OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS **4726**

Further Pure Mathematics 2

Monday **16 JANUARY 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- There is an **insert** for use in Question **4**.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages and an insert.

1 **(i)** Write down and simplify the first three non-zero terms of the Maclaurin series for $\ln(1 + 3x)$. [3]

(ii) Hence find the first three non-zero terms of the Maclaurin series for

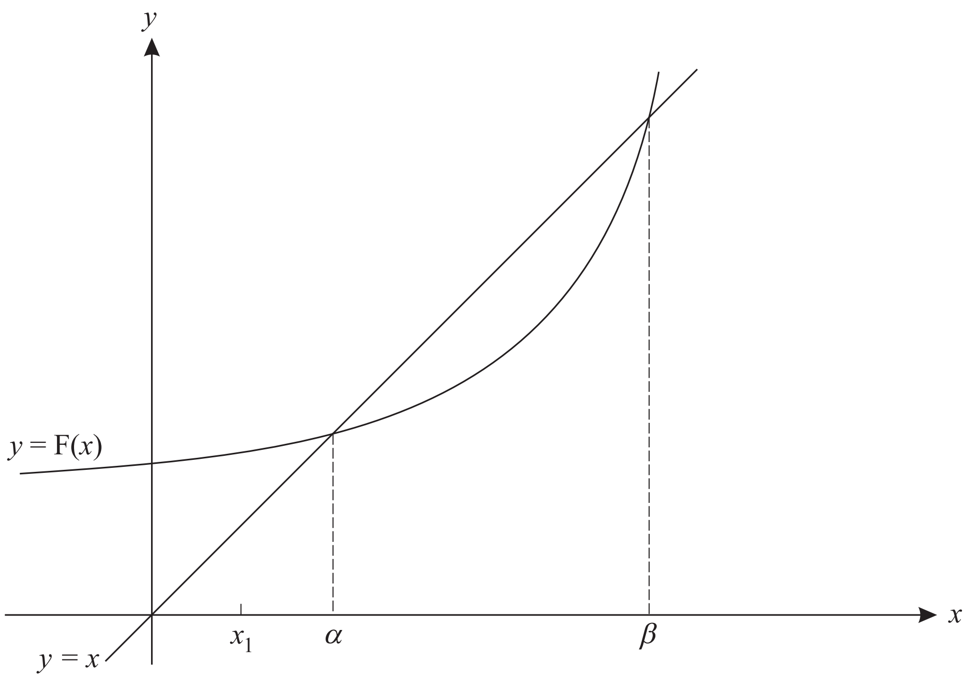
$$e^x \ln(1 + 3x),$$

simplifying the coefficients. [3]

2 Use the Newton-Raphson method to find the root of the equation $e^{-x} = x$ which is close to $x = 0.5$. Give the root correct to 3 decimal places. [5]

3 Express $\dfrac{x+6}{x(x^2+2)}$ in partial fractions. [5]

4 **Answer the whole of this question on the insert provided.**



The sketch shows the curve with equation $y = F(x)$ and the line $y = x$. The equation $x = F(x)$ has roots $x = \alpha$ and $x = \beta$ as shown.

(i) Use the copy of the sketch on the insert to show how an iteration of the form $x_{n+1} = F(x_n)$, with starting value x_1 such that $0 < x_1 < \alpha$ as shown, converges to the root $x = \alpha$. [3]

(ii) State what happens in the iteration in the following two cases.

(a) x_1 is chosen such that $\alpha < x_1 < \beta$.

(b) x_1 is chosen such that $x_1 > \beta$.

[3]

5 **(i)** Find the equations of the asymptotes of the curve with equation

$$y = \frac{x^2 + 3x + 3}{x + 2}.$$ [3]

(ii) Show that y cannot take values between -3 and 1. [5]

