A1 **3**	Integrate x^5, limits 0 and 1, allow with n Subtract $\left(\frac{4}{5}\right)^2$ Answer $\frac{2}{75}$ or a.r.t. 0.027
	(iv)	$N\left(\frac{4}{5}, \frac{2}{7500}\right)$	B1 B1 B1√ **3**	Normal stated Mean $\frac{4}{5}$ or $\frac{n+1}{n+2}$ Variance their (iii)/100, a.e.f., allow √
	(v)	Same distribution, translated Mean 0 Variance $\frac{2}{75}$	M1 A1√ B1√ **3**	Can be negative translation; *or* integration, must include correct method for integral (Their mean) – $\frac{4}{5}$, c.w.d. Variance same as their (iii), or $\frac{2}{75}$ by integration

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STATISTICS 3

1	(i)	$p_S \pm z\sigma_{est}$	M1		Use formula,σ involving p_S and 400.
		p_s=186/400(0.465)	A1		
		$\sigma_{est}=\sqrt{\dfrac{0.465\times0.535}{400}}$	B1		
		z=1.96	A1		
		(0.416,0.514)	A1	**5**	
	(ii)	Councillor statement implies p=0.5. CI does contain 0.5 but only juat so councillor probably correct.	B1	**1**	Any justifiable comment **Not too assertive**
2	(i)	σ^2 unknown	B1	**1**	
	(ii)	H_0: μ=2000 (or ≥), H_1:μ <2000	B1		
		$\bar{x}$ =1958.2, s=115.57	B1B1		or 1958,115.6
	EITHER:	Test statistic = $\dfrac{1958.2-2000}{115.57/2}$	M1		
		=-0.7234	A1		art –0.723
		Critical value –1.638	B1		
		Test statistic not in CR, accept H_0		M1	Or equivalent
		Accept that specification is being met	A1		Conclusion in context
	OR:	Critical region:			
		$\dfrac{\bar{x}-2000}{115.57/2} < t$	M1		
		t=-1.638		B1	
		$\bar{x}$ < 1905.2	A1		art 1900 or 1910
		As above	M1A1	**8**	Conclusion in context
3	(i)	Use of $\int_{20}^{a}$ f(t)dt	M1		With limits and f(t) substituted
		$\left[-\dfrac{2}{3}\cos\dfrac{\pi t}{60}\right]_{20}^{a}$	A1		
		AG	A1	**3**	Properly obtained
	(ii)	3×(i) + 2×(1-(i))	M1		Idea of expectation
			A1		All correct
		Equate to 2.80 and attempt to solve		M1	From equation in a, 2 or 3
		a=44.8	A1	**4**	Accept 45
					SR: ⅓(1-2cos..)= 0.8 give max3/4

_

4	(i)	Use Poisson distribution	M1			Po(5.5) or Po(55) seen
		With μ=55	B1			
		σ^2=55	A1			
		$(39.5-55)/\sqrt{55}$	A1			Standardising, with ,without or wrong cc
		-2.09	A1			
		art 0.982		A1	**6**	
	(ii)	E(*X*- *Y*)=37	B1√			ft μ above
		Var(*X*-*Y*)=55+18		M1		
		=73	A1√		**3**	ft μ above
	(iii)	EITHER: Expectation not equal to variance OR: *X*-*Y* could be negative OR: Difference of two Poisson variables could have a negative expectation		M1		Any one
		So *X*-*Y* does not have a Poisson distn	A1		**2**	
5	(i)EITHER:	Use ⅛$(3-1)^2$=a(3-2)				
	OR:	a(4-2)=1		M1		Continuity of F
		a=½	A1		**2**	
	(ii)	F(1.8)=⅛$(0.8)^2$ =0.08	M1			Appropriate use of F
		$C_X(8)$=1.8	A1		**2**	
	(iii)	G(*y*)=P(*Y*≤*y*)=P($(X-1)^{1/2}$≤*y*)	M1			
		=P(*X* ≤ y^2+1)		A1		
		=F(y^2+1)	A1			
		$G(y)=\begin{cases}\frac{1}{8}y^4 & (0\le y\le\sqrt{2}),\\ \frac{1}{2}(y^2-1) & (\sqrt{2}<y\le\sqrt{3}).\end{cases}$	A1			
		Ignore others, A1 for both ranges of y	B1		**5**	
	(iv)	Use G(*y*) to find $C_Y(8)$	M1			
		Obtain √(0.8)	A1			
		Correct verification	B1		**3**	

6	**(i)**	s^2 =(8×0.7400+9×0.8160)/17	M1		Formula for pooled estimate
		=0.7802	A1		At least 4DP shown
		0.780 AG		**2**	
	(ii)	Assumes braking distances have normal distributions	B1		
		Use $\bar{x}_A - \bar{y}_B \pm t\sigma$	M1		Must be *t* value
		t=2.567	A1		
		σ=√[0.7802(1/10+1/9) (0.40584)	B1		Allow 0.780
		(1.518,3.602)	A1	**5**	art (1.52, 3.60)
	(iii)	$H_0{:}\mu_A\text{-}\mu_B=2$, $H_1{:}\mu_A\text{-}\mu_B > 2$	B1		For both hypotheses
		Use of CV, 1.740	B1		
	EITHER:	Test statistic=(2.56-2)/σ	M1		Standardising, σ as above
		=1.38	A1		Rounding to 1.38
	OR:	Critical region			
		$\bar{x}_A - \bar{x}_B > 2 + 1.74 \times 0.4054$	M1		
		=2.7054	A1		2.70 or 2.71
		Indication that test statistic is not in critical region	M1		Not from different signs test statistic
		and Insufficient evidence to accept claim and H_1	A1	**6**	critical value. A1 dep on correct H_0
7	(i)	Use $\int_1^\infty x\alpha x^{-\alpha-1}dx = (\int_1^\infty \alpha x^{-\alpha}dx)$	M1		
		$\left[\dfrac{-\alpha x^{-\alpha+1}}{\alpha-1}\right]_1^\infty$	B1		Correct limits not required
		= $\alpha/(1-\alpha)$ AG	A1	**3**	Properly obtained
	(ii)	$\alpha/(1-\alpha)$=1.92 giving 2.087 AG	B1	**1**	
		(iii) Integral of $2.087x^{-3.087}$ from 2 to 3		M1	
		$[-x^{-2.807}]_2^3$	A1		
		×200	A1		
		Obtain AG	A1	**4**	Evidence required
	(iv)	Combine last 3 cells	B1		
		$X^2 = 6.9^2/152.9 + 6.1^2/26.9$ $+ 4.9^2/9.1 + 4.1^2/11.18$	M1		Accept one error
			A1		All correct
		=5.847…	A1		art 5.8
		Use CV 5.991	B1 √		ft number of sells used.
		Accept that data supports Zipf's law	B1	**6**	
		SR: From 6 cells: B0M1A1 (for 9.34) then B1 for 9.488, B1 Max 4/6			

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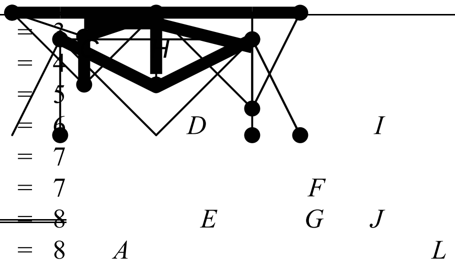
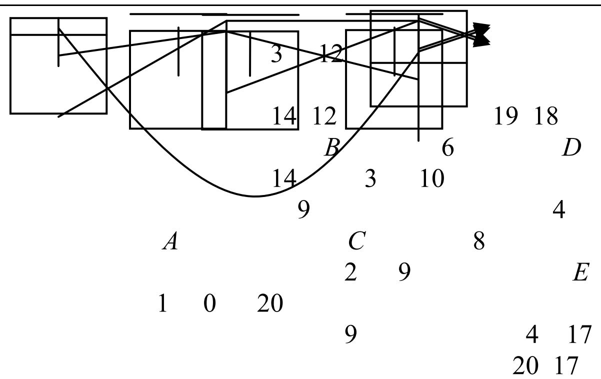
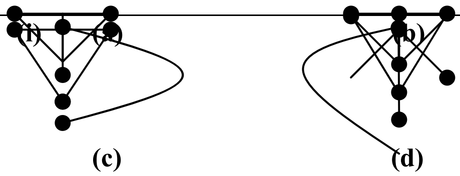
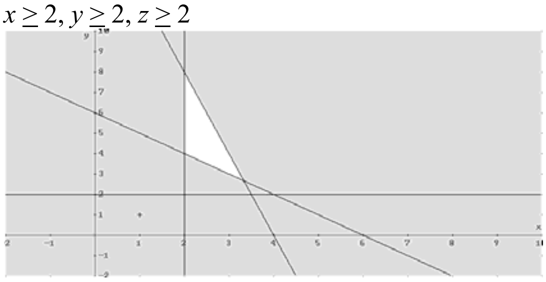
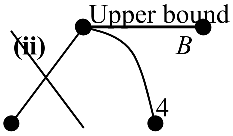
1	*BC* = 3 *FG* = 4 *JL* = 5 *EG* = 6 *AE* = 7 *BG* = 7 ~~*AB* = 8~~ *CH* = 8 *DF* = 8 *GJ* = 8 *HK* = 8 ~~*AC* = 9~~ ~~*DE* = 9~~ *FI* = 9 ~~*GH* = 9~~ ~~*IJ* = 9~~ ~~*JK* = 9~~ ~~*AD* = 10~~ ~~*DG* = 10~~ ~~*GK* = 10~~ ~~*IL* = 10~~ ~~*KL* = 10~~ ~~*GI* = 11~~ ~~*CG* = 12~~ ~~*DI* = 12~~  Total weight = 73	M1 A1 M1 A1 B1 **5**	For selecting all arcs up to *AB* and deleting *AB* in list For deleting *AC*, *DE* in list and selecting arcs for tree correctly, indicated in any way For a spanning tree drawn For correct (minimum) spanning tree drawn For total = 73
2		M1 M1 A1 B1 M1 A1 **6**	For temporary labels at *B* correct, no extras For temporary labels at *E* correct, no extras For permanent labels correct at *B*, *C* and *E* (dependent on both M marks) For order of labelling correct at *C*, *B* and *E* For temporary labels at *D* correct For no permanent label at *D*
3	**(i)**  **(ii)** $2n$ if n is even $2n + 1$ if n is odd	B1 B1 B1 B1 (4) M1 (2) A1 **6**	For a correct graph for (a) For a correct graph for (b) For a clearly correct graph for (c) For a clearly correct graph for (d) For treating the cases *n* odd and *n* even separately For both rules correct

4	**(i)**	P x y z s t 1 -5 4 3 0 0 0 0 2 -3 4 1 0 10 0 6 5 4 0 1 60	M1 A1 (2)	For overall structure correct, including two slack variable columns For a correct initial tableau, with no extra constraints added
	(ii)	Pivot on 2 in x column r1 = r1 + 5npr r2 = r2 ÷ 2 r3 = r3 - 6npr 1 0 -3.5 13 2.5 0 25 0 1 -1.5 2 0.5 0 5 0 0 14 -8 -3 1 30	M1 A1 M1 A1	For the correct pivot choice for their tableau For dealing with the pivot row correctly (formula or numerical) For dealing with the other rows correctly (formulae or numerical) For a correct tableau (not ft)
		$x = 5$, $y = 0$, $z = 0$ $P = 25$	B1 (6) B1 **8**	For reading off x, y and z from their tableau For reading off P from their tableau

5	**(i)**	x = number of lengths swum using breaststroke y = number of lengths swum using backstroke z = number of lengths swum using butterfly	B1	For defining variables as 'number of lengths swum' using each stroke, or equivalent
		Maximise $2x + y + 5z$	B1 (2)	For a correct expression using their variables
	(ii)	$x + y + z \geq 8$ $2x + 0.5y + z \leq 10$ $x \geq 2$, $y \geq 2$, $z \geq 2$	B1 B1 B1 (3)	For a correct expression using their variables For a correct expression using their variables For correct expressions using their variables
	(iii)		M1 A1	For plotting the sloping lines correctly For completely correct shading
		(2, 4), (2, 8), (3.3, 2.7)	M1 A1	For two correct vertices from their graph For all three vertices correct to at least 1 dp
		2×2 + 8 = 12 2×3.33 + 2.67 = 9.33	M1	For calculating P at vertices or using a valid line of constant profit or writing down their max point
		So maximum is when $x = 2$ and $y = 8$	A1 (6)	For the correct values
	(iv)	Swim 2 lengths using breaststroke, 8 lengths using backstroke and 2 lengths using butterfly	B1	For interpreting their solution in the context of the original problem (at least for x and y)
		Total = 22 style marks	B1 (2) **13**	For calculating the number of marks for their solution

6	(i)	*A-B-D-E-G-F-C-A* 42 minutes *A-B-D-C-F-G-E-A* 46 minutes Upper bound = 42 minutes	M1 A1 B1 B1 B1ft(5)	For *A-B-D-E-G-F-C*, with or without closing tour For 42 For *A-B-D-C-F-G-E*, with or without closing tour For 46 For the smaller of their two times
	(ii)	 *B* 6 *E* 4 5 *A* *D* 6 *C* 10 *F* eg } *AB* *BD* *AC* *BE* *CF* *A B D C E F* or A B D E C F Total weight of tree = 31 minutes Two least weight arcs from *G* have weight 5+5 =10 minutes Lower bound = 31 + 10 = 41 minutes	M1 A1 B1 A1 ft M1 A1 (6)	For a diagram or listing showing a tree connecting the vertices *A*, *B*, *C*, *D*, *E* and *F*, but not *G* For a diagram showing this tree (vertices need to be labelled, but arc weights are not needed) For a valid vertex or arc order For the total weight of their tree stated For stating or using *GE*, *GF* or 5+5 or 10 For 41 or 10 + their 31 calculated
	(iii)	Odd nodes: *B D E F* *BD* = 5 *BE* = 6 *BF* = 16 *EF* = <u>10</u> *DF* = <u>14</u> *DE* = <u>7</u> 15 20 23 120 minutes Travel *BD*, *EG* and *FG* twice (accept *BD*, *EGF*) 3 times	B1 M1 A1 B1 (5) B1 **[16]**	For identifying or using *B D E F* For calculating 5+10 or 6+14 or 16+7 (may be implied from correct pair chosen) For 120 (unsupported 120 scores 0 marks) For correct arcs listed and no others For 3

