OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS **4736**

Decision Mathematics 1

Monday   **23 JANUARY 2006**   Afternoon   1 hour 30 minutes

Additional materials:
 8 page answer booklet
 Graph paper
 List of Formulae (MF1)

**TIME** 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

* Write your name, centre number and candidate number in the spaces provided on the answer booklet.
* Answer **all** the questions.
* There is an **insert** for use in Questions **1** and **2**.
* Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
* You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

* The number of marks is given in brackets [ ] at the end of each question or part question.
* The total number of marks for this paper is 72.
* Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
* **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 6 printed pages, 2 blank pages and an insert.

1 **Answer this question on the insert provided.**
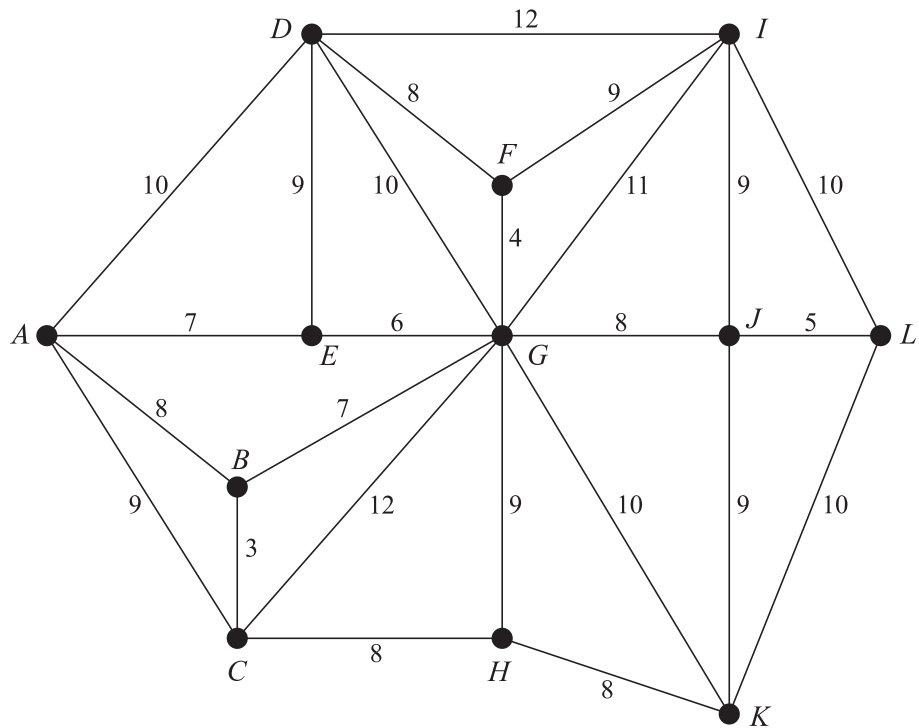
This diagram shows a network. The insert has a copy of this network together with a list of the arcs, sorted into increasing order of weight. Use Kruskal's algorithm on the insert to find a minimum spanning tree for this network. Draw your tree and give its total weight. [5]

2 **Answer this question on the insert provided.**
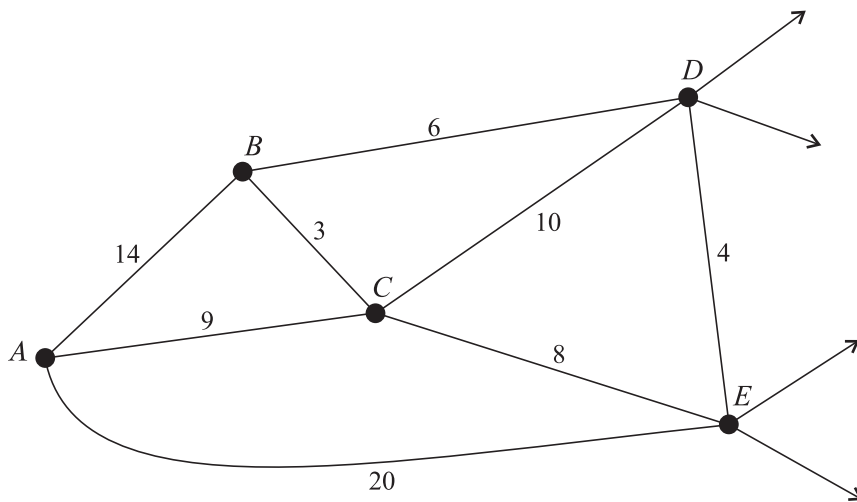
This diagram shows part of a network. There are other arcs connecting *D* and *E* to other parts of the network. Apply Dijkstra's algorithm starting from *A*, as far as you are able, showing your working. Note: you will not be able to give permanent labels to all the vertices shown. [6]

