OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4727

Further Pure Mathematics 3

Wednesday **25 JANUARY 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

1 Find the acute angle between the skew lines

$$\frac{x+3}{1} = \frac{y-2}{1} = \frac{z-4}{-1} \qquad \text{and} \qquad \frac{x-5}{2} = \frac{y-1}{-3} = \frac{z+3}{1}.$$ [4]

2 The tables shown below are the operation tables for two isomorphic groups G and H.

G	a	b	c	d
a	d	a	b	c
b	a	b	c	d
c	b	c	d	a
d	c	d	a	b

H	2	4	6	8
2	4	8	2	6
4	8	6	4	2
6	2	4	6	8
8	6	2	8	4

(i) For each group, state the identity element and list the elements of any proper subgroups. [4]

(ii) Establish the isomorphism between G and H by showing which elements correspond. [3]

3 **(i)** By using the substitution $y^3 = z$, find the general solution of the differential equation

$$3y^2\frac{\mathrm{d}y}{\mathrm{d}x} + 2xy^3 = \mathrm{e}^{-x^2},$$

giving y in terms of x in your answer. [6]

(ii) Describe the behaviour of y as $x \to \infty$. [1]

4 **(i)** By expressing $\cos\theta$ and $\sin\theta$ in terms of $\mathrm{e}^{\mathrm{i}\theta}$ and $\mathrm{e}^{-\mathrm{i}\theta}$, or otherwise, show that

$$\cos^2\theta\sin^4\theta = \tfrac{1}{32}(\cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2).$$ [6]

(ii) Hence find the exact value of

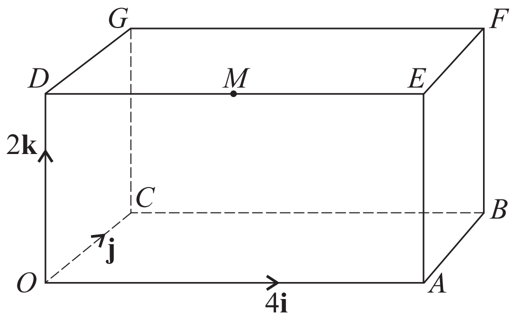
$$\int_0^{\frac{1}{3}\pi} \cos^2\theta\sin^4\theta\,\mathrm{d}\theta.$$ [3]

5 **(i)** Solve the equation $z^4 = 64(\cos\pi + \mathrm{i}\sin\pi)$, giving your answers in polar form. [2]

(ii) By writing your answers to part **(i)** in the form $x + \mathrm{i}y$, find the four linear factors of $z^4 + 64$. [4]

(iii) Hence, or otherwise, express $z^4 + 64$ as the product of two real quadratic factors. [3]

6



The cuboid $OABCDEFG$ shown in the diagram has $\overrightarrow{OA} = 4\mathbf{i}$, $\overrightarrow{OC} = \mathbf{j}$, $\overrightarrow{OD} = 2\mathbf{k}$, and M is the mid-point of DE.

(i) Find a vector perpendicular to $\overrightarrow{MB}$ and $\overrightarrow{OF}$. [3]

(ii) Find the cartesian equations of the planes CMG and OEG. [5]

(iii) Find an equation of the line of intersection of the planes CMG and OEG, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. [3]

7 A group G has an element a with order n, so that $a^n = e$, where e is the identity. It is given that x is any element of G distinct from a and e.

(i) Prove that the order of $x^{-1}ax$ is n, making it clear which group property is used at each stage of your proof. [6]

(ii) Express the inverse of $x^{-1}ax$ in terms of some or all of x, x^{-1}, a and a^{-1}, showing sufficient working to justify your answer. [3]

(iii) It is now given that a commutes with every element of G. Prove that a^{-1} also commutes with every element. [2]

8 **(i)** Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + 4x = 0,$$

where k is a real constant, in each of the following cases.

(a) $|k| > 2$

(b) $|k| < 2$

(c) $k = 2$

[8]

(ii) **(a)** In the case when $k = 1$, find the solution for which $x = 0$ and $\dfrac{dx}{dt} = 6$ when $t = 0$. [4]

(b) Describe what happens to x as $t \to \infty$ in this case, justifying your answer. [2]

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