7	(i)	Original list: 34 42 27 31 12 48 24 37 1st pass: 34 27 31 12 42 24 37 <u>48</u> 2nd pass: 27 31 12 34 24 37 <u>42 48</u> 3rd pass: 27 12 31 24 34 <u>37 42 48</u> 4th pass: 12 27 24 31 <u>34 37 42 48</u> 5th pass: 12 24 27 <u>31 34 37 42 48</u> 6th pass: <u>12 24 27 31 34 37 42 48</u> Swaps = 5+5+2+2+1 =15 Comparisons = 7+6+5+4+3+2 = 27	M1 M1 M1 A1 B1 B1 (6)	nb decreasing or numbers misread $\Rightarrow$ M only For result of first pass correct (underlined entries may be omitted) For second and third passes correct, must be using bubble sort For fourth and fifth passes correct, must be using bubble sort For sixth pass correct, from correct method For 15, from correct method For 27, from correct method
	(ii)	Original list: 95 74 61 87 71 82 53 57 1st pass: 74 95 <u>61 87 71 82 53 57</u> 2nd pass: 61 74 95 <u>87 71 82 53 57</u> 3rd pass: 61 74 87 95 <u>71 82 53 57</u> 4th pass: 61 71 74 87 95 <u>82 53 57</u> 5th pass: 61 71 74 82 87 95 <u>53 57</u> 6th pass: 53 61 71 74 82 87 95 <u>57</u> 7th pass: 53 57 61 71 74 82 87 95 Swaps = 1+2+1+3+2+6+6 = 21 Comparisons = 1+2+2+4+3+6+7 = 25	M1 M1 M1 A1 B1 B1 (6)	nb decreasing or numbers misread $\Rightarrow$ M only For result of first pass correct (underlined entries may be omitted) For second and third passes correct, must be using shuttle sort For fourth and fifth passes correct, must be using shuttle sort For seventh pass correct, from correct method For 21, from correct method For 25, from correct method
	(iii)	Each script is looked at once so the time taken is roughly proportional to the number of scripts	B1 B1 (2)	For 'each script is looked at once', or equivalent For 'proportional', or equivalent
	(iv)	} Splitting 100 scripts takes 50 seconds so splitting 500 scripts takes about 250 seconds Sorting 50 scripts takes 250 seconds = 0.1×50^2 Sorting 250 scripts takes about 0.1×250^2 = 6250 seconds Total = 6500 seconds or 108 minutes 20 seconds	M1 M1 A1 (4) A1 **[18]**	250 (but not for 250 + 50) $(500\div2)^2$, $(250)^2$, $(100\div2)^2$ or equivalent For 6250, dependent on previous M only For 6500 or equivalent

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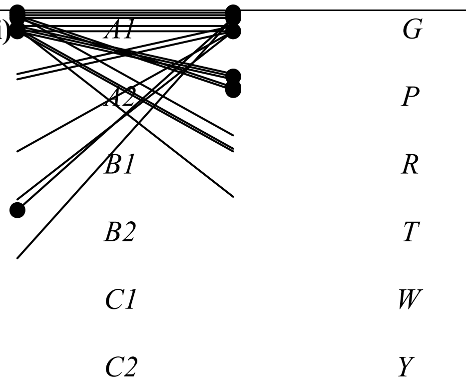
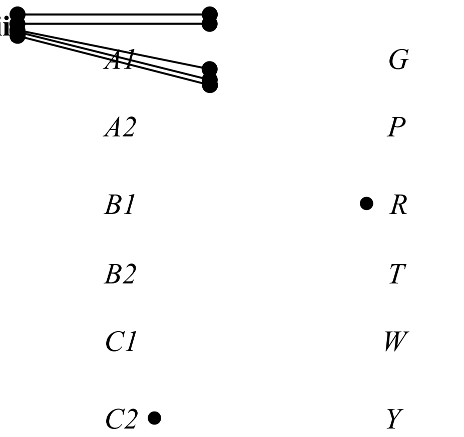
1 **(i)** 	M1 A1	For *A1* or *A2* or both joined to *G*, *P* and *R* *B1* or *B2* or both joined to *G*, *T*, *W* and *Y* and *C1* or *C2* or both joined to *P*, *W* and *Y* For both *A1* and *A2* joined to *G*, *P* and *R* both *B1* and *B2* joined to *G*, *T*, *W* and *Y* and both *C1* and *C2* joined to *P*, *W* and *Y*
(ii) 	B1	For *A1* paired with *G* and *A2* with *P*, or vice versa; *B1* paired with *T* and *B2* with *W*, or vice versa; one of *C1*, *C2* paired with *Y* and the other left unpaired
(iii) $R - A2 - P - C2$ or in reverse Accept $R - A - P - C$ or in reverse Amanda gets green and red Ben gets turquoise and white Carrie gets yellow and pink	M1 A1 B1	For a valid alternating path for their diagram (need not be minimum) For this alternating path For this matching (may use *G*, *R*, etc.)
(iv) Amanda gets pink and red Ben gets green and turquoise Carrie gets white and yellow	B1 7	For this matching (may use *G*, *R*, etc.)

2

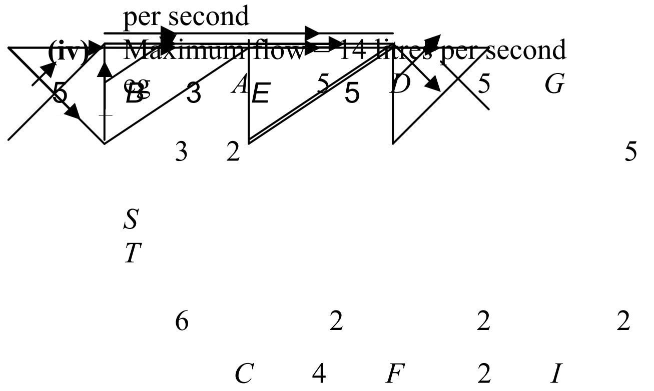
Stage	State	Action	Cost	Minimum
1	0	0	3	3
	1	0	4	4
	0	0	6+3 = 9	
2		1	4+4 = 8	8
	1	0	6+3 = 9	9
		1	7+4 = 11	
3	0	0	5+8 = 13	
		1	3+9 = 12	12

Shortest route: (3; 0) – (2; 1) – (1; 0) – (0; 0)
Length of shortest route = 12 km

Mark	Guidance
M1	For completing cost column for stage 2
A1	For completing cost column for stage 3
M1	For completing minimum column for stage 2
A1	For completing minimum column for stage 3
B1	For this route (not in reverse)
B1 [6]	For 12

3

	Answer	Mark	Guidance
(i)	6+3+2+4 = 15 litres per second	B1	For 15
(ii)	6+2+3+6+2+2 = 21 litres per second	M1	For showing how 21 (given) was worked out
	Do not use arc *DG* as it is crossed twice or X = {*S,A,B,C,E,F*} Y = {*D,G,H,I,T*}	A1	For explaining why arc *DG* is not used
(iii)	*SC* cannot be full since the most that can leave it is 2+4 = 6 litres per second	B1	For 6 and 'out' or equivalent
	AD cannot be full since the most that can enter it is 2+3 = 5 litres per second	B1	For 5 and 'in' or equivalent
	The most that can flow in *SB* is 2+3 = 5 litres per second	B1	For *SB* = 5
(iv)	Maximum flow = 14 litres per second	B1	For 14
	eg	M1	For a feasible flow (may imply vertex labels)
		A1	For a feasible flow of 14 litres per second (directions must be shown for the A mark)



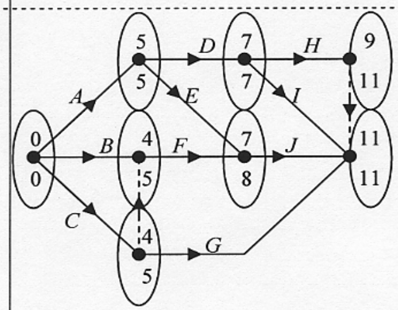
Answer	Mark	Guidance
Cut X = {*S,B,C*} Y = {*A,D,E,F,G,H,I,T*}	M1	For this cut described or drawn on diagram
This cut has capacity 14 litres per second	A1	For explicitly stating that this cut = 14
Maximum flow ≥ this flow = 14 Minimum cut ≤ this cut = 14 But maximum flow = minimum cut so 14 is the maximum flow and the minimum cut.	B1 [12]	For explaining how maximum flow = minimum cut shows that 14 is the maximum here (at least referring to 'this flow' and 'this cut', not just stating 'max flow = min cut)

4	(i)	£260	B1	For correct answer with units
	(ii)	Reduce rows 0 60 30 10 0 90 90 60 0 80 70 40 10 10 20 0	M1	Or scaled throughout by 10 For correct method for reducing rows
		Reduce columns 0 50 10 10 0 80 70 60 0 70 50 40 10 0 0 0	M1 A1	For correct method for reducing columns For a correct reduced cost matrix
		Cross through 0's using as few lines as possible 0 50 10 10 0 80 70 60 0 70 50 40 10 0 0 0	M1	For correct crossing out for their reduced cost matrix. Likely to be shown on reduced cost matrix.
		Augment by 10 0 40 0 0 0 70 60 50 0 60 40 30 20 0 0 0	M1 A1	For correct augmenting from their reduced cost matrix and their crossing out For a correct solution after first augmenting
		Cross through 0's using as few lines as possible 0 40 0 0 0 70 60 50 0 60 40 30 20 0 0 0	M1	For correct crossing out for their augmented matrix. Likely to be shown on matrix.
		Augment by 30 30 40 **0** 0 **0** 40 30 20 0 30 10 **0** 50 **0** 0 0	M1 A1	For correctly augmenting their matrix in one step by an amount greater than 10 (or greater than 1 if scaling has been used) For correct final matrix from completely correct method
		Allocation *A* = *Y* *B* = *W* *C* = *Z* *D* = *X* Cost = £180	B1 B1	For this allocation For £180
	(iii)	Hungarian algorithm finds the minimum cost complete matching	B1 **13**	For 'minimum cost' or equivalent

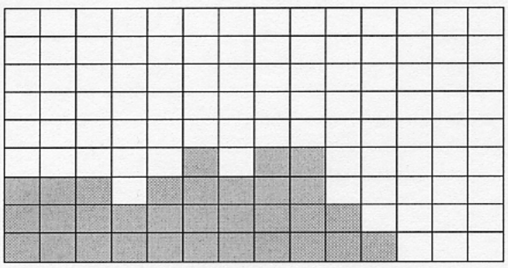
5	(i)	The dummy activity after C is needed because G follows C only but F follows both B and C.	B1	For explaining the relationships between B,C,F,G
		The dummy activity after H is needed because both H and I directly join the same pair of vertices (events).	B1	For identifying H and I
	(ii)	(table below)	B1	For A,B,C having no predecessors
			B1	For D,E,F,G correct
			B1	For H,I,J correct

Activity	Duration	Immediate predecessors
A	5	-
B	3	-
C	4	-
D	2	A
E	1	A
F	3	B,C
G	5	C
H	2	D
I	4	D
J	3	E,F

(iii)



	(iii)	(network above)	M1	For carrying out forward pass
			A1	For all early event times correct
			M1	For carrying out backwards pass
			A1	For all late event times correct
		Minimum completion time = 11 days	B1	For 11 with units
		Critical activities: A, D, I	B1	For A, D, I only
	(iv)	1 day	B1	For 1 (not 'more than 1')
	(v)	(histogram below)	B1	For the first four days correct
			M1	For a resource histogram with no 'hanging' cells
			A1	For days five onwards correct
		Number of workers required = 4	A1	For 4 or follow through their histogram if possible
	(vi)	Delay activity E by 2 days, so the start changes from 5 to 7 (it happens on day 8).	B1	For stating how much to move E
		This causes activities H and J to shift.	M1	For saying that H and J(only) will need to shift
		J shifts by 1 day and H by 2 days.	A1 19	For describing how much H and J move



6	**(i)**	In column Y: -3 < 5 so A does not dominate B In column X: -3 < 2 (or column Z: 1 < 4) so B does not dominate A	B1 B1	For -3 < 5 or equivalent For -3 < 2 or 1 < 4 or equivalent
	(ii)	The worst outcomes for Maria are: X lose 4, Y lose 5, Z lose 4	M1 A1	For finding column maxima For rejecting 5 as being bigger than 4, or using a word like 'lose' or '-4, -5, -4'
	(iii)	If Lucy plays B she could win as much as 5	B1	For '5 is the most she can win' or equivalent
	(iv)	Need to add 3 throughout matrix to make values non-negative, this removes the 3 again	B1	For 'add 3 throughout matrix' or equivalent
	(v)	Having added 3 throughout, the expected number of points win by Lucy when Maria chooses strategy X is $5p_1 + 0p_2 + 7p_3$, and similarly the second expression is the expected number of points won by Lucy when Maria chooses strategy Y and the third expression is the expected number of points won by Lucy if Maria chooses strategy Z	M1 A1	For showing where one of the expressions came from, or for referring to 'when Maria plays each of her strategies' or equivalent in a non-specific way For specifically linking the expressions to Maria choosing strategy X, strategy Y and strategy Z in that order
	(vi)	The number of points that Lucy can expect to win cannot be less than the worst of the three expressions, so it is less than or equal to each of them.	M1 A1	For reference to 'number of points won by Lucy' or equivalent For reference to 'the worst outcome' or equivalent
	(vii)	$2(p_1) + 4(p_3) = 2p_1 + 4(1-p_1) = 4-2p_1$ (given) $4p_1 - 3(1-p_1) = 7p_1 - 3$	B1 B1	For $2p + 4(1-p)$ For $7p - 3$
	(viii)	$4 - 2p_1 = 7p_1 - 3 \Rightarrow p_1 = \frac{7}{9}$ $p_1 = \frac{7}{9} \Rightarrow 2\frac{4}{9}$, $p_1 = 0 \Rightarrow \min(4, -3) = -3$, $p_1 = 1 \Rightarrow \min(2, 4) = 2$ Maximin is when $p_1 = \frac{7}{9} \Rightarrow$ choose randomly between A and C so that A is chosen with probability $\frac{7}{9}$	B1 M1 A1 **15**	For solving $4 - 2p_1$ = their expression to get a probability For evaluating $4-2p_1$ at their p_1 and the values -3 and 2 For reference to maximin, or equivalent, leading to selection of $p = \frac{7}{9}$, or in context

Report on the Units
January 2006

Pure Mathematics

Chief Examiner's Report

General Comments

All seven Core and Further Pure modules were available at this session and each unit proved to be a suitable test for candidates. The preparation of many candidates had clearly been very thorough and much excellent work was seen. The best candidates showed not only competence and insight in dealing with the various topics, but clarity and accuracy in their communication of mathematical ideas. In particular, good candidates were always precise in their use of algebraic notation.

The *List of Formulae* booklet which is available to candidates contains formulae which are potentially of use in all the units except Core Mathematics 1. It was evident, however, that many candidates did not make effective use of the booklet and so sometimes used formulae which were wrong or presented solutions which were longer than they would have been if a relevant result from the booklet had been used or adapted. It is expected that candidates will be thoroughly familiar with the booklet, knowing which results are there as well as those which are not.

Multiple attempts at questions.

In recent sessions examiners have noted an increasing number of candidates making two, or more, attempts at a question, and leaving the examiner to choose which attempt to mark. Examiners have been given this instruction.

'If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt, and ignore the others.'

Please inform candidates that it is in their best interest to make sure that, when they have a second attempt at a question, they make it clear to the examiner which attempt is to be marked. The obvious way for candidates to do this is to make sure they cross out any attempts which they do not regard as their best attempt at the question.