3   Consider the following algorithm.

> Step 1 : Input a positive integer n.
>
> Step 2 : Draw a graph consisting of n vertices and no arcs.
>
> Step 3 : Create a new vertex and join it directly to every vertex of order 0.
>
> Step 4 : Create a new vertex and join it directly to every vertex of **odd** order.
>
> Step 5 : Stop.

   (i) Draw separate diagrams to show the result of applying this algorithm in the following cases.

   (a) $n = 1$

   (b) $n = 2$

   (c) $n = 3$

   (d) $n = 4$

[4]

   (ii) State how many arcs are in the graph that results when the algorithm is applied to a set of n vertices.   [2]

4   **(i)** Represent the linear programming problem below by an initial Simplex tableau.   [2]

> Maximise $P = 5x - 4y - 3z,$
>
> subject to $2x - 3y + 4z \leqslant 10,$
> $6x + 5y + 4z \leqslant 60,$
>
> and $x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0.$

   (ii) Perform **one** iteration of the Simplex algorithm and write down the values of x, y, z and P that result from this iteration.   [6]

[Turn over

5   Findlay is trying to get into his local swimming team. The coach will watch him swim and will then make his decision. Findlay must swim at least two lengths using each stroke and must swim at least 8 lengths in total, taking at most 10 minutes.

Findlay needs to put together a routine that includes breaststroke, backstroke and butterfly. The table shows how Findlay expects to perform with each stroke.

Stroke	Style marks	Time taken
Breaststroke	2 marks per length	2 minutes per length
Backstroke	1 mark per length	0.5 minutes per length
Butterfly	5 marks per length	1 minute per length

Findlay needs to work out how many lengths to swim using each stroke to maximise his expected total number of style marks.

 (i) Identify appropriate variables for Findlay's problem and write down the objective function, to be maximised, in terms of these variables. [2]

 (ii) Formulate a constraint for the total number of lengths swum, a constraint for the time spent swimming and constraints on the number of lengths swum using each stroke. [3]

Findlay decides that he will swim two lengths using butterfly. This reduces his problem to the following LP formulation:

$$\text{maximise} \qquad P = 2x + y,$$

$$\text{subject to} \qquad x + y \geqslant 6,$$
$$4x + y \leqslant 16,$$
$$x \geqslant 2, \ y \geqslant 2,$$

with x and y both integers.

 (iii) Use a graphical method to identify the feasible region for this problem. Write down the coordinates of the vertices of the feasible region and hence find the integer values of x and y that maximise P. [6]

 (iv) Interpret your solution for Findlay. [2]

