

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

**Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education**

MATHEMATICS

4723

Core Mathematics 3

Thursday **16 JUNE 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

- 1 The function f is defined for all real values of x by

$$f(x) = 10 - (x + 3)^2.$$

(i) State the range of f . [1]

(ii) Find the value of $ff(-1)$. [3]

- 2 Find the exact solutions of the equation $|6x - 1| = |x - 1|$. [4]

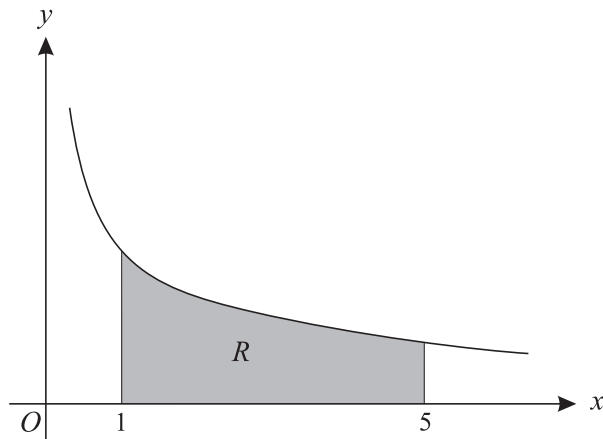
- 3 The mass, m grams, of a substance at time t years is given by the formula

$$m = 180e^{-0.017t}.$$

(i) Find the value of t for which the mass is 25 grams. [3]

(ii) Find the rate at which the mass is decreasing when $t = 55$. [3]

- 4 (a)



The diagram shows the curve $y = \frac{2}{\sqrt{x}}$. The region R , shaded in the diagram, is bounded by the curve and by the lines $x = 1$, $x = 5$ and $y = 0$. The region R is rotated completely about the x -axis. Find the exact volume of the solid formed. [4]

- (b) Use Simpson's rule, with 4 strips, to find an approximate value for

$$\int_1^5 \sqrt{x^2 + 1} \, dx,$$

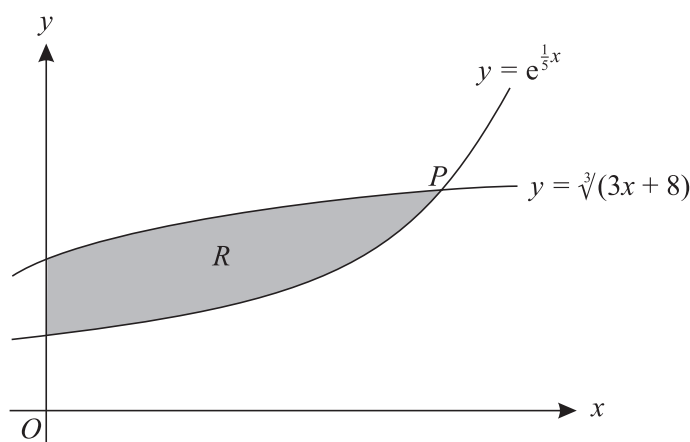
giving your answer correct to 3 decimal places. [4]

- 5 (i) Express $3 \sin \theta + 2 \cos \theta$ in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

(ii) Hence solve the equation $3 \sin \theta + 2 \cos \theta = \frac{7}{2}$, giving all solutions for which $0^\circ < \theta < 360^\circ$. [5]

- 6 (a) Find the exact value of the x -coordinate of the stationary point of the curve $y = x \ln x$. [4]
- (b) The equation of a curve is $y = \frac{4x + c}{4x - c}$, where c is a non-zero constant. Show by differentiation that this curve has no stationary points. [3]
- 7 (i) Write down the formula for $\cos 2x$ in terms of $\cos x$. [1]
- (ii) Prove the identity $\frac{4 \cos 2x}{1 + \cos 2x} \equiv 4 - 2 \sec^2 x$. [3]
- (iii) Solve, for $0 < x < 2\pi$, the equation $\frac{4 \cos 2x}{1 + \cos 2x} = 3 \tan x - 7$. [5]

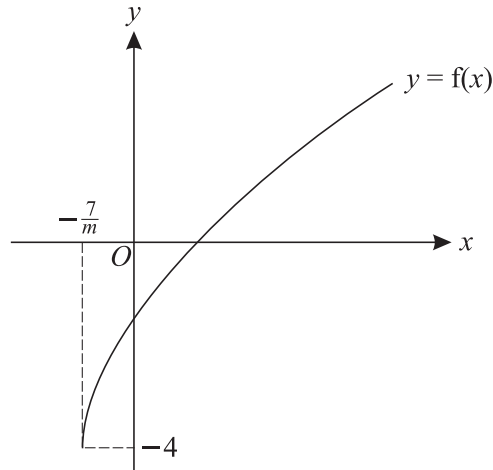
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The diagram shows part of each of the curves $y = e^{\frac{1}{5}x}$ and $y = \sqrt[3]{(3x + 8)}$. The curves meet, as shown in the diagram, at the point P . The region R , shaded in the diagram, is bounded by the two curves and by the y -axis.

- (i) Show by calculation that the x -coordinate of P lies between 5.2 and 5.3. [3]
- (ii) Show that the x -coordinate of P satisfies the equation $x = \frac{5}{3} \ln(3x + 8)$. [2]
- (iii) Use an iterative formula, based on the equation in part (ii), to find the x -coordinate of P correct to 2 decimal places. [3]
- (iv) Use integration, and your answer to part (iii), to find an approximate value of the area of the region R . [5]

[Question 9 is printed overleaf.]



The function f is defined by $f(x) = \sqrt{mx + 7} - 4$, where $x \geq -\frac{7}{m}$ and m is a positive constant. The diagram shows the curve $y = f(x)$.

- (i) A sequence of transformations maps the curve $y = \sqrt{x}$ to the curve $y = f(x)$. Give details of these transformations. [4]
- (ii) Explain how you can tell that f is a one-one function and find an expression for $f^{-1}(x)$. [4]
- (iii) It is given that the curves $y = f(x)$ and $y = f^{-1}(x)$ do not meet. Explain how it can be deduced that neither curve meets the line $y = x$, and hence determine the set of possible values of m . [5]

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