4721: Core Mathematics 1

General Comments

The majority of candidates were able to make a good attempt at this paper, with a significant proportion achieving 60 marks or more. Most candidates attempted all questions and many scored well on the longer, more structured questions towards the end of the paper.

As in previous sessions, candidates demonstrated excellent understanding of differentiation but still made errors in calculations involving fractions and/or negative numbers.

Some candidates had failed to learn formulae correctly, particularly the formula for solving a quadratic equation, but also those needed to find the mid-point of a line and the radius of a circle.

Candidates appeared to have enough time to finish the paper and there was some evidence of checking of answers.

Comments on Individual Questions

1 This question was generally well answered.

In part (i), most candidates stated $x = 8$ although $x – 6$ was also seen quite often.

Part (ii) was also answered well, $t = 0.1$ being the most common wrong answer. A few candidates used logarithms but they usually failed to evaluate their final expression, not realising that log 1 = 0 (or that log 10 = 1).

In part (iii), even the very brightest candidates failed to appreciate that $y = – 3$ was a solution. Many of the candidates who scored 71 marks overall on the paper lost the single mark here.

2 In part (i), almost all candidates scored the first 2 marks for correctly squaring one of the bracketed expressions. Only the very weakest omitted the middle term when squaring. However, even able candidates were careless with signs, thus losing the final mark. It was also disappointing to see $2(2x – 3)^2$ become $(4x – 6)^2$ in a significant minority of scripts.

Part (ii) was often done by expanding the expression fully, although good candidates saved themselves time by identifying the required x^3 terms. As in part (i), a mark was too often lost due to a lack of care with signs.

3 Both parts of this question were very well answered with a large majority of candidates scoring all 5 marks. Almost all candidates realised the need to differentiate in each part and were able to rewrite $\sqrt{x}$ correctly in index form. It was pleasing to see confident handling of negative fractional indices in many scripts.

4 Once again, far too many candidates lost marks through poor graph sketching. Some candidates produced a casual pen drawing without using a ruler to draw axes; others laboriously plotted points on graph paper. Centres need to emphasise the importance of sketching graphs, as many candidates seem to lack an understanding of this technique.

In part (i), although the majority of candidates knew the general shape of the curve, marks were frequently lost by the lack of quality in drawing curves approaching asymptotes. Other than this, the common error was to draw the curve $y = \frac{1}{x}$.

In part (ii), most candidates knew that they needed to translate their curve parallel to an axis. Of those who chose the correct direction, a significant proportion lost a mark by omitting to indicate how far the curve had moved.

Part (iii) was poorly answered, although there was much variation by centre. Most candidates knew that it was a stretch but were let down by a lack of precision in their descriptions; for example, 'in the *y*-axis' instead of 'parallel to the *y*-axis' was common, as was the word 'squash'.

5 This question was confidently tackled by most candidates with many scoring full marks, although not all perceived the link between the first two parts and the final two parts of the question and restarted in part (iii).

In parts (i) and (ii), completing the square was well done although, as in last year's papers, there was evidence of some candidates' inability to manipulate fractions. For example, in part (i), $\left(\frac{3}{2}\right)^2$ was simplified to $\frac{9}{2}$ or $\frac{6}{4}$. In part (ii), $-4-\frac{11}{4}$ often became $\frac{-15}{4}$ or, in a few cases, $4\frac{11}{4}$.

Part (iii) was generally answered well although (–3, 4) was a common wrong answer, as was $\left(\frac{3}{2},-2\right)$.

Part (iv) proved more challenging with many candidates including the $\frac{-11}{4}$ term twice in their calculation of the radius, while others thought that the radius was simply $\sqrt{\frac{11}{4}}$. Once again, there were errors in arithmetic involving negative signs.

6 Most candidates did the first two parts of this question extremely well, gaining full marks.

Part (i) was answered well by most, with almost every candidate differentiating correctly. However, it was disappointing to see so many errors in the solution of the two-term quadratic equation. Some candidates divided by x, omitting the solution $x = 0$. Others made slips in factorising. Those who used the quadratic formula generally fared well, but some were not able to use it correctly with the constant term being zero. A very small number of candidates factorised the original equation instead of differentiating, giving the coordinates of intersections with the x-axis instead of the stationary points.

For part (ii), the vast majority of candidates used the second derivative, although alternative methods such as a sketch of the graph or consideration of gradients either side of the x values found in (i) were also seen. These candidates usually scored well.

As in other questions, candidates often did not understand the link between the final part of the question and previous working. Part (iii) was often omitted completely. Some candidates sketched the curve correctly but then gave a wrong, or wrapped, inequality. Others checked a few integer values for x and then guessed at the solution. Only the strongest candidates scored both marks here.

7 This question discriminated well between candidates. Many candidates gained full or almost full marks while there were some who were unable to make any progress at all.

In part (i), most candidates were able to recall the quadratic formula correctly and obtained $\dfrac{8 \pm \sqrt{20}}{2}$. After this, many made errors in simplifying this fraction, the most frequently seen wrong answers being $4 \pm \sqrt{10}$ and $4 \pm 2\sqrt{10}$. A minority of candidates completed the square in this part, usually successfully.

In part (ii), most candidates recognised that the graph was a parabola although other shapes were seen and there were quite a few parabolas with a maximum rather than a minimum point. Some graphs showed negative roots which were inconsistent with the (often correct) roots identified in part (i). It was common for one mark to be lost because the intercept (0. 11) was not shown on an otherwise correct curve. As in Q4, a few candidates worked out the coordinates of points and then plotted the curve on graph paper.

Few candidates appeared to notice that part (iii) was linked to part (i) although some realised that it involved a disguised quadratic. Only the very best candidates managed to progress beyond the initial statement $y = x^2$ to earn more than the first mark. Having calculated the roots of the equation in (i) again, wrong working included trying to find the square roots of $4 \pm \sqrt{5}$ or, less commonly, $\left(4 \pm \sqrt{5}\right)^2 = 16 \pm 5$. A few candidates rearranged the equation in part (iii) to make $y^{\frac{1}{2}}$ the subject and then squared both sides. This was often successful. However, it was clear that many candidates had no idea at all of how to tackle this part of the question. Many started by 'squaring' to get either $y^2 - 8y + 121 = 0$ or $y^2 - 64y + 121 = 0$ and then attempted to solve.

8 There were many completely correct solutions to this question and even the less able candidates scored well.

Parts (i) and (ii) were done correctly by most candidates. Only a tiny number used the square root in finding the discriminant.

Part (iii) was done less well, with many candidates simply stating 'one real root' or referring to the line and/or the curve touching the x-axis. Very many stated that the line crossed the curve at one point and thus did not earn the mark.

Although many candidates did not link part (iv) to earlier work in part (i), most were able to gain full marks nevertheless.

As in previous papers, part (v) on the equation of a line was done well, although a few good candidates lost the final mark by not giving the equation with integer coefficients, as required by the question. Again, some candidates were let down by careless errors involving negative number manipulation. A significant minority differentiated to get the gradient expression $2x - 5$ and then stated that the gradient was $\frac{5}{2}$.

9 Candidates used a wide variety of methods to tackle part (i) of this question. Although some candidates found the algebraic approach too demanding, most made some progress, often using a geometric approach involving use of Pythagoras' theorem or, in a few cases, vectors. These candidates often found at least one correct value for p, although their thinking was not always easy to follow. Many other candidates started well, obtaining correct expressions for AC and AB, but were unable to solve the resulting equation. In most cases, this was because they had equated their expression for AB to $\sqrt{20}$ leading to a quadratic equation with no real roots. Other common errors included using $AC = 2AB$ or squaring one expression but not the other. More worryingly, however, were those candidates who showed little understanding of square roots by stating, for example, that $\sqrt{36+(p-5)^2} = 6 + p - 5$.

In part (ii), most candidates could find the mid-point of the line joining two points correctly but many made no use of the line $y = 3x - 14$. Some failed to justify their choice of p value from part (i) and these candidates scored only 2 marks from 4.

4722: Core Mathematics 2

General Comments

This paper was accessible to the vast majority of candidates, and overall the standard was very good. There were a number of straightforward questions where candidates who had mastered routine concepts could demonstrate their knowledge; other questions had aspects that challenged even the most able candidates.
As on previous papers, only the most able candidates could manipulate logarithms accurately, though candidates are becoming more proficient in using logarithms to solve exponential equations. A number of candidates also demonstrated poor algebraic skills, particularly order of operations, use of indices and manipulating indices.
This paper contained several parts where candidates were asked to prove a given statement. It is very important in this type of question that sufficient detail is provided to convince the examiner that the principle has been fully understood. Candidates must also ensure that any intermediate values in their working are accurate enough to justify the final answer to the specified degree of accuracy. A significant number failed to gain full marks due to using a rounded value from a previous part of the question.
Whilst some scripts contained clear and explicit methods, on others the presentation was poor making it difficult to follow methods used and decipher answers given. This is especially true if a candidate starts a second attempt at a question. Some candidates squashed all their work on to a few pages of the answer booklet, making it difficult for examiners to follow. Whilst numerical methods were usually easy to follow, the same could not be said of questions requiring an explanation. This was particularly noticeable in Q8(iii), where a number of candidates gave vague and confusing explanations that did not involve the correct mathematical terminology. When asked for a sketch graph, candidates must appreciate that there is no need for graph paper, and this can even sometimes be counter-productive. A simple sketch on the lined paper is all that is required, but they must still ensure that the intention is clear. Sloppiness when sketching a curve could imply a lack of understanding, such as overlapping parts on a $y = \tan x$ curve when the behaviour should be asymptotic.

Most candidates seemed familiar with the necessary formulae and could quote them accurately, but there were a worrying number of slips on formulae that are actually given in the booklet supplied to candidates. It is important that candidates have access to the formulae booklet throughout the course, and that they become fully conversant with its contents.

Comments on Individual Questions

1 (i) This question was generally well done, with candidates recognising the need to formulate and solve two simultaneous equations. These equations were generally correct, though some candidates used $a + nd$, and others confused n and u_n. A number of candidates employed a more informal method to find d ($60 \div 30$), but then sometimes struggled to find a.

(ii) Most candidates could quote a correct formula for the sum, and then substitute in their values from (i). Candidates with incorrect values for a and/or d did not seem perturbed that their sum was not equal to zero, and did not reconsider previous answers. Those candidates who used the formula $\frac{1}{2}n(a + l)$ sometimes lost marks by not showing explicitly how they arrived at their value for u_{29}.

2 (i) Most candidates made a good attempt at the area of the triangle, though some assumed that it was either a right-angled or an isosceles triangle. Some candidates did not seem familiar with the formula and first found the perpendicular height of the triangle.

(ii) Despite the appearance of the cosine rule in the formula book, a number could not quote it accurately, though most appreciated that this was the required method. A few candidates struggled to evaluate the formula correctly, with 149 – 140 cos80 becoming 9 cos80.

(iii) Most candidates realised the need to use the sine rule and attempted to do so correctly, though some attempted angle *A*, despite calling it angle *C*. However, many candidates failed to gain the final mark due to not using a sufficiently accurate value from part (ii). Whilst intermediate steps may be written down to 3 significant figures, candidates must appreciate the need to use the full value in subsequent calculations.

3 (i) Most candidates could make a good attempt at the binomial expansion, but algebraic insecurity led to a number of errors. Most candidates could obtain the first two terms of 1 and $24x$, though the latter was not always of the correct sign. In most solutions the third term was a product of the correct binomial coefficient and an attempt at squaring $-2x$ but a lack of brackets led to errors, with $132x^2$ being the most common mistake. A few candidates found the first three terms in descending not ascending powers.

(ii) This part of the question was generally done well, and two of the three marks were available for correct working using incorrect values from part (i). A number of candidates identified the relevant terms and produced concise solutions, whilst others carried out a full expansion. Some candidates did not appreciate that the answer was derived from two terms, and there were a number of sign errors.

4 (i) This question was generally very well done, with candidates being able to quote the correct formula and use it accurately to produce efficient and concise solutions. A few candidates found only one arc length, or found the difference in the perimeters of the two sectors. Some candidates attempted to use degrees instead, mostly with little success, which was also the case for those attempting to find fractions of the circumference. A minority of candidates continue to think that 1.8 radians means an angle of 1.8π.

(ii) This was also very well done, with the majority of candidates attempting to find the difference of two areas. Most could quote and use the correct formula, though some omitted the ½ and just used $r^2\theta$, and others confused the area of a sector with the area of a segment.

5 (i) This question was very well done, with most candidates gaining both of the marks available.

(ii) Whilst many candidates could attempt an expression for the sum of *n* terms, and relate it to 124 using an inequality, there were very few who could prove the given inequality convincingly. $5(1 - 0.96^n)$ often became $5 - 4.8^n$, and many could not cope with manipulating the inequality sign. A surprising number did not even attempt to prove the given inequality but went straight into solving it. Candidates are becoming increasingly proficient at using logarithms to solve equations and many could obtain the value of 118.3 but then struggled to make the correct final conclusion. Few appreciated that log0.96 is negative, necessitating a change in the sign when it is used to divide, leading to $n < 118.3$. Only the most astute candidates realised that the smallest integer value of *n* was required, with 118 and 118.3 being the most common answers. Even those who recognised that 119 was the required final value often concluded with an inequality rather than a statement of $n = 119$.

6 (a) This question was very well done, with many candidates gaining all the marks available. Most could obtain the correct index in the first term, though a few lost a mark by having a coefficient of $^3/_2$ not $^2/_3$. Others omitted the constant of integration, or still had an integral sign remaining in their final answer.

(b)(i) Once again, the integration was very well done, with most candidates obtaining an integral of the form kx^{-1}. Many then also managed the correct coefficient, though in some cases $-4x^{-1}$ subsequently became $-(4x)^{-1}$. Virtually all candidates then appreciated the need to use the limits a and 1, though an insecurity with indices then often led to $4(1)^{-1}$ becoming ¼. A large number of candidates did not then stop at the required answer, but equated the answer to zero or introduced inequalities, possibly because they had been told that $a > 1$ and they felt that this had to be used in some way.

(ii) This part of the question caused more problems, with some candidates being unable to attempt it at all, and others leaving their final answer still containing ∞. Only the most astute appreciated the link between this and the previous part, and most started the entire question again.

7 (i)(a) Virtually all candidates gained this mark, but many then struggled to gain further credit in this question.

(b) Most candidates could split the given expression into two terms, but failed to take the further, crucial step of separating the 10 and the x^2. The most common error by far was for $\log(10x^2)$ to become $2\log(10x)$, an approach that gained no credit.

(ii) Most candidates recognised the link with previous work and could gain a mark for using their two previous results in the given equation, but there were slips with both signs and the factor of 2. Many then struggled to make further progress, as previous errors meant that their equation did not simplify to one in $\log y$ only. Even those who were using two correct expressions often made a sign error leading to $3\log y = 2$. Whilst a number of candidates could then make further progress with the question, many did not appreciate how to obtain a solution for y. e was sometimes used as the inverse of $\log_{10}$ and others viewed $\log y$ as a product and divided by just the log part. Logarithms continue to be an area of uncertainty for many candidates. Whilst some misapplied correct logarithm rules, others appeared to be completely confused by the entire topic.