6 The network represents a railway system. The vertices represent the stations and the arcs represent the tracks. The weights on the arcs represent journey times between stations, in minutes. The sum of all the weights is 105 minutes.
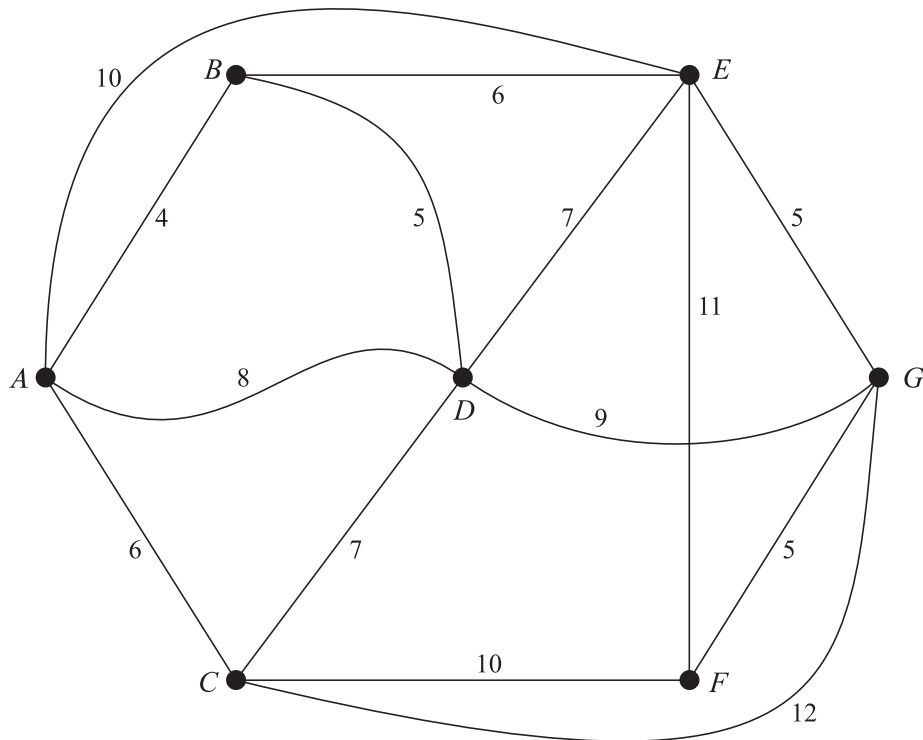
Norah wants to travel around the system visiting every station. She wants to start and end at *A* and she wants to complete her journey in the shortest possible time.

(i) Apply the nearest neighbour method starting at *A* to find two suitable tours and calculate the journey time for each of these tours. Which of these answers gives the better upper bound for Norah's journey time? [5]

(ii) Construct a minimum spanning tree by using Prim's algorithm on the reduced network formed by deleting vertex *G* and all the arcs that are directly joined to *G*. Draw a diagram to show the arcs in your tree. Hence calculate a lower bound for Norah's journey time. [6]

Norah now decides that she wants to use every section of track in her journey. She still wants to start and end at *A* and to complete her journey in the shortest possible time.

(iii) Calculate the journey time for Norah's new problem. Show your working; quickest times between stations may be found by inspection. State which arcs Norah will have to travel twice and how many times she will pass through station *D*. [5]

[Question 7 is printed overleaf.]

[Turn over

7 Mr Rank and Miss File need to sort a pile of examination scripts into increasing order of mark. Mr Rank first goes through the pile of scripts and puts each script into one of two piles, depending on whether the mark is below 50 or not. He then sorts the scripts in the 'below 50' pile and Miss File sorts the scripts in the '50 and above' pile. At the end they put the two sorted piles together again.

(i) The scripts in the 'below 50' pile have the following marks, starting from the top of the pile.

$$34 \quad 42 \quad 27 \quad 31 \quad 12 \quad 48 \quad 24 \quad 37$$

Use **bubble** sort to sort this list into increasing order. Clearly indicate the list that results at the end of each pass through the algorithm. Give the number of swaps and the number of comparisons that were used in sorting this list. [6]

(ii) The scripts in the '50 and above' pile have the following marks, starting from the top of the pile.

$$95 \quad 74 \quad 61 \quad 87 \quad 71 \quad 82 \quad 53 \quad 57$$

Use **shuttle** sort to sort this list into increasing order. Clearly indicate the list that results at the end of each pass through the algorithm. List the number of swaps and number of comparisons that were used in sorting this list. [6]

(iii) Explain why splitting the original list into two piles is a linear order algorithm. [2]

(iv) Both bubble sort and shuttle sort are quadratic order algorithms. Mr Rank and Miss File use their method to sort a pile of 100 scripts. It takes about 50 seconds to split the pile and about 250 seconds to do each sort. As the sorts are done at the same time, this gives a total time taken of about 300 seconds, or 6 minutes. Approximately how long would Mr Rank and Miss File take to split a pile of 500 scripts into two roughly equal piles and sort the piles? Show all your working. [4]

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Candidate Name	Centre Number	Candidate Number

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RECOGNISING ACHIEVEMENT

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Decision Mathematics 1
INSERT for Questions 1 and 2

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INSTRUCTIONS TO CANDIDATES

- This insert should be used to answer Questions **1** and **2**.
- Write your name, centre number and candidate number in the spaces provided at the top of this page.
- Write your answers to Questions **1** and **2** in the spaces provided in this insert, and attach it to your answer booklet.

