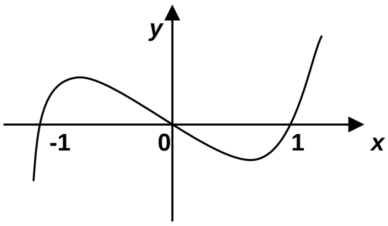
1. (i)	$11^{-2} = \frac{1}{121}$	B1 1	$\frac{1}{121}$
(ii)	$\sqrt{50} + \frac{6}{\sqrt{3}}$ $= 5\sqrt{2} + \frac{6\sqrt{3}}{3}$ $= 5\sqrt{2} + 2\sqrt{3}$	B1 M1 A1 3 4	$5\sqrt{2}$ Attempt to rationalise $\frac{6}{\sqrt{3}}$ $2\sqrt{3}$
2.	$qx^2 - 2qrx + qr^2 + 10$ $= 2x^2 - 12x + p$	B1 B1 M1 A1 $\sqrt{}$ 4 4	$q = 2$ $r = 3$ $qr^2 + 10 = p$ $p = 28$ $\sqrt{}$ on both q and p values
3.	$y = 5\sqrt{2x} - 3$	M1 A1 M1 A1 4 4	$\sqrt{2x}$ or $\sqrt{\frac{x}{2}}$ seen $5\sqrt{2x}$ ± 3 to y = ( ) $y = a\sqrt{kx} - 3$, $k > 0$
4. (a)	$\tan 2x = 1$ $2x = 45, 225$ $x = 22\frac{1}{2}, 112\frac{1}{2}$	B1 M1 A1 3	45 or 225 seen or implied Evidence of angle being halved $22\frac{1}{2}$ and $112\frac{1}{2}$ and no others between 0^{o} and 180^{0}

(b)	$\tan\theta + \dfrac{1}{\tan\theta}$ $\equiv \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$ $\equiv \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ $\equiv \dfrac{1}{\sin\theta\cos\theta}$	M1 M1 A1 3 <u>6</u>	Use $\dfrac{\sin}{\cos} = \tan$ Use $\sin^2 + \cos^2 = 1$ $\dfrac{1}{\sin\theta\cos\theta}$ AG CWO
5.	Either $y = 2x + 1$ or $y = \dfrac{x^2 + 11}{3}$ $x^2 - 6x + 8 = 0$ $(x-2)(x-4) = 0$ $x = 2 \quad x = 4$ $y = 5 \quad y = 9$ OR $x = \dfrac{y-1}{2}$ $\dfrac{(y-1)^2}{4} - 3y + 11 = 0$ $y^2 - 14y + 45 = 0$ $(y-5)(y-9) = 0$ $y = 5 \quad y = 9$ $x = 2 \quad x = 4$	M1 M1 M1 A1 A1 <u>5</u>	Substitute for x/y Obtain 3 term quadratic = 0 Correct method to solve 3 term quadratic $x = 2/4 \quad y = 5/9$ $y = 5/9 \quad x = 2/4$

6.(i)	$9-4\times7\times1=-19$	B1	-19 or 9 – 28
	0 real roots	B1 √ 2	0 real roots
(ii)	$\left(p+1\right)^2-64=0$ or $2\left[\left(x+\frac{p+1}{4}\right)^2-\frac{\left(p+1\right)^2}{16}+4\right]=0$	M1	Attempts b^2-4ac (involving p) or attempts to complete square (involving p)
		A1	$\left(p+1\right)^2-64=0$ aef
	$p=-9,7$	B1	$p=-9$
		B1 4	$p=7$
		<u>6</u>	
7.(i)	Gradient DE $=-\frac{1}{2}$	B1 1	$-\frac{1}{2}$
(ii)	Gradient EF $=\frac{4}{2}=2$	B1	2
	$-\frac{1}{2}\times2=-1$	B1 2	$m_1m_2=-1$ o.e.
(iii)	$y-3=-\frac{1}{2}\left(x-2\right)$	M1	Correct equation for straight line, gradient *DE*, point *F*.
	$\frac{1}{2}x+y-4=0$ $x+2y-8=0$	A1 √	$y-3=-\frac{1}{2}\left(x-2\right)$ aef
		A1 3	$x+2y-8=0$ (this form but can have fractional coefficeints.)
		<u>6</u>	
8.(i)	$\frac{dy}{dx}=4x$	B1	$4x$
	At $x=3$, $\frac{dy}{dx}=12$	B1 2	12
(ii)	Gradient of tangent $=-8$ $4x=-8$	M1	$\frac{dy}{dx}=-8$
	$x=-2$	A1	$x=-2$
	$y=8$	A1 √ 3	$y=8$