8 (i) This question was generally well answered. Most candidates attempted f(–1), though some lost a mark through not explicitly equating this to zero. Some candidates used more cumbersome methods, such as division or matching coefficients, but these had varying degrees of success. There was a pleasing degree of success in finding the quadratic factor and a variety of methods were used. Whilst many then continued and correctly found the full factorisation, a number could not progress beyond the quadratic factor.

(ii) Most candidates could correctly integrate the given expression and then attempt F(2) – F(–1), but a significant number then struggled to evaluate the expression correctly. Sign errors were abundant.

(iii) The standard of the sketch graphs was very varied, both in quality and content. Whilst an accurate drawing on graph paper was not required, it was important that candidates took enough care to ensure that their intention was clear. Many candidates attempted to sketch a cubic curve, but these were not always a positive cubic, with three distinct non-zero roots, as required. Examiners were disappointed at the quality of the explanations, with many candidates unable to use clear, unambiguous words. Some felt that the area under the x-axis was ignored and others described **how** to find the actual area, without explaining **why** the answer to (ii) did not give the correct area of the region.

9 (i) There were many good sketches seen, though a significant number offered accurate drawings on graph paper. Whilst the general shape of both graphs was usually correct, some had incorrect periods. $y = \tan x$ did not always pass through the origin and often only had a range of $-1 < y < 1$, despite $y = 4\cos x$ being correctly drawn. A number of candidates did not identify the intercept on the y-axis.

(ii) Assuming that the graphs had two points of intersection, most candidates attempted to identify α and β, though a significant number had them the wrong way around on the graph. This was probably due to assuming that $\alpha < \beta$ applied to the y-coordinates and not the x-coordinates. Only the most able candidates could express β in terms of α explicitly, though a few more could deduce some kind of relationship.

(iii) Proving the given equation was generally very well done, with most candidates being familiar with the two identities and able to apply them accurately. Actually solving the equation caused more problems. Most recognised it as a quadratic in $\sin x$, but many then attempted to factorise it. Those who attempted to use the formula could usually do so correctly, but there were a number of slips with both the discriminant and the denominator. Of the candidates who got this far, many assumed that the roots were the actual values for α and β, with only the most able candidates appreciating that these were the solutions for $\sin x$. A number could then find 62° correctly, but couldn't find the second angle, either because they had an incorrect relationship between α and β, or because they expected the second angle to come from the second root of the quadratic. Despite all of this, a pleasing number of correct solutions were seen.

4723: Core Mathematics 3

General Comments

This paper enabled candidates to display their mathematical ability. Several of the questions were accessible to all but a few whereas there were a few requests which challenged the most able of candidates. There were not too many candidates with very low marks and, pleasingly, a good number of candidates recorded full marks. Time was not a problem for candidates.

One aspect of algebraic technique which was not always handled competently was the use of brackets. When the brackets were already present in the question, such as in Q3(a), there were no problems. However, when candidates had to incorporate brackets into their expressions, problems sometimes arose, with necessary brackets being either absent or incorrectly located. Questions where problems were noted included Qs 3(b), 5 and 9. Sometimes a candidate rescued the situation by proceeding as if the missing brackets were present but, on other occasions, the candidate was led astray by the absence of brackets. For example, in Q9(i), it was not uncommon for candidates to write $2\sin\theta.1-\sin^2\theta+1-2\sin^2\theta.\sin\theta$ and to follow this with the incorrect $2\sin\theta-\sin^2\theta+1-2\sin^3\theta$.

Questions in which it is required to confirm a given result obviously encourage candidates to check their solution when they do not reach the result. There are other situations when candidates would benefit by being aware of the nature of the answer to be expected. In Q5, awareness of integration should have led candidates to expect that one definite integral would be positive and the other negative. In Q8, consideration of the diagram should have led to the expectation that the gradient of *PQ* would be negative and that the area of *A* would be less than the area of the triangle *OPQ*. In the majority of cases where candidates obtained an unreasonable answer, there was little evidence that they had checked their working to find and correct an error.

Comments on Individual Questions

1 This question was answered well and most candidates demonstrated the appropriate use of logarithm properties to obtain the given answer convincingly. Some included a step such as $\ln 512-\ln 8=\dfrac{\ln 512}{\ln 8}$ in their solution; full credit was not available for solutions containing such an error. A number of candidates correctly reached $3\ln 8-3\ln 2$ but then resorted to their calculators and the use of decimal approximations; the exact result cannot then be claimed legitimately.

2 Many candidates used the appropriate identity to form an equation quadratic in $\tan\theta$. There were some slips with the identity or when simplifying or solving the equation but it was common for the four correct angles to be obtained without difficulty. The first step of some candidates was to rewrite the given equation in terms of $\sin\theta$ and $\cos\theta$. Such attempts seldom made any progress (although there are at least two approaches – each much more difficult than the expected method of solution – which can be used successfully).

3 This was a straightforward question for many candidates and they obtained all six marks without difficulty. The product rule is required in part (a); this was not always realised and solutions such as $12x(x+1)^5$ were not uncommon. A few candidates used the binomial theorem to rewrite the given expression as the sum of seven separate terms before attempting differentiation. No simplification of the derivative was requested in the question but many candidates did show confidence with algebraic skills to express their correct derivative as $2x(4x+1)(x+1)^5$.

A statement of the quotient rule is given in the *List of Formulae* but there was a significant number of candidates who applied the rule incorrectly, confusing numerator and denominator or making sign errors. Some candidates omitted to substitute the value 1 and others made errors in their simplification of a correct derivative and thus ended with an incorrect value.

4 Many candidates had no difficulty in writing down the correct answer for the range in part (i). But many other responses did raise doubts about candidates' understanding of range. The answer $f(x) \geq 2$ appeared often, contradicting the clear evidence provided by the given graph; presumably this was the result of an unconsidered step whereby $x \geq 0$ for the domain became $f(x) \geq f(0)$ for the range.

The vast majority of candidates answered part (ii) correctly. However, correct responses to part (iii) were rare. Some candidates considered the graph of $y = |f(x)|$ and readily deduced the correct answer but, in the majority of cases, the approach involved squaring both sides to obtain $4 - 4\sqrt{x} + x = k^2$ and no sensible progress was made.

5 Most candidates earned some credit for this question but correct solutions needed a combination of accurate integration, an understanding of how to deal with the part of the area below the x-axis and the retention of exact values. Many excellent solutions were seen but it was disappointing that many candidates struggled with one or more of these aspects of the question. Some candidates adopted the approach of integrating $(1-2x)^5 - (e^{2x-1} - 1)$ but sign errors were sometimes a problem. Other candidates evaluated the two areas separately but did not always deal appropriately with the negative definite integral. The integration steps were often incorrect with results such as $\frac{1}{6}(1-2x)^6$ and $2e^{2x-1} - x$ among the more common errors. A few candidates tried the use of $\int x \, dy$ but found the integrals too demanding and a few thought that $\int \pi y^2 \, dx$ was required.

6 There were two common correct approaches to part (a). Some candidates realised that exponential increase meant that the quantity X was increasing by a factor 1.6 over time intervals of length 10 and obtained the correct answer without difficulty. Other candidates attempted to set up a formula for X in terms of t. This approach was less successful as there were problems in using an appropriate formula or in retaining sufficient accuracy in the solution to reach an acceptable value of X. A common incorrect approach led to the answer 605, the result of assuming that the values of X followed an arithmetic progression as t increased by equal steps.

The two requests in part (b) were answered very well with the majority of candidates answering both correctly. Some faulty work with logarithms was seen in part (i) and some candidates substituted $t = 30$ in part (ii) before attempting differentiation, but most candidates displayed confidence in dealing with this topic.

7 In part (i), candidates generally recognised that the curve to be drawn was the result of a stretch and translation of the given curve. Most had the details of these transformations correct although, in some cases, the stretch was parallel to the x-axis or parallel to the y-axis but with a scale factor $\frac{1}{3}$ or the translation was in the negative x direction. The response to part (ii) was disappointing with many candidates not recognising $y = x$ as the straight line to be drawn. Some drew no line and common incorrect responses included a line parallel to the y-axis, a line parallel to the x-axis and a line joining the points (2, 0) and (0, 3π).

Many attempts at parts (iii) and (iv) were marred by the use of angles measured in degrees instead of in radians. In part (iii), most candidates indicated that they were expecting to see a change in sign but those working in degrees and those substituting into $3\cos^{-1}(x-1)$ looked in vain. For those candidates working with radians, the iteration in part (iv) was handled well with the individual iterates shown and the correct value of α found. The final part of the question was not answered well, with many candidates referring vaguely to one equation being the inverse of the other or carrying out a numerical calculation involving their value of α. A clear mathematical explanation, showing a step-by-step rearrangement of $x = 1 + \cos(\frac{1}{3}x)$ to $3\cos^{-1}(x-1) = x$, or *vice-versa*, was not noted often.

8 Correct differentiation of $\ln(5 - x^2)$ was a problem for many candidates and wrong derivatives such as $\frac{1}{5 - x^2}$ and $\frac{2x}{5 - x^2}$ were frequently noted. Most candidates knew how to find the equation of the tangent though a few formed an equation in which the gradient remained an expression involving x.

Part (ii) was answered well with most candidates adopting a systematic process to establish the correct value. There were some errors with the value of h, some slips with the evaluation of the relevant y values and some instances of an incorrect application of Simpson's rule in which the factors 4 and 2 were associated with the wrong y values.

Many answered part (iii) correctly but it was surprising how often this apparently simple request was answered incorrectly. Obviously there were problems if the answers to part (i) or part (ii) or both were wrong although both marks in part (iii) were available for correct work following plausible earlier answers. However, some candidates seemed unsure about how their results from part (i) and (ii) related to the diagram. Some claimed the coordinates of Q as (0, ln 5) and others found only the area of the triangle.

9 The vast majority of candidates earned the first mark in part (i) by expressing sin 3θ as $\sin 2\theta \cos\theta + \cos 2\theta \sin\theta$. Most were aware of the identities then needed to complete the solution and many accurate and clear solutions were seen. Other attempts were not so assured; dealing with cos 2θ was a problem for some.

Progress with parts (ii) and (iii) depended on an appreciation that the identity from part (i) was needed but that appreciation was absent in many cases and no progress was made. In part (ii), the two required values followed readily once the expression had been rewritten as $3\sin(10\alpha)$. Part (iii) proved to be a challenging final request but a number of candidates showed excellent technique to express the equation in terms of $\sin 2\beta$ and to find the two values of β. Many other candidates did not use the identity from part (i) and their solutions tended to meander without much purpose and often contained errors such as $\frac{\sin 6\beta}{\sin 2\beta} = \sin 3\beta$.

4724: Core Mathematics 4

General Comments

An important indicator of a good candidate is the presentation of the answers; an able mathematician has clarity of thought and a consequent awareness of giving clear, concise presentation. Candidates who followed this dictum produced reliable work of a high standard; others, whose work was untidy and unclear, often made mistakes and their mathematics was generally of poor quality. However, the vast majority of the work was excellent.

Comments on Individual Questions

1 This question based on the syllabus item "Simplify rational expressions, including factorising and cancelling" gave most candidates a good start; very few simplified too far by cancelling the correct answer to an inappropriate one. A few candidates attempted long division with just a few making satisfactory progress.

2 The techniques of implicit differentiation were well known; a few careless errors crept into the solution e.g. the mis-reading of '*y*' for 'sin y', the derivative of sin *y* being-cos y, and the transference of '*xy*' to the left hand side of the equation with a derivative of $-x\,\frac{\mathrm{d}y}{\mathrm{d}x}+y$ instead of $-\left(x\,\frac{\mathrm{d}y}{\mathrm{d}x}+y\right)$.

3 The methods of long division and the use of an identity were both prevalent in this question and the first part was generally done well. The answers to the second part could be easily deduced from the remainder in the first part but the majority effectively started again.

4 The first stage of the integration by parts was rarely effected incorrectly so it was surprising that, in quite a few cases, the next stage integral of $\tan x$ became $\sec^2 x$ (particularly as the result is quoted in the *List of Formulae*). Needless to say, the writing of $\int \tan x$ was much more common than $\int \tan x\,\mathrm{d}x$.

In the second part, a connection between $\tan^2 x$ and $\sec^2 x$ was generally used (though, as this is not in the *List of Formulae*, sign errors were not rare) either directly or by another application of integration by parts.

5 In the first part, the direct production of $\frac{\mathrm{d}y}{\mathrm{d}x}$ from $\frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}$ was very common and few mistakes occurred other than the simplification of $\frac{2}{2t}$ to *t*; however, the obvious lack of confidence with parameters shown by a number of candidates led them to convert parametric to cartesian, so missing the simpler work involved with the parameters.

The second part often had confusion between *t* and *p* although, needless to say, the given answer was always present on the final line.

In the final part it was heartening to see that most candidates realised that *p* was –5 and not 5; most obtained the result from solving $p^2 = 25,\ \ 2p = -10$ simultaneously though some, unusually, substituted (25, –10) into the equation of the tangent. On the other hand, it was surprising how many worked out the equations of the tangents again even though the format had been given in part (ii).

6 In part (i) it was rare to see $\mathrm{d}x$ being just replaced by $\mathrm{d}\theta$ but a little surprising to see $\sin^2\theta$ being changed into a $\cos 2\theta$ variant before differentiation took place (generally satisfactorily) – perhaps candidates had already looked at part (ii). The reduction to $2\sin^2\theta$ was not always totally convincing and candidates must realise that, when an answer is given, very close inspection will be made of the working. In part (ii), most changed the limits to 0 and $\frac{\pi}{2}$ but some used 0 and 90; a very few tried re-substituting but this device was rather unsuitable and rarely produced the correct answer.

7 Again, it was heartening to see the most usual identity in correct form on nearly every occasion and part (i) usually only failed because of carelessness or writing $\left(1+x^2\right)$ instead of $\left(1+x\right)^2$. In part (ii), the expansion of $\left(2-x\right)^{-1}$ caused most trouble, often being changed to $2\left(1-\frac{x}{2}\right)^{-1}$ or $2\left(1-x\right)^{-1}$. The expansions of $B\left(1+x\right)^{-1}$ and $C\left(1+x\right)^{-2}$ were frequently spoilt by the candidate trying to do too much all at once: e.g. the unsimplified expansions of $\left(1+x\right)^{-1}$ and $\left(1+x\right)^{-2}$ would be simplified in the head <u>and</u> multiplied by B or C simultaneously instead of performing the two operations in separate stages.

8 The technique of separation of variables in part (i) was generally understood but many candidates failed to show the arbitrary constant, thus making nonsense of the substitution of (5,4). However, they were still able to (and generally did) complete the squares and produce some reasonable format in part (ii). In part (iii), it would have been helpful if candidates had stated that their locus, of sometimes indeterminate shape, was intended to be a circle.