6 **(i)** It is given that, for non-negative integers n,

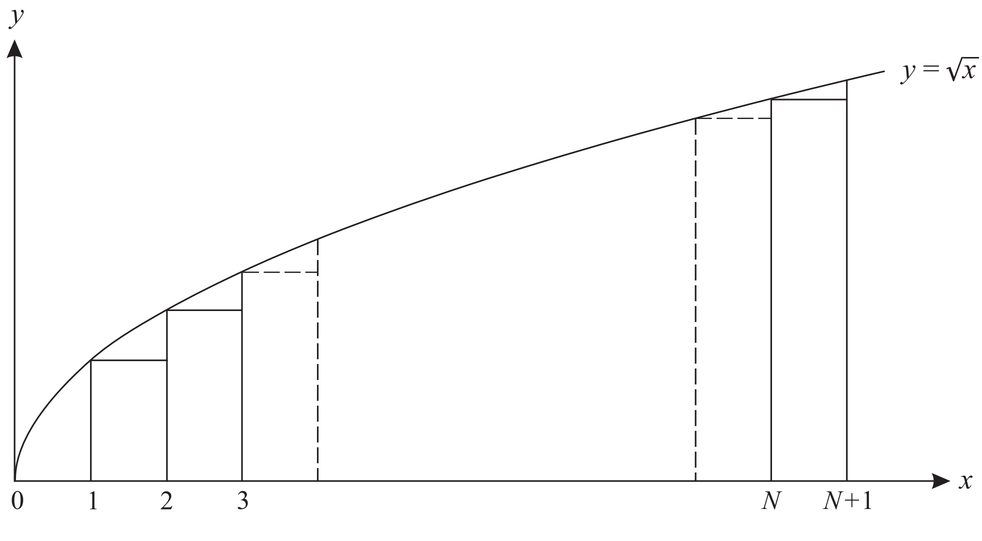
$$I_n = \int_0^1 e^{-x} x^n \, dx.$$

Prove that, for $n \geqslant 1$,

$$I_n = nI_{n-1} - e^{-1}.$$ [4]

(ii) Evaluate I_3, giving the answer in terms of e. [4]

7



The diagram shows the curve with equation $y = \sqrt{x}$. A set of N rectangles of unit width is drawn, starting at $x = 1$ and ending at $x = N + 1$, where N is an integer (see diagram).

(i) By considering the areas of these rectangles, explain why

$$\sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots + \sqrt{N} < \int_1^{N+1} \sqrt{x} \, dx.$$ [3]

(ii) By considering the areas of another set of rectangles, explain why

$$\sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots + \sqrt{N} > \int_0^{N} \sqrt{x} \, dx.$$ [3]

(iii) Hence find, in terms of N, limits between which $\sum_{r=1}^{N} \sqrt{r}$ lies. [3]

[Turn over

8 The equation of a curve, in polar coordinates, is

$$r = 1 + \cos 2\theta, \quad \text{for } 0 \leqslant \theta < 2\pi.$$

(i) State the greatest value of r and the corresponding values of θ. [2]

(ii) Find the equations of the tangents at the pole. [2]

(iii) Find the exact area enclosed by the curve and the lines $\theta = 0$ and $\theta = \frac{1}{2}\pi$. [5]

(iv) Find, in simplified form, the cartesian equation of the curve. [4]

9 **(i)** Using the definitions of $\cosh x$ and $\sinh x$ in terms of e^x and e^{-x}, prove that

$$\sinh 2x = 2 \sinh x \cosh x.$$ [4]

(ii) Show that the curve with equation

$$y = \cosh 2x - 6 \sinh x$$

has just one stationary point, and find its x-coordinate in logarithmic form. Determine the nature of the stationary point. [8]

Candidate Name	Centre Number	Candidate Number

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INSERT for Question 4

Monday **16 JANUARY 2006** Morning 1 hour 30 minutes

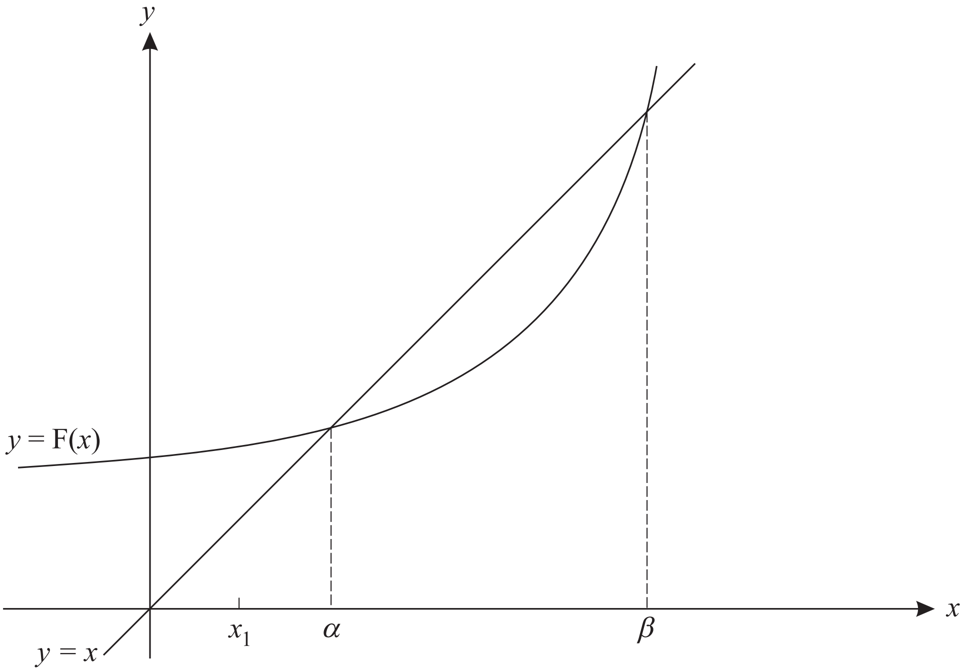
INSTRUCTIONS TO CANDIDATES

- This insert should be used to answer Question **4**.
- Write your name, centre number and candidate number in the spaces provided at the top of this page.
- Write your answers to Question **4** in the spaces provided in this insert, and attach it to your answer booklet.

This insert consists of 2 printed pages.

[Turn over

4 (i)



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(ii) (a) ....

(b) ....