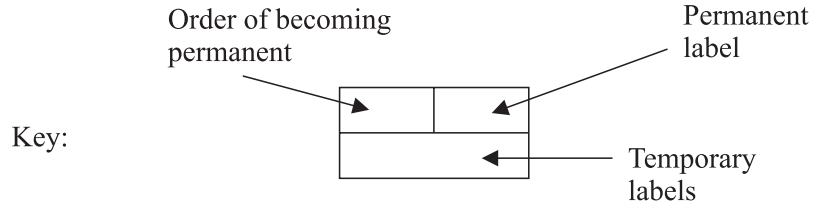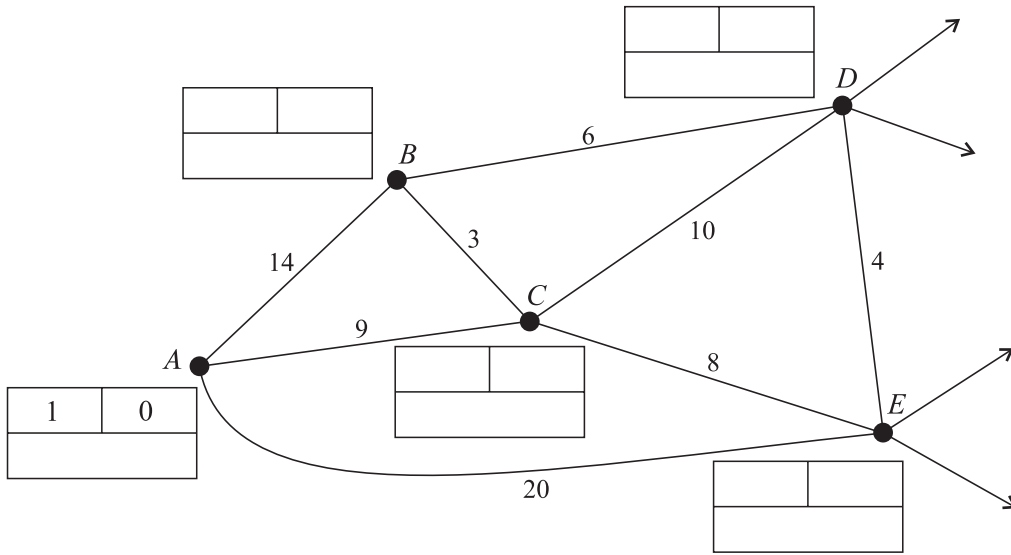**This insert consists of 3 printed pages and 1 blank page.**