9 The vast majority chose the correct vectors to determine the angle between the lines. In part (ii), it was good to see a systematic approach in the solution of the simultaneous equations.

4725: Further Pure Mathematics 1

General comments

In general, most candidates worked sequentially through the paper and few seemed to be under time pressure. Completely correct solutions to all questions were seen and no question proved to be inaccessible. A good number of candidates scored high marks on this paper.

The level of algebraic skill shown was pleasingly high, but errors at any early stage in, for instance, Qs 4 and 5 led to a considerable proportion of the marks being lost.

Qs 2, 6 , 9 and 10 proved to be the most demanding but candidates found the remaining questions, in general, more straightforward.

Comments on Individual Questions

1 Most candidates answered part (i) correctly, the most common error being to make $2 - 8i^2 = -6$.
In (ii) the majority of candidates saw how to use the conjugate, but the error, $(2 - i)(2 + I) = 3$, was quite common.

2 As in June 2005, the presentation of the Induction proof was not of the standard required by the examiners. A demonstration that the result is true when $n = 1$ was often very vague. The assumption that the result is true for some particular value leading to the truth that the result is true for the next value was sometimes similarly vague and the mark for the explanation of the Induction conclusion was not earned in a high proportion of scripts.

3 The method of evaluating the determinant was known by the majority of candidates, with sign errors being the most frequent loss of marks.

Part (ii) showed that the definition of singularity was not clearly understood. Many candidates thought that a non-zero determinant meant a singular matrix, or that a positive determinant meant a non-singular matrix, while others gave information about the inverse matrix, with no mention of non-singularity.

4 Algebraic errors in the expansion of $(u + 2)^3$ and the simplification of the equation in u, caused many candidates to lose marks. A surprisingly large number of candidates found the correct value for u, but failed to state the corresponding value for x. Very few candidates gave decimal answers, which was pleasing as the exact value was requested.

5 This question was answered correctly by most candidates, with few making algebraic errors or failing to show sufficient working to justify the given answer.

6 The inverse matrix in part (i) was generally correct, with omission of the determinant being the most common error.

Quite a large proportion of candidates either used no matrix algebra or incorrect pre- or post-multiplication in trying to find either $\mathbf{B}^{-1}$ or $\mathbf{B}$. Very few candidates used the result $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$. Candidates who used a general matrix for $\mathbf{B}$ and then solved simultaneous equations to find the elements invariably made an error in their solution.

7 In part (a) most knew how to find the modulus and argument of a complex number, the most frequent error being to give the argument of w instead of w^*.

In part (b) most candidates found the real and imaginary parts of u, but a significant number did not give their answer as a complex number.

A considerable number of candidates thought the locus in part (c) was a circle. A rather high proportion of candidates who sketched a vertical straight line, made no indication of scale, which was penalised.

8 The transformation geometry was generally well understood, with many candidates scoring full marks. As in Q7, omission of scales on diagrams, when there was no evidence of coordinates being found elsewhere was penalised. There was confusion in some descriptions between a stretch in one direction and an enlargement.

9 It was pleasing to see that a good proportion of candidates wrote down sufficient terms to see that four elements from the differences remained rather than just assuming that the first and last elements were the ones required. Most found the sum to infinity correctly, but many did not see that part (iii)(b) required the difference between the sum to infinity and the answer to part (ii).

10 This question proved to be quite demanding, and many candidates scored only the mark for each of parts (i) and (iii). A significant number did not appreciate that the complex roots were a conjugate pair and produced a considerable amount of work that made no progress. Those who used the conjugate roots were generally able to find correct answers in one form or another for both parts (ii) and (iv), while some candidates used the fact that $\Sigma\alpha\beta = 27$ to find q in terms of α, some finding the simplified value $\sqrt{3}/_2\left|\alpha - 3\right|$.

4726:Further Pure Mathematics 2

General Comments

Most candidates found the examination accessible, answering the questions in the order set. Candidates appeared to be well-prepared for the range of questions, and many candidates produced sound answers. There were many excellent scripts showing a variety of methods.

The main area of concern was in questions in which either the answer to be proved was given or where an explanation was called for. Candidates should appreciate that in these cases a full and detailed answer is expected. The lack of such precision was particularly noticeable in Q7 where the use of such words as "obviously" or "clearly" led to a loss of marks for many candidates.

A few candidates appeared to have problems with time. This was apparently due to overlong methods, where instructions in questions were not read carefully enough or were ignored. Examples include "write down" in Q1 and "state" in Q4.

Comments on Individual Questions

1 The majority of candidates successfully "wrote down" the expansion for $\ln(1+3x)$, with few errors. Some marks were lost by the lack of brackets around $(3x)$ or by using factorials in the denominator. In part (ii), candidates tended to expand the brackets beyond the x^3 that was required. Candidates who opted for expansion by repeated differentiation were often successful but wasted some time, particularly if it was applied to part (ii). Candidates should be aware of the standard expansions given to them in the *List of Formulae* booklet.

2 Most candidates showed a good awareness of Newton-Raphson, ensuring that an $f(x)$ of the form $\pm(\mathrm{e}^{-x} - x)$ was given as an initial step. Other interesting choices, such as $x\mathrm{e}^{x} - 1$, were also used. The use of calculators was usually good enough to give the correct answer, although some candidates did not give sufficient decimal places in their early estimates to justify the final answer correct to 3 decimal places. Candidates who used methods other than Newton-Raphson were given no credit.

3 This proved to be a straightforward question in which very few errors were seen.

4 (i) The use of the insert caused few problems to candidates, although a surprising number of centres failed to attach it to the script concerned. The three lines given on the insert were not always used for the explanation to accompany the diagram, and although candidates generally went from x_1 to the curve and then to $y = x$, they did not always continue in a convincing manner. Those who mentioned "step" or "staircase" were given credit. The best solutions showed x_2, x_3 … and explained the process.

(ii) Even candidates who showed an understanding in part (i) believed that the iteration converged to β in (a). In (b), any form of words was allowed which indicated that the iteration diverged, although there were some very convoluted ways of putting it.

5 (i) A variety of methods was seen for the oblique asymptote, including dividing out and equating coefficients. Marks were awarded for the correct equation even with minor algebraic errors.

(ii) Methods were firmly in two camps. The first involved the consideration of the quadratic $x^2 + x(3 - y) + (3 - 2y) = 0$ and the condition for real values of x. This was an example of an answer being given, and so the final solution of the inequality for y should have been justified and not merely copied down. It was apparent that some candidates were not sure why they were considering $b^2 - 4ac$, but marks were awarded for a reasonable attempt at the set method.

The second method involved differentiation, either of the original y or of the y developed in dividing out in part (i). Many candidates found the turning points and stopped. Better solutions involving testing for maximum and minimum and sketching, possibly including obvious points and/or approaches to asymptotes, were rare.

6 (i) Again, this question included a given answer. Candidates who showed that they were attempting to use the limits in the first part of the integral were not penalised if they merely wrote down $-e^{-1}$. A surprising number of candidates did not use the limits at all to produce this term.

(ii) Errors in this part were mainly algebraic, with some confusion as to the meaning, and then the evaluation, of I_0 or I_1.

7 Candidates in general did not "explain" at all well. Statements such as "Rectangles = $\sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots$" were given no credit. The best solutions explained the left-hand side of the inequality in terms of the total area or summation of the areas of rectangles (up to N) compared to the area under the graph (from 1 to $N + 1$), together with an indication of the area of a single rectangle as $1 \times \sqrt{x}$ for the correct values of x. Others noted that translating the original rectangles one unit to the left helped in explaining part (ii). Candidates should monitor the number of marks awarded for each part of each question. In part (iii), it was surprising to see the inequalities the wrong way round and/or the integrals not attempted.

8 (i) It was sufficient to "state" $r = 2$ for $\theta = 0, \pi$ without any working. Answers outside the given values were ignored, but extra answers within the given values were penalised.

(ii) Most candidates solved $r = 0$, sometimes producing too many "solutions". A minority were less successful employing differentiation.

(iii) It was pleasing that the majority of candidates could deal with the integral involved, with a lot of correct solutions. There were a few candidates who produced a wrong formula for $\cos^2 2\theta$, but, for minor errors, four out of five marks were possible.

(iv) Candidates were less successful in this part, wrongly using $x = r \cos 2\theta$ or merely replacing r with $\sqrt{x^2 + y^2}$ and θ with $\tan^{-1}(y/x)$. Others attempted a final equation with y as the subject. As long as these candidates showed a simplified form at some stage of their working, they were penalised by time constraints and not by losing marks.

9 (i) Only minor errors were ever seen in this part.

(ii) It may be that candidates were getting to the end of the paper, but again there was evidence of a lack of care in how many read the question. For full marks, it was necessary to show that there was "just one" turning-point. Many candidates showed that there was at least one turning-point but did not justify it as the only one.Most candidates used the hint in part (i) to solve $2\sinh 2x - 6\cosh x = 0$, whilst those who opted for an immediate substitution for $\sinh x$ and $\cosh x$ in terms of e^x and e^{-x} had more problems. Most used the second differential test for the type of turning-point, but accurate numeric evidence was required for both marks.

4727: Further Pure Mathematics 3

General Comments

The entry for this session was small because the majority of centres use the whole of Year 13 to prepare their candidates for this unit. There was some evidence that some candidates had not fully covered all the topics in the specification. As all four sections of the specification are always tested in each paper, it is desirable for candidates to have had as much experience as possible before taking the examination. The paper appeared to be of the correct length, and there was no indication that candidates had either run out of time or had much time to spare.

Comments on Individual Questions

1 This was a straightforward question which required the use of the scalar product to find the angle, and candidates' answers were almost entirely correct.

2 (i) Candidates had no difficulty in stating the identity element in each group, and the proper subgroups were usually correct.

(ii) Although the request to show corresponding elements had not been set recently, most candidates knew what was expected and answered confidently. A few gave only the orders of the elements, without identifying the correspondence between them. As the corresponding elements can easily be deduced from the orders, some credit was given for this.

3 A majority of candidates scored well on this question. Among others there was some uncertainty about differentiating and using the substitution, and little more credit could then be obtained. In part (ii) the fact that y approaches 0 as $x \to \infty$ was not given as often as expected.

4 (i) This part required careful attention to detail, but it was done quite well. Not all candidates used the suggested method of expanding $\left(e^{i\theta} + e^{-i\theta}\right)^2$ and $\left(e^{i\theta} - e^{-i\theta}\right)^4$ and then grouping terms, but all other valid methods could gain full credit.

(ii) This was an easy application of the result of the first part and the correct answer was usually found.

5 Although most candidates gained some marks on this question, only the best were able to complete all three parts. The majority of answers to part (i) were correct, and a fair number managed to find the equivalent cartesian values to give the linear factors of $z^4 + 64$. But the concept of combining complex conjugate pairs of factors to give the quadratic factors was seen in only a few scripts.

6 This standard type of vector question is usually done confidently, but in this session it was evident that some candidates lacked the experience required, or even had not completed the work for the topic. Among those who knew what to do, part (i) was found straightforward. In part (ii) some answers found the two-parameter form of the equation of the planes without attempting to find the cartesian form. Vector problems often have several different possible methods, and it requires practice to know which is the best to use in a particular case. When the cartesian form of the equation of a plane is requested, changing from the parametric form without using a vector product usually takes longer than more direct methods. Answers to part (iii) by candidates who had progressed that far were good.

7 This question was the most demanding on the paper, and only a small number of good answers were seen. Very careful reasoning was required in the proof of part (i). To gain full credit here it was necessary to note that a having order n means not only that $a^n = e$, as given, but that $a^m \neq e$ when $m < n$. In contrast, parts (ii) and (iii) were easier, and some fair answers were given.

8 (i) This question should have been found reasonably straightforward, but in fact candidates had more trouble than expected with distinguishing between the three cases. The auxiliary equation was usually solved correctly, and correct answers to part (a) were given. However there seemed to be problems with writing $\sqrt{k^2 - 4}$ as $\mathrm{i}\sqrt{4 - k^2}$ in order to obtain the trigonometrical part of the solution in part (b). Part (c) was usually correct, with a few slips in writing the variables as x and t.

(ii) Those candidates who had coped with the trigonometrical solutions in part (i)(b) were able to make fair attempts at this part.

(iii) Although it was easy to state that $x \to 0$, it was only the best candidates who gave a fully correct explanation, referring not only to e^{-t} approaching 0, but also to the fact that the sine function is bounded.

Mechanics

Multiple attempts at questions.

In recent sessions examiners have noted an increasing number of candidates making two, or more, attempts at a question, and leaving the examiner to choose which attempt to mark. Examiners have been given this instruction.

‘If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt, and ignore the others.’

Please inform candidates that it is in their best interest to make sure that, when they have a second attempt at a question, they make it clear to the examiner which attempt is to be marked. The obvious way for candidates to do this is to make sure they cross out any attempts which they do not regard as their best attempt at the question.

4728: Mechanics 1

General Comments

The quality of the candidates' work was high, and very few poor scripts were seen. Indeed the commonest outcome for all questions, save one, was the award of full marks. Nevertheless, some candidates would have found their ability better rewarded by:

- reading questions more carefully, either to assimilate all the information given, or appreciate better the results being required in their answers (Qs 1(ii), 3(iii) and 4(iv)),
- selecting swifter methods of solution, and not recalculating earlier results needlessly (Qs 1(ii) and 7(ii)),
- having greater facility with problems involving two dimensions (Qs 2, 4(iv), 6).

Specific references to these matters are made in the comments on individual questions below.

Comments on Individual Questions

1 (i) This question was convincingly answered by candidates who clearly used their a in the direction specified in the question. Examiners are entitled to withhold marks from scripts if a candidate has used a letter with a defined meaning in an idiosyncratic way.

(ii) The most common error in this question was to calculate only the time taken for particle P to come to rest. The most common way to answer part (ii) was to start by finding the time taken to come to rest, then calculate the distance P descends in this time, and then find the time required for the particle to ascend to its initial position. Solutions based on substituting $s = 0$ into $s = ut + at^2/2$ were extremely rare.

2 (i) This part of the question was well answered, though a few candidates included g in their term for ma.

(ii) The second part of the question was frequently completed successfully, though some candidates who obtained the correct equation by resolving vertically were unable to solve their two correct simultaneous equations. More often candidates resolved parallel to the string or incorrectly included the acceleration.

3 (i) This was very well attempted and almost all candidates scored full marks.

(ii) Nearly all candidates successfully completed this.

(iii) Most candidates started this part of the question with the correct integration of vdt, though a few then deleted their correct expression and used the equation $s = [(u+v)/2]t$ instead, perhaps not appreciating that it is a constant acceleration formula.

Though very many candidates completed the question correctly, a large minority simply substituted 31 as the value for t, not understanding how there were two stages in the motion of the motorcycle.

4 (i) Nearly all solutions were based on the correct trigonometric ratio, and this part of the question was well answered, though some candidates calculated the weight of the block.

(iii) Again almost all candidates were successful with the calculation.

(iii) Nearly all candidates employed the correct method for finding a coefficient of friction. However, there were frequently problems associated with the correct degree of accuracy. Candidates who used values from parts (i) and (ii) rounded to three significant figures gave wrong answers, and it was very common for the initial zero in the answer treated as a significant figure, so 0.70 (and 0.7) were often seen.

(iv) Though correct solutions to the entire problem were often seen, very many candidates noted only the increase in the force from 12 N to 20 N, and overlooked the change in its direction. It was also common for the effect of the change(s) on the normal component of the contact force to be overlooked, and such candidates continued to use the frictional force found in part (i). Even candidates who did recalculate the frictional force might use an excessively approximate value for the coefficient of friction, so obtaining an inaccurate value for the acceleration.

5 (i) Only a small minority of candidates interpreted "sketch" in an artistic sense, but it was common to see "straight lines" drawn without the aid of a straight edge. Many candidates were unaware that the horizontal "walk" section of the diagram should have been below the *t*-axis, and not linked to it by any line segments, either vertical or inclined.

(ii) This part of the question was well answered. Although nearly all candidates could find the time spent walking, the time spent on the motor-cycle was seen less often.

6 Candidates found this the most taxing question on the paper. It was the question with the most uniform spread of marks.

(i) Although candidates could set up an equation for the vertical equilibrium of the system, it was unusual for the behaviour of the cosine function to be correctly used in the solution.

(ii)(a) Candidates who attempted this part of the question were likely to spend too much effort in calculating angles, which in turn rendered an exact value for *T* approximate.

(ii)(b) Although the solution for *W* requires the same essential work as (i), it was common to see progress made in one part of the question but not the other. Some scripts were marred at this stage by multiplying together *W* and *g*.

(iii) The most frequently seen answers were that the ring would be at *B*, consistent with an elastic string, and that the tension would be zero, the string being slack. The most common non-zero value for tension was the weight of the ring.

7 (i) Very many scripts included a correct calculation for the velocity of *B* after the collision. The next part of this question was frequently completed correctly, the most usual difficulty being finding the correct value for the acceleration of *B* on the inclined plane.

(ii) Again, many instances were seen of candidates correctly analysing the motion of the particles on the plane, leading to the award of full marks. In most cases, candidates found the time for *B* to ascend and descend separately. There were few instances of a candidate using $s = ut + at^2/2$, to obtain the time in a single calculation, as observed in the comments on Q1. Many scripts showed the recalculation of the acceleration of *A* and *B* for each stage of the motion.

4729: Mechanics 2

General Comments

The majority of candidates showed reasonable understanding of all topics. Unusually, the questions which caused most difficulty were the ones for which diagrams were given (1, 4 and 8). Few candidates appeared to run out of time and many scored very highly. Only a small percentage of the candidature was totally unprepared. As general hints, candidates must be spatially aware and be familiar with geometry, trigonometry and specifically the components of forces.

Comments on Individual questions

1 Many candidates showed either a lack of thought or understanding in answering the first question of the paper. Resolving forces to find the tension was not an appropriate method as the force at the hinge was not known and was simply ignored by the majority of candidates. However, there were many perfect solutions gained through taking moments about the hinge at *A*.

2 This question was generally well answered. Only a few candidates used an incorrect method in part (i) and found half the required time from *O* to *A*. Part (ii) was well answered.

3 This question was also well answered. Most candidates followed the hint and used work/energy in solving part (ii).

4 This question was found challenging. Very few candidates were successful in drawing the correct directions for the horizontal components of forces at *E* and *F*. Part (ii) was not well answered. Very few candidates demonstrated understanding in their methods. Part (iii) was found to be more straightforward but there was some confusion about the distances required for taking moments about the axis *AD*.

5 This question was found to be straightforward. However, a significant number of candidates complicated their calculations in part (i) by using the conservation of kinetic energy rather than the fact that $e = 1$. A large number of candidates failed to state the direction of the impulse in part (ii). In this part, the *m* was often missing from the answer $4.8m$. Part (iii) was well answered with few sign fiddles.

6 Part (i) was well answered with very few fiddles seen to gain the given answer. In part (ii), most candidates used the given formula with *y* set to the value -20. Part (iii) was well answered.

7 Most candidates understood this question well and answered it accurately. Surprisingly, a fairly common fault was in part (iii) where many lost one mark out of five by quoting their answer as 0.05 thinking that this was to 3 sig. fig. accuracy. Balancing the work/ energy equation in part (iv) was often found tricky. A hint for tackling this is to first consider the situation without any resistance.

8 This question was found to be the most difficult one of the examination. The main difficulty was in identifying forces, components and resolving in appropriate directions. In part (i) many either confused sin/cos or incorrectly derived the reaction to be $mg\cos 30°$. The one mark in part (iii) was gained by the majority of candidates but part (iv) was answered poorly. Many assumed that the reaction force from part (i) remained the same for the new set up.

4730: Mechanics 3

General Comments

Candidates were very well prepared for examination and this is reflected in the marks obtained. Only a small number of candidates scored fewer than half of the total marks available.

Some candidates, a significant minority, were content to use a 'recipe' method in Qs 6(i) and 7(ii). This approach worked better in Q7(ii) than in Q6(i). Candidates should be aware of the limitations of a particular recipe, and the recipe should not be regarded as an alternative to an understanding of the underlying mechanics principle.

Comments on Individual Questions

1 This was generally well attempted although there was a number of sources of error including; omission of mass (usually accidentally after a correct statement in symbols), obtaining $\theta = 38.0^{\circ}$ without making clear how this relates to the required direction (indication of θ in a diagram would have been sufficient), incorrect triangle (representing impulse as the sum of momenta instead of the difference), obtaining the value of 83.0° for an angle in the correct triangle instead of the corresponding obtuse angle value of 97.0°.

2 This question was very well attempted, although some candidates inappropriately sought to use $a = v\dfrac{\mathrm{d}v}{\mathrm{d}x}$ instead of $a = \dfrac{\mathrm{d}v}{\mathrm{d}t}$ in one or both parts. It was disappointing to see errors of transposition such as transferring 0.5 on the left hand side of the equation of motion, to the right hand side, as a multiplier instead of a divisor, and similarly basic errors in solving $2 = 1 + k/6$. Obtaining $t = 2$ from $t^2(t^2 + 1) = 20$ by inspection was deemed acceptable, but intending candidates should be aware that this is the case here only because the uniqueness of $t = 2$ as a viable answer is immediately apparent ($X[X + 1]$ is clearly greater than 20 for $X > 4$ and less than 20 for $0 < X < 4$).

3 (i) This was very well attempted and almost all candidates scored full marks.

(ii) This was also very well attempted, most candidates equating the energy when the particle is 4.5m below *O* with that when the particle is in its initial position, rather than when it is at *O*. In this case there are however two terms representing elastic potential energy in the resulting equation, one of which was often omitted.

4 This question was very well attempted and errors in method were rare. However many errors were made by weaker candidates in constructing and solving the relevant equations.

5 (i) This was well attempted and most candidates took moments of the forces on *BC*, about *B*, to obtain the given answer correctly. However some candidates took moments of the forces on the whole system, about *A*. Candidates who did this correctly could not make progress because of the presence of a term representing a horizontal force of unknown magnitude, and candidates who omitted the moment of the horizontal force at *C* could not obtain the given answer.

(ii) This was fairly well attempted, although many candidates made errors of sign, and other errors arising from unnecessarily long methods (e.g. taking moments rather than resolving forces to find the vertical component of the force exerted on *BC* at *B*).

(iii) This was well attempted although some candidates had the values of *F* and *R* reversed on substitution.

6 (i) Almost all candidates applied the principle of conservation of energy and many did so successfully. However some candidates made errors in dealing with the GPE, particularly those who chose a datum level other than the horizontal through O. A significant minority used the formula $v^2 = 2ga(1 - \cos\theta)$, failing to recognise that this applies only in cases equivalent to that for which the particle moves on the outer surface of a smooth circular cylinder with initial conditions $v = 0$ and $\theta = 0$.

(ii) This part of the question was very well attempted. However some candidates used the wrong sign for the term representing the component of the weight, and some candidates had $\sin\theta$ in this term instead of $\cos\theta$.

(iii) Almost all candidates used $T = 0$ appropriately but relatively few reached the correct answer for the angle turned through.

7 (i) Most candidates scored all four marks available although there was a considerable variation in the level of mathematical elegance used in obtaining the marks.

(ii) Most candidates recognised the need to use Newton's second law although many of the attempts to obtain $\ddot{x} = -400x$ were flawed. Almost all candidates realised the need to obtain this equation because the formula $T = 2\pi / \omega$ was well known, so that (with $T = 0.314$ given) the equation is effectively a given answer. A significant minority of candidates used the formula $\omega = \sqrt{\frac{1}{m}\left(\frac{\lambda_1}{L_1} + \frac{\lambda_2}{L_2}\right)}$ in lieu of Newton's second law. This is acceptable here because the question informs the candidate that the motion is simple harmonic, but intending candidates should be aware that Newton's second law cannot be avoided in all (or even most) questions of this type.

(iii) As in part (ii) there was a tendency to work back from the given answer, the formula $u = A\omega$ (or more generally $v^2 = \omega^2(A^2 - x^2)$) being well known. This approach produced a lot of confusion between u as the maximum value of v, and the maximum possible value of u for which the equation $\ddot{x} = -400x$ can apply continuously. Thus $A = 0.3$ was rarely satisfactorily justified, or even distinguished as the greatest possible amplitude.

(iv) Many candidates equated $0.3\sin 20t$ with 1.3, and among those who used –0.2 or +0.2 relatively few were able to obtain the correct answer.

Probability and Statistics

Chief Examiner's Report

As before, the general standard of work on the Probability and Statistics part of the examination is pleasingly high. There are many candidates who can answer relatively taxing questions. However, verbal questions continue to reveal that candidates' understanding is not always on a par with their ability to obtain correct numerical answers. It is also clear that, while some centres have clearly taken note of the remarks in these Reports, others have not, with the same mistakes and misconceptions continuing to appear from one year to the next.

Examiners report that many candidates do not complete the grid on the front of the first side of their answer booklet. This grid is used to display the marks for each individual question on the whole paper; the question-numbers of questions answered in subsequent answer booklets or loose sheets should be included on the front of the *first* booklet (and not on the subsequent booklet or loose sheets). Question numbers should not be subdivided; question numbers such as "3(i), 3(ii)" etc should *not* be given.

Centres are particularly asked to note the following points which have been agreed by the Examiners responsible for these papers in order to encourage good practice.

- Answers given to an excessive number of significant figures (such as "probability = 0.11853315"), which have not in the past lost marks, may in future be penalised.
- Hypothesis tests are likely no longer to include the explicit instruction "stating your hypotheses clearly"; any answer to a question involving hypothesis tests should include a statement of hypotheses unless they are already given in the question.
- Likewise, questions that involve critical regions (particularly for significance tests using discrete distributions) may not ask explicitly for the relevant probabilities to be quoted from tables, but candidates should always write down the values of these probabilities.
- Conclusions to hypothesis tests should be stated in terms that acknowledge the uncertainty involved. Thus "the mean height is 1.8" is too assertive and may not gain full credit; a statement such as "there is insufficient evidence that the mean height is not 1.8" is much to be preferred.

<u>Multiple attempts at questions.</u>

In recent sessions examiners have noted an increasing number of candidates making two, or more, attempts at a question, and leaving the examiner to choose which attempt to mark. Examiners have been given this instruction.

'If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt, and ignore the others.'

Please inform candidates that it is in their best interest to make sure that, when they have a second attempt at a question, they make it clear to the examiner which attempt is to be marked. The obvious way for candidates to do this is to make sure they cross out any attempts which they do not regard as their best attempt at the question.

4732: Probability and Statistics 1

General Comments

Most candidates showed a good understanding of a fair proportion of the mathematics in this paper. There were some very good scripts.

A few candidates (fewer than usual) ignored the instruction on page 1 and rounded their answers to fewer than three significant figures, thereby losing marks. Some rounded answers to varying degrees of accuracy, apparently at random. In some cases, marks were lost through premature rounding of intermediate answers.

In certain calculations a calculator can give the answer immediately, without the need to show working (e.g. Q8(ii)). However, candidates need to be aware that they risk losing all the available marks if the answer is incorrect but no working is shown. It is wise to show some working or at least to double-check the calculation.

In questions requiring written answers, candidates commonly failed to gain the marks because they gave only statistical responses, without interpretation and/or reference to the relevant context.

Only a few candidates seemed to run out of time.

Most candidates failed to fill in the question numbers on the front page of their answer booklet, despite the clear instruction to do so.

In order to understand more thoroughly the kind of answers which are acceptable in the examination context, centres should refer to the published mark scheme.

Use of statistical formulae

Use of the formula booklet MF1 was generally better than in June 2005. Nevertheless there were still a few candidates who made no use of the formula booklet, but used formulae from memory. More often than not, these formulae were incorrect. Others tried to use the given formulae, but clearly did not understand how to use them properly. Some candidates used the less convenient versions of formulae from the formula booklet, e.g. $\Sigma(x - \bar{x})^2$. The volume of arithmetic involved in these versions led to errors in most cases. Even the more convenient versions were sometimes misinterpreted, e.g. $\Sigma x^2 - \frac{\Sigma x^2}{n}$.

Candidates would benefit from direct teaching on the proper use of the formula booklet, particularly in view of the fact that text books give statistical formulae in a huge variety of versions. Much confusion could be avoided if candidates were taught to use exclusively the versions given in MF1. They need to understand which formulae are the simplest to use and also how to use them.

A few candidates used the binomial tables as if the probabilities were individual, rather than cumulative.

Comments on Individual Questions

1 (i) This part was successfully answered by most candidates, although a few just multiplied the two given probabilities. A tree diagram is possibly helpful here. However, some candidates drew extra branches, showing Jenny passing on her first attempt and then either passing or failing on a fictitious second attempt. These candidates sometimes included working from these incorrect branches in their calculation, achieving the correct answer by a rather suspect route.

(ii) Many candidates found $^5/_6 - ^2/_3$ but went no further. Some found the correct answer of $^1/_2$, but then went on to find P(John fails on his first attempt and passes on his second attempt) $= ^1/_3 \times ^1/_2 = ^1/_6$. A few simply combined the two given probabilities in some fashion. A few arrived at the correct answer by luck from $^1/_6 + ^1/_3$. These scored no marks.

2 (i) The effect of a linear transformation on μ and σ^2 were understood by most candidates. However, many candidates attempted to use methods appropriate for combining groups, thus wasting much time. These were sometimes successful for the mean but never for the variance. A common incorrect calculation for the mean was 24.5/50 + 100 = 100.49. Some candidates gave 104.8 for the variance.

(ii) Interpretation of the greater standard deviation was sometimes weak. Some candidates thought that a greater standard deviation meant that the heights were greater. Many stated that the "range" of heights was greater. This did not score the mark. Answers such as "Greater spread" or "Less consistency" were accepted. It was also pleasing to see the occasional answer such as "The difference from the mean is greater on average". A few candidates confused the two groups of heights.