1
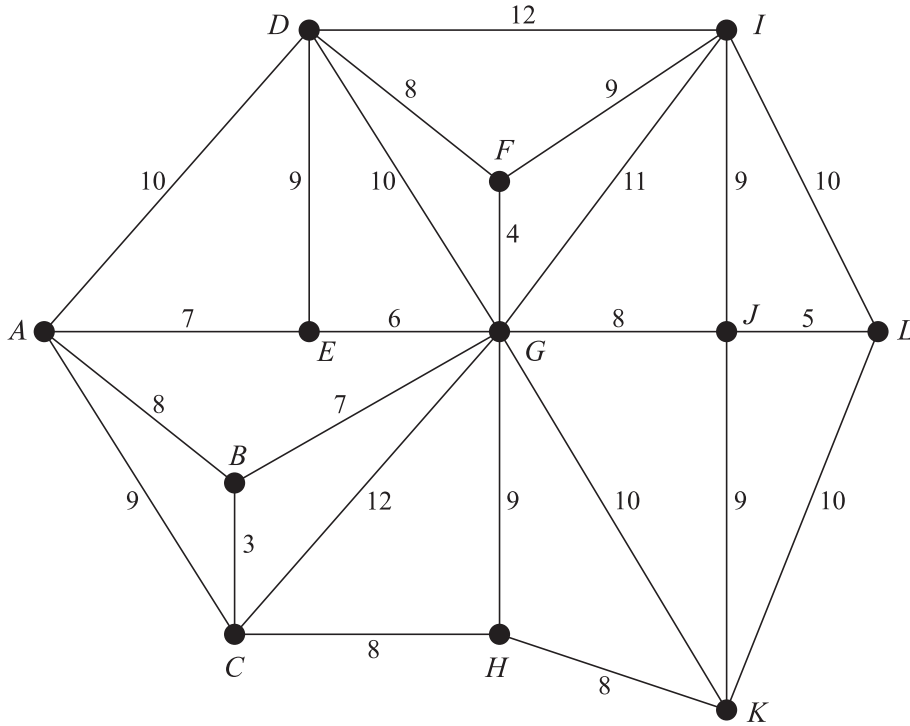
BC = 3
FG = 4
JL = 5
EG = 6
AE = 7
BG = 7
AB = 8
CH = 8
DF = 8
GJ = 8
HK = 8
AC = 9
DE = 9
FI = 9
GH = 9
IJ = 9
JK = 9
AD = 10
DG = 10
GK = 10
IL = 10
KL = 10
GI = 11
CG = 12
DI = 12
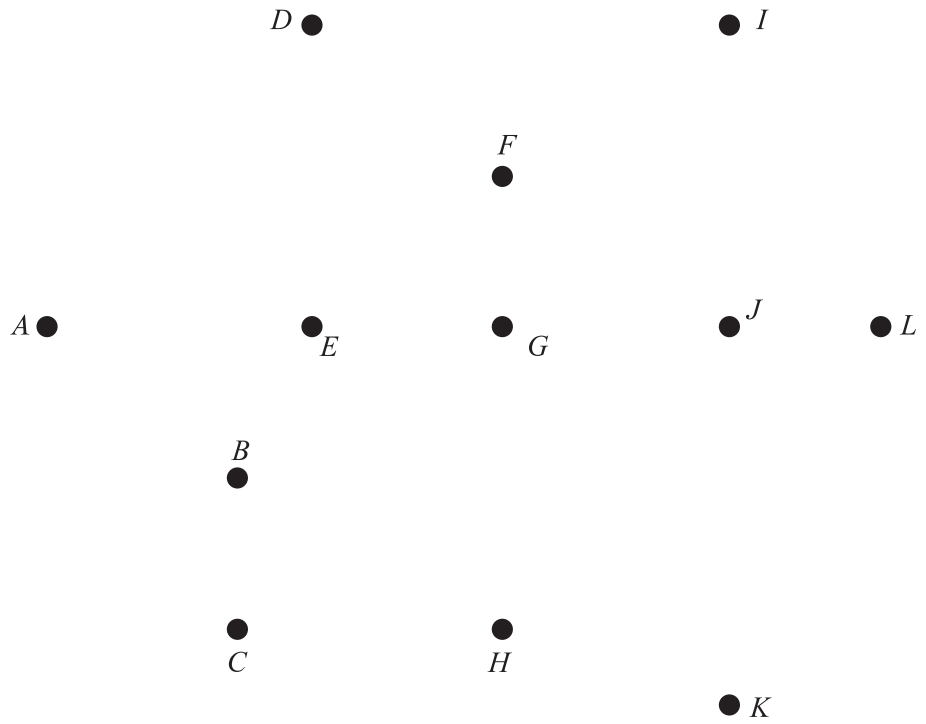
Total weight of arcs in minimum spanning tree = ...

2

Key:

Order of becoming permanent

Permanent label

Temporary labels

Do not cross out your working values (temporary labels)

14

3

6

10

4

9

8

20

B

D

C

A

E

1	0

4

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