(iii) Many candidates found the unweighted mean of the two means.

3 (i) The use of a geometric or binomial distribution was common. Many candidates arrived at the correct answer by the incorrect method: $^3/_5 \times ^1/_2 \times ^2/_3$. This scored no marks. Some candidates found P(BPP or PBP or PPB). A few resorted to using the values given in the table and $\Sigma p = 1$.

(ii) This was one of the better answered parts in the paper. Some errors were seen, e.g. $(\Sigma xp)/4$, Σxp^2, $\Sigma (xp)^2$. Some candidates who obtained an incorrect answer for E(X) or E(X^2) then arrived at a negative value for Var(X), but this appeared to give them no concern. Some omitted to subtract $(\mathrm{E}(X))^2$.

4 (i)(a) Many candidates used the standard approach to histograms:

Frequency density = Frequency / Class width.

However, very few of these candidates actually arrived at a value for the frequency density represented by the height of one square. More commonly this approach led to confusion and/or incorrect answers. Many candidates did not know what to do with their numbers such as 15 000 and 300. Some combined the total frequency with a bogus "total class width" of 6000 – 1000 = 5000. A far more fruitful approach was to find the frequency represented by the area of one square. Candidates taking this approach generally answered this part correctly, taking only one or two lines of working to do so.

(i)(b) There was a similar contrast between the clumsy and the elegant methods in this part. A further difficulty was that some candidates interpreted the phrase "calculate an estimate" to mean that an approximate method was adequate. These candidates estimated the equivalent number of whole squares which would represent the frequency for £2700 to £3200. Some estimates were close enough to gain partial credit but most were not. Some candidates ignored the difference in height between the two blocks. The most efficient method was to calculate exactly the equivalent number of whole squares.

(ii) There appeared to be a common misunderstanding of the properties and advantages of different statistical diagrams. Many candidates gave a cumulative frequency graph as the answer to (a) rather than (b). Some gave a pie chart or frequency polygon in (b).

5 This question required candidates to distinguish between situations giving rise to a geometric distribution, a binomial distribution and neither of these. Some candidates failed to do this. It was also common for correct working to be followed by incorrect answers.

(i)(a) This part was generally well answered, although a few candidates included a binomial coefficient.

(i)(b) Many candidates appeared to be unfamiliar with questions on the Geometric distribution involving "less than". Just $(^3/_4)^5$ was common. Some tried to use a binomial method. Some of those who understood the question correctly, used the long method, adding terms. A few of these omitted a term or added an extra one. Only a disappointingly small number of candidates used the most efficient method, and even some of these used a power of 3 or 5 instead of 4.

(ii)(a) Many candidates omitted the binomial coefficient, probably because they were still thinking in terms of a Geometric distribution. Some candidates recognised the binomial situation, but surprisingly, most used the formula rather than the binomial tables.

(ii)(b) This unusual question was well answered by a good number of candidates, although some multiplied their answer to part (i)(b), instead of part (ii)(a), by $^2/_5$. On the other hand, despite the clear lead-in given by part (ii)(a), the use of B(6, $^2/_5$) was common.

6 This was the least successfully answered question. A few used an incorrect formula, ${}^nC_r = \frac{n!}{r!}$. Some candidates used permutations. Incorrect combinations such as ${}^{11}C_7$ were common.

(i) A very large number of candidates found the correct two combinations, but added them.

(ii) ${}^7C_4$ was often seen but used incorrectly. As in part (i), many candidates added instead of multiplying. Candidates who attempted direct probability methods often used $^1/_4$ or $^1/_7$ or $^6/_7$, each of which is incorrect.

(iii) Many candidates calculated the number of ways of <u>not</u> choosing A1 by ${}^4C_3$ instead of ${}^3C_2$. Others used ${}^6C_4$ instead of ${}^6C_3$ for the number of ways of choosing B1. Others considered (A1 and not B1) but not (not A1).

7 (i) Almost all candidates correctly gave B(10, 0.9). For the assumptions, very many candidates just listed parrot-fashion the conditions for the use of the binomial. Because no context was referred to, these scored no marks. Most candidates seemed confused between "conditions" and "assumptions". In this case, two of the conditions (Repeated trials and Only two outcomes to each trial) are not assumptions that have to be made, but are inherent in the situation as described. The other two conditions are not inherent and have to be assumed to be true if a Binomial distribution is to be used. (The probability of each seed germinating is the same, and Germination of each seed is independent of the others). Some candidates gave the first of these in the form "Each seed must be planted in the same conditions", which was accepted as a good answer in context.

(ii) This very easy binomial probability can be read directly from the tables, but many candidates failed to realise this. Some found $P(X = 8)$ or $1 - P(X = 8)$ or $1 - P(X \leq 8)$. Some attempted a very long (but correct) method, adding eight different probabilities using the formula. Some used the tables, but found $P(X \leq 8)$.

(iii) Some candidates failed to recognise a binomial distribution and attempted methods using, e.g., $^{19}/_{20}$. Others used B(20, 0.9) or B(200, 0.9). Some did not use their answer from part (ii), but started from scratch, usually incorrectly. A few used the correct distribution, but found only one term instead of two. Others used the wrong tail, reading the question as "fewer than 8", as in part (ii). Some found the correct terms but evaluated them incorrectly. Another method which displayed some misunderstanding was $0.9298^{20} + 0.9298^{19}$. The number of correct solutions to this part was small.

8 (i)(a) This was answered correctly by most candidates. A few found the differences of the raw data pairs instead of their ranks. Some made arithmetical errors. Others used an incorrect formula, either $\dfrac{6\Sigma d^2}{n(n^2-1)}$ or $\dfrac{1-6\Sigma d^2}{n(n^2-1)}$. A few found r instead of r_s.

(i)(b) Most candidates quoted, parrot-fashion, answers such as "Good correlation between the populations of these countries and their capital cities" or "A strong relationship between the populations of these countries and their capital cities" or "The ranks of country populations and capital city populations are similar". Answers such as these did not score the mark. To score the mark, answers had to interpret the value of r_s in context, e.g. "Counties with large populations tend to have capital cities with large populations." Another common answer which showed partial understanding was "As the population of the country increases, so does the population of the capital city." A common wrong answer was that the populations of countries and capitals were "in proportion".

(ii) This was well answered, except for the few candidates who did not use the formula booklet or who misinterpreted the given formulae or who chose the less convenient versions for S_{xx} etc.

(iii) This part was also generally well answered. Some candidates referred to "skew", which is irrelevant. Others thought that because the values of y for France and the UK were large, their removal would decrease r.

(iv)(a) Most candidates chose the correct regression line and gave a correct reason. Some described y as the "controlled" variable, which is incorrect, but shows some understanding. A few chose y on x, and gave as a reason that an x value was to be estimated from a y value.

(iv)(b) Many candidates recognised the inappropriateness of applying the regression line in such a different context. Some ignored the context and stated that because r is fairly high, and because the estimate would be an interpolation, it would be fairly reliable. These gained partial credit. Some candidates referred neither to context nor to r but only to interpolation. These scored no marks. The use of the word “accurate” rather than “reliable” resulted in the loss of a mark for many candidates.

4733 Probability and Statistics 2

General Comments

This paper was found generally accessible and it produced a good range of marks. There were many very good candidates who produced high quality scripts; however, it seemed that some would have benefited from more preparation, especially in terms of detail. As usual there were some seriously under-prepared candidates.

The usual problems arose with this specification. Mistakes that have been repeatedly identified in the past include wrong or omitted continuity corrections (and the converse, the insertion of continuity corrections when they are wrong); the similar omission, or wrong inclusion, of a factor of $\sqrt{n}$ in the standard deviation; incorrect use of tables, particularly with errors such as $P(X \geq 5) = 1 - P(X \leq 5)$; and poor understanding of verbal questions. As usual, numerical calculations were better done than verbal interpretations, and the impression persists that many candidates do not really understand what they are doing, even when they get correct numerical answers.

As was mentioned in the report on the Summer 2005 examination, in future it is expected that questions will not explicitly include instructions to "state hypotheses" or "show all relevant probabilities"; it will be assumed that these are an essential part of full solutions. Candidates may also in future be penalised for giving too many significant figures in their answers if this is clearly inappropriate.

Rather too many candidates think that "$P(X < 9)$" means "$P(X \leq 8)$", even for a continuous distribution.

Comments on Individual questions.

1 Most could get the two calculations correct, though use of Po(0.7) tables for the distribution $\text{Po}\left(\frac{2}{3}\right)$ is not acceptable. For part (ii), in order to score both marks it was necessary to give a reason why one of the standard conditions for the validity of a Poisson distribution might not hold, and to state that reason. Thus answers such as "Foxes may go around in groups so they are not independent" or "some parts of the region might be more favourable to foxes than others, so the average rate would not be constant" would score full marks. This was a comparatively rare example of a context in which "foxes might not occur singly" would have been an appropriate reason, though as explained in previous reports neither "singly" nor "occurring at random" is generally a safe reason to give.

2 Most correctly used a normal approximation. Some used *nq* instead of *npq* as the variance, but most good candidates scored at least 5 out of 7 here. The continuity correction was often wrong, with several instances of candidates saying that $P(X > 13) = P(X \geq 12) = P(X > 11.5)$.

3 As usual the significance test using a binomial distribution was poorly done. Many could not state the hypotheses correctly. (It is good practice to state the meaning of the letter used in the hypothesis test – e.g. "where p is the population proportion of chocolates with hard centres" – but very few candidates do this.) As usual many seemed unable to do an exact binomial test and attempted to use a normal approximation. In this particular question there was a further problem in that many did not seem to realise that they had to test for the *upper* tail (because 5 is more than the expected value of 3); solutions such as "$P(X \leq 5) = 0.9887 > 0.975$ so do not reject H_o" were incorrect.

4 Here the Poisson approximation was essential; the normal approximation is very poor and scored few marks. In part (ii), it was very disappointing that, despite the clear instruction, many candidates failed to write down any of the necessary probabilities such as "$P(X \leq 4) = 0.9763$, $P(X \leq 5) = 0.9940$". Without at least one of these probabilities, full marks could not be scored, and if neither was shown and the final answer was not 6, no credit at all could be given.

5 This question was poorly answered. In part (i), errors included inclusion of a spurious continuity correction, failure to square-root $\frac{1}{4}\mu^2$, failure to simplify $\mu/(\frac{1}{2}\mu)$ to 2, and failure to obtain the final numerical answer as well as to state that it was independent of μ.

In the second part, many attempted to use the mean of three years (with a variance of 9/3), rather than to find the probability for one year and cube it.

The final part produced many impressive (and often long) responses, but the answer "because the years are not chosen randomly" was unacceptable and would probably have been so even with a different wording of the question. As has been stated before, clear thinking on this issue is not encouraged by concentrating on the word "random".

6 Many scored full, or nearly full marks on this question with apparent ease, with omission of the 50/49 factor the commonest error. But many others betrayed serious confusion; it is essential to use the hypothesis value of μ, here 32, as μ in the normal distribution calculation. Those who wrote "N(32.3, …)" and found $P(\overline{X} < 32)$ scored few marks, as did those who omitted the $\sqrt{n}$ factor. Most of the marks for a question of this sort are given for method, not calculation. Again, hypotheses were poorly stated; in the absence of a verbal definition, only the symbol μ should be used, and those who used $\overline{X}$ (or $\overline{w}$) were on completely the wrong track.

7 The main problems in parts (i) and (iii) here were omission of the $\sqrt{12}$ factor, and getting the wrong sign. Quite a few candidates rounded off their answers to the nearest integer, which is incorrect. A surprising number failed to realise that the answer to part (ii)(b) was that the test produced the correct result; the perhaps automatic response of "Type II error" was common. In part (iii), using 80 rather than the value of *c* was a common error.

8 This question produced a very wide range of responses. At one extreme, a good many dealt very confidently with the first four parts and showed excellent algebraic and calculus skills; some of those who did not realise that the answers to part (v) could simply be written down obtained the correct values by integration, either by expanding $\left(y+\frac{4}{5}\right)^3$ or by using parts. At the other extreme, it was disappointing to find many candidates (presumably including some taking Further Maths) who thought that $x \times x^n = x^{2n}$ or that the value of kx^{n+1} when $x = 1$ was k^{n+1}.

Most realised that the distribution in part (iv) was normal.

It should be noted that, in part (v), use of the integrals $\int (y+\frac{4}{5})^4\,dx$ and $\int (y+\frac{4}{5})^5\,dx$, although they give apparently correct answers, are wrong as these find the expectation and variance of $\left(Y+\frac{4}{5}\right)$, rather than *Y*.

4734: Probability and Statistics 3

General Comments

The general performance of candidates was similar to those of the January 2005 paper. Fewer, however, scored very high marks. This was, perhaps, mainly due to the unusual hypotheses in Qs 2 and 6.
Some candidates believe that a test statistic should always be positive, and this can lead to an incorrect conclusion in a one-tailed test. z values should be obtained from $z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}}$, as recommended in previous reports.

Comments on Individual Questions

1 (i) This proved to be a question in which most of the candidates could score well and the required interval was usually obtained correctly.

(ii) The comment was not expected to be too assertive with statements such as "the councillor may be right".

2 (i) Many referred only to the small sample size whereas the t-test requires an unknown population variance.

(ii) The hypothesis "At least 2000" would not be rejected if $\mu \geq 2000$ were correct, and rejected if $\mu < 2000$ were correct. The hypotheses should then be H_0:μ = 2000, H_1:μ < 2000. It is also acceptable to have H_0:$\mu \geq 2000$. Despite this not always being known, candidates did usually score well.

3 This was the least well-answered question on the paper, in both parts although, with the given answer, part (i) was found to be easier.

4 Most candidates could pick up some marks but it was generally poorly answered. It was often realised that a normal approximation to a Poisson distribution was required, but the required continuity correction was not always used. The principles for finding the mean and variance in part (ii) were usually known but their relevance in part (iii) was not always appreciated.

5 The finding of cdfs, and their relevance to finding pdfs of related variables, is a topic that more candidates are becoming familiar with. It was pleasing to find so many excellent solutions.

6 Candidates seemed very familiar with the independent sample test, but it was not always applied correctly. The required formula was usually known for finding the unbiased estimate of variance, but since the answer was given, a value to more than 3 decimal places was required before rounding in order to obtain full credit.

In part (ii) some candidates used a z-value in their test despite the hint in part (i) .

The hypotheses in part (iii) were often incorrect and the further assumption asked for was often shortened to "normal distributions" without any context. It should always be made clear which distributions are involved.

7 (i) Most knew which integral was required to obtain E(X) but there were some problems with the limits.

(ii) Candidates were expected to obtain the value of α from a relevant equation, but some verified the given value, which was not acceptable.

(iii) This was the most difficult part, but the given answer meant that those unsuccessful here could still obtain full credit in the final part.

(iv) Most candidates could obtain several marks here, the main error was to give 3 for the number of degrees of freedom, instead of 2 (from 4 cells).

Decision Mathematics

Multiple attempts at questions.

In recent sessions examiners have noted an increasing number of candidates making two, or more, attempts at a question, and leaving the examiner to choose which attempt to mark. Examiners have been given this instruction.

‘If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt, and ignore the others.’

Please inform candidates that it is in their best interest to make sure that, when they have a second attempt at a question, they make it clear to the examiner which attempt is to be marked. The obvious way for candidates to do this is to make sure they cross out any attempts which they do not regard as their best attempt at the question.

4736: Decision Mathematics 1

General Comments

A good spread of marks was achieved with only a few very poor scripts. Even the weaker candidates could usually score some marks on every question. Most candidates were able to complete the examination in the time allowed, although some spent far too long on writing out every swap and comparison in the bubble and shuttle sorts, giving too many iterations of the Simplex algorithm or making multiple attempts at the linear programming question.

In general, the quality of the presentation of the candidates' answers was good. Some candidates were unable to read their own working and created extra work for themselves as well as forfeiting accuracy marks.

Comments on Individual Questions

1 This was a straightforward start for most candidates. Some candidates did not use the ordered list of arc weights to show the order in which they had selected or rejected the arcs, but most were able to construct an appropriate minimum spanning tree. A few candidates had used Prim's algorithm instead of Kruskal's.

2 Many very good answers. The weaker candidates had extra temporary labels (there is no need to record a temporary label if it is larger than the current temporary labels at a vertex) or gave permanent labels to vertex *D*, or both. Some candidates left both *D* and *E* without permanent labels and so also lost the mark for the order of assigning the permanent labels.

3 (i) Most candidates were able to follow the algorithm accurately; a few drew out every step and some seemed to extrapolate from the first two diagrams to give incorrect answers for parts (c) and (d). A common mistake was to only have six arcs in the diagram for $n = 3$.

(ii) The better candidates realised that the odd and even values of n behaved differently and were able to give a rule for the number of arcs in these two cases. The weak candidates assumed that the formula would be $\frac{1}{2}n(n+1)$, despite the fact that this did not fit their values.

4 (i) Most candidates were able to set up an initial Simplex tableau. Some candidates omitted the column corresponding to *P* and a few had sign errors in the objective row.
When slack variables have been added, the problem becomes:

maximise P

subject to $P - 5x + 4y + 3z = 0$

$2x - 3y + 4z + s = 10$

$6x + 5y + 4z + t = 60$

and $x, y, z, s, t \geq 0$

This is represented in matrix form using the tableau:

1	-5	4	3	0	0	0
0	2	-3	4	1	0	10
0	6	5	4	0	1	60

(ii) Some good answers, candidates who showed the operations they were using were able to gain method marks even when they had made arithmetic slips. Most candidates could find the appropriate pivot element (the 2 in the second row) and were able to carry out pivot operations of the form *current row* $\pm$ *multiple of pivot row* to get a basis column with a 1 replacing the pivot value and a zero in the other rows.

5 (i) Very few candidates were able to identify the variables correctly. Many just said 'breaststroke', 'backstroke' and 'butterfly', instead of 'x = number of lengths of breaststroke', and so on. The identification of the objective function often appeared in amongst the constraints, even when the appropriate combination of the variables was selected. Several candidates claimed that the objective was $x + y + z$ (which represents the total distance) instead of maximising the style marks.

(ii) Many candidates just wrote down every combination of x, y, z and numbers that they could see despite the question having set out quite clearly where the constraints were to come from.

(iii) Most candidates were able to draw the boundary lines for the constraints given, but some assumed that the feasible region would be the quadrilateral with the origin as one vertex. Quite a few of the graphs were not sufficiently accurate for the candidates to be able to read off the vertices with any confidence. The question asked candidates to write down the coordinates of the feasible region but some chose to ignore the non-integer valued point completely.

(iv) Some candidates were able to interpret their solution from part (iii) in the context of the original problem to describe the number of lengths of each stroke that Findley should swim and a few gave the total number of points that he could expect as being 22 points.

6 (i) Most candidates were able to write down the order of visiting the vertices for at least one of the tours obtained by using the nearest neighbour method although some then chose the longer route as the better upper bound. If a diagram is used, the direction of travel around the tour should be indicated.

(ii) Part of Prim's algorithm is to show the order in which the vertices are included and to state the weight of the minimum spanning tree. Several candidates showed the two shortest arcs from *G* on their tree and consequently lost the credit for the minimum spanning tree. Some candidates calculated the length of the spanning tree but did not use it to find a lower bound for the journey time.

(iii) Most candidates gained marks on this question but several who correctly identified *BD* and *EF* as the routes to be repeated did not state that the arcs that needed to be travelled twice were *BD*, *EG* and *GF*. Several candidates claimed that *D* was travelled through six times, instead of three times.

7 (i) Many candidates gave incoherent answers in which they just showed pages of swaps with no clear indication of where the passes ended. It was only necessary to show the results at the end of each pass. A large number of candidates made errors in copying their own figures from one row or column to the next and many did not seem to know the bubble sort algorithm. Even those who did perform the algorithm correctly often went on to carry out an unnecessary seventh pass or stopped at the end of the fifth pass. The algorithm can be stopped early only when no swaps have been made in an entire pass.

(ii) Shuttle sort was somewhat more successfully carried out than bubble sort, but even so many candidates were not able to count the total number of comparisons and the total number of swaps accurately. Some candidates sorted into decreasing order, despite the question clearly asking for a sorted list in increasing order.

(iii) Only a few candidates were able to explain that each script needed to be looked at once and so the time taken is, roughly, proportional to the number of scripts. Some candidates seemed to have the right sort of idea but were not able to convincingly explain their thoughts, several just missed the point completely.

(iv) Quite a few candidates were able to calculate the total time as being 250 + 6250 = 6500 seconds, some candidates did not appreciate that the sorting of the two piles took place simultaneously. The majority of the candidates just scaled the total time for splitting and sorting the 100 scripts by a factor of 5^2.

4737: Decision Mathematics 2

General Comments

A good spread of marks was achieved and most candidates were able to complete the examination in the time allowed.

In general, the quality of the presentation of the candidates' answers was good. A few candidates submitted messy work that was squeezed into a tiny space; this invariably led to a loss of accuracy and a consequent loss of marks.

Comments on Individual Questions

1 (i) Most candidates were able to draw the correct bipartite graph, a few did not realise that both *A1* and *A2* needed to be matched to *G*, *P* and *R*.

(ii) Nearly all the candidates completed the second diagram to show the incomplete matching. A few then superimposed their alternating paths and almost obscured the matching in the process.

(iii) Most candidates gave a valid alternating path, but several did not give a path consisting of just three arcs (the shortest alternating path), indeed some candidates seemed determined to make the longest alternating path they could.

(iv) Almost all the candidates were able to find the matching, even those who had lost their way in constructing the alternating path in part (iii).

2 Most candidates scored full marks on this question, including reading off the route. A few candidates gave the route backwards, but most started at (3; 0), and a few omitted either the start or the finish vertex. A few candidates made numerical errors but were usually saved by having the network given in the question.

3 (i) Most candidates were able to calculate the capacity of the cut with $X = \{S, A, B, C\}$, $Y = \{D, E, F, G, H, I, T\}$ although some found the capacity of the cut α instead.

(ii) Several candidates realised that they did not need to include the arc *DG* in their calculation, but their explanations of why were often confused. The best answers referred to the fact that *D* and *G* are both on the sink side of the cut.

(iii) Many good answers, although many candidates did not give the maximum flow in the arc *SB*.

4 (iv) Most candidates found a flow of 14 litres per second, although they did not always show their flow clearly and often omitted the direction arrows. A few candidates were concerned that the diagram in the question paper did not show flow directions, and some did not appreciate that the fluid could flow in either direction.
Although some candidates correctly found the cut of 14 litres per second, rather more just used the flows instead of the capacities and so thought that every cut had capacity 14.

Only a small minority of candidates were able to state that since they had found a flow of 14 litres per second and a cut of 14 litres per second this must be the maximum flow and the minimum cut. One or two candidates went as far as explaining that each cut causes a restriction to the flow so every flow is less than or equal to every cut and hence maximum flow ≤ every cut and thus having found a flow equal to a cut it must be the maximum flow (and the minimum cut).
The majority of the candidates either assumed that the cut they had found was the minimum cut with no reasoning given or they just said maximum flow equals minimum cut but did not refer to the flow and the cut that they had found.

(i) Most candidates calculated the value 260, but several omitted the units.

(ii) Many correct or very nearly correct solutions. Some candidates stopped before they had reached an augmented reduced cost matrix with a zero cost complete matching. A few candidates did not augment efficiently and went up in four single steps instead of just using two augmentations and some candidates only reduced the rows before beginning to augment.

(iii) Most candidates were able to gain the mark but only the best gave a completely correct description by using words such as 'minimum cost allocation'.

5 (i) Few candidates were able to explain the necessity for the dummies, often just giving very general descriptions of what dummy activities are. The dummy between *C* and *F* is needed because *F* follows both *B* and *C* whereas *G* follows just *C*, the dummy after *H* is needed because otherwise activities *H* and *I* would share a common start point and a common end point. This would mean that when the algorithm is run (mechanically, rather than using 'common sense') there would be ambiguity over which value to use.

(ii) Most candidates understood what was required to complete the precedence table. A few listed the activities that followed the activities instead of those that they followed from.

(iii) Most candidates were able to make a good attempt at the forward and backward passes although quite a few ignored the dummy and gave a late event time of 6 instead of 5 after activity *C* and rather more did not give an early or a late event label times for the activities instead of the events (vertices) between them. The minimum project completion time needed units of days.

(iv) Many correct answers, but also several candidates who thought that *C* would need to increase by 2 days as a consequence of earlier errors.

(v) The resource histogram was not done well; a large number of candidates drew a graph with 'holes' in it or 'overhanging blocks' instead of a histogram. A resource histogram should look like a histogram in statistics, except that the vertical axis shows 'number of workers'. It is not necessary to label the activities *A*, *B*, *C*, etc. on the histogram.

(vi) Many candidates tried to move activity *E* to after activity *J* without realising that this contravened the original precedences. Several candidates realised that activity *H* needed to be delayed but fewer realised that they also needed to move *J*. It was sometimes difficult to interpret where an activity was being moved to, in particular there was confusion between the reading on the scale and which day it referred to (e.g. day 1 is from time 0 and time 1). Some candidates gave a schedule listing the order of the activities carried out by each worker.

6 (i) Most candidates identified the appropriate values to show that *A* does not dominate *B* ($-3 < 5$ in column *Y*) and *B* does not dominate *A* ($-3 < 2$ in column *X* or $1 < 4$ in column *Z*). Some candidates just listed inequality signs without explaining how this showed the dominance.

(ii) A few candidates unnecessarily redrew the pay-off matrix from Maria's perspective. Some candidates gave column minima of giving column maxima and then explaining why column *Y* does not give the minimax (play-safe) strategy.

(iii) Most candidates realised that the row minima were all equal and hence each is a maximin and there is no preference between *A*, *B* and *C* in terms of play-safe (cautious play), however if Lucy plays row *B* then she could win as many as 5 points (the maximum possible) if Maria plays column *Y* (rather than playing safe).

(iv) Most candidates realised that 3 has to be subtracted from *m* because previously 3 has been added throughout the matrix. Some candidates thought that *M* stood for the matrix or just said that 3 was the most Lucy could lose.

(v) To justify the expressions given, candidates needed to write out the matrix with 3 added throughout and then explain how column *X* led to an expected gain of $5p_1 + 0p_2 + 7p_3$ and similarly for column *Y* and column *Z*. Some candidates seemed to think that *A*, *B* and *C* had 'become' p_1, p_2 and p_3 and some candidates used rows instead of columns.

(vi) Candidates' explanations were generally poor. The expressions in part (v) give the expected gain for Lucy for each of Maria's choices and for each combination of the probabilities p_1, p_2 and p_3. *m* represents Lucy's minimum expected gain for each combination of the probabilities, which is equal to (at least) one of the expressions from part (v) and is lower than the others.

Even if Maria does her worst, Lucy can choose the probabilities to maximise her minimum expected gain, and she may even do better than this in the long run if Maria does not play optimally.

Many candidates discussed the probabilities needing to sum to 1 or talked vaguely about 'it' needing to be like this so that the Simplex method would work.

(vii) Most candidates were able to recover to show where $4 - 2p_1$ came from and to find the other expression as $7p_1 - 3$. Some candidates used the augmented matrix and so got the expressions $7 - 2p_1$ and $7p_1$ without realising that they needed to then subtract 3.

(viii) Most candidates were able to solve their equations from part (vii) to give a probability p_1 and some put this back into one or both equations to find the expected number of points won by Lucy in this case. Some candidates also calculated the number of points won when $p_1 = 0$ and when $p_1 = 1$ for at least one of the two expressions. Only a few candidates realised that the smaller number of points for each value of p_1 corresponded to the vertices of the lower boundary and thus selected the value of p_1 to maximise the minimum expected number of points won.

Advanced GCE Mathematics (3890, 3892, 7890)
January 2006 Assessment Session

Unit Threshold Marks

Unit		Maximum Mark	a	b	c	d	e	u
4721	Raw	72	57	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4722	Raw	72	57	49	41	34	27	0
	UMS	100	80	70	60	50	40	0
4723	Raw	72	55	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
4724	Raw	72	63	55	47	39	31	0
	UMS	100	80	70	60	50	40	0
4725	Raw	72	59	51	43	36	29	0
	UMS	100	80	70	60	50	40	0
4726	Raw	72	59	51	43	36	29	0
	UMS	100	80	70	60	50	40	0
4727	Raw	72	58	50	43	36	29	0
	UMS	100	80	70	60	50	40	0
4728	Raw	72	55	48	41	35	29	0
	UMS	100	80	70	60	50	40	0
4729	Raw	72	58	50	43	36	29	0
	UMS	100	80	70	60	50	40	0
4730	Raw	72	57	50	43	36	29	0
	UMS	100	80	70	60	50	40	0
4732	Raw	72	56	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4733	Raw	72	58	50	43	36	29	0
	UMS	100	80	70	60	50	40	0
4734	Raw	72	54	47	40	33	27	0
	UMS	100	80	70	60	50	40	0
4736	Raw	72	56	49	42	35	29	0
	UMS	100	80	70	60	50	40	0
4737	Raw	72	55	48	41	34	28	0
	UMS	100	80	70	60	50	40	0

Specification Aggregation Results

Overall threshold marks in UMS (i.e. after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
3890	300	240	210	180	150	120	0
3892	300	240	210	180	150	120	0
7890	600	480	420	360	300	240	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
3890	14.9	31.4	52.6	72.5	90.3	100	475
3892	50.0	50.0	100	100	100	100	2
7890	21.9	65.6	84.4	87.5	93.8	100	54

For a description of how UMS marks are calculated see;
www.ocr.org.uk/OCR/WebSite/docroot/understand/ums.jsp

Statistics are correct at the time of publication

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