(iii) (iv)	Gradient $=6$ $\frac{dy}{dx}=2kx$ $x=1$ $\frac{dy}{dx}=2k$ $k=3$	B1 1 M1 M1 A1√ 3 9	Gradient = or (approaches) 6 (tends to) (converges to) $\frac{dy}{dx}=2kx$ $\frac{dy}{dx}=2k$ = 6 – equating derivative at x = 1 or at midpoints of chords to appropriate gradient value $k=3$ SR $\frac{y_2-y_1}{x_2-x_1}$ = gradient of chord M1 A1 k = 3 A1
9 (i) (ii)	 (-1,0) (0,0) (1,0) $\int_{-1}^{0} x(x^2-1)\,\mathrm{d}x$ $=\left[\frac{x^4}{4}-\frac{x^2}{2}\right]_{-1}^{0}$ $=\frac{1}{4}$	M1 B1 A1 3 M1 A1√ A1 M1 A1 5 8	Positive cubic (-1,0) (0,0) (1,0) Any one point stated or marked on sketch All 3 and no extras stated or marked on sketch $\int x\left(x^2-1\right)\mathrm{d}x$ $\int_{-1}^{0}$ $\frac{x^4}{4}-\frac{x^2}{2}$ Substitute limits (top-bottom) $\frac{1}{4}$
10.(i) (ii)	$2[10+x+x]>64$ $x(x+10)<299$ $x^2+10x-299<0$ $(x-13)(x+23)<0$	B1 1 B1 B1 2	$20+4x>64$ o.e. $x(x+10)<299$ Correctly shows $(x-13)(x+23)<0$ AG CWO SR Working backwards form AG $x^2+10x<299$ B1 $x(x+10)<299$ B1

(iii)	$x>11$ $(x-13)(x+23)<0$ $-23<x<13$ $\therefore 11<x<13$	B1 M1 B1 A1 B1 $\sqrt{\ }$ 5 <u>8</u>	$x>11$ Correct method to solve $(x-13)(x+23)<0$ e.g. graph 13 and -23 seen $-23<x<13$ $11<x<13$ $\sqrt{\ }$ on $x > 11$

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1 (i) State $3\ln x$	**B1**	**1**	[or equiv such as $3\ln 3x$ or $3\log_e x$]
(ii) State answer of the form $k\,\mathrm{e}^{\frac{1}{2}x}$	**M1**		[… where k is (unsimplified) constant]
Obtain $8\,\mathrm{e}^{\frac{1}{2}x}$	**A1**	**2**	

2 (i) Obtain 256 + …	**B1**		[2^8 needs to be evaluated]
Obtain … + $1024x$	**B1**		[coefficient must be simplified]
Attempt term of the form $k\,.\,2^t\,.\,x^2$	**M1**		[where k is attempt at binomial coeff]
Obtain … + $1792\,x^2$	**A1**	**4**	[coefficient must be simplified]
(ii) Recognise that third term is needed	**M1**		
Obtain 448	**A1**√	**2**	[… following their coeff of x^2 from part **(i)**; accept $448y^4$]

3 (i) Either:			
Equate attempt at at least one of f(–1), f(1), f(–3), f(3) to zero	***M1**		
Obtain $-1-p+q=0$ and $27+3p+q=0$	**A1**		[or equivs; maybe implied]
Attempt solution of pair of simultaneous equations	**M1**		[dep ***M1**]
Obtain $p=-7$ and $q=-6$	**A1**	**4**	
Or:			
Consider x^2 term from $(x+1)(x-3)(x+a)$	***M1**		
Obtain $x+2$ as third factor	**A1**		
Consider coefficient of x and constant term	**M1**		[dep ***M**]
Obtain $p=-7$ and $q=-6$	**A1**	**(4)**	
(ii) State -1 and 3	**B1**		
State -2	**B1**√	**2**	[…following their value of q]

4 Obtain derivative of the form $k\,\mathrm{e}^{0.4t}$	**M1**		[any constant k different from 3]
Obtain correct $1.2\,\mathrm{e}^{0.4t}$	**A1**		[or unsimplified equiv]
Substitute 7 to obtain 19.7	**A1**		[or 20 or greater accuracy (19.7335…)]
Obtain derivative of the form $kt^3(2t^4+9)^n$	**M1**		[any constant k; any non-zero $n < \frac{1}{2}$]
Obtain correct $4t^3(2t^4+9)^{-\frac{1}{2}}$	**A1**		[or unsimplified equiv]
Substitute 7 to obtain 19.8	**A1**	**6**	[or 20 or greater accuracy (19.7804…)]

5	**(i)**	State expression of form $k(y_0 + 2y_1 + y_2)$ or equiv	**M1**		[involving attempts at *y* values corresponding to 0, 0.5, 1 – exact or decimal; any constant *k*]
		State $\frac{1}{2}\times\frac{1}{2}(\ln 16 + 2\ln 13 + \ln 4)$ or decimal approx	**A1**		[0.25(2.77 + 2 × 2.56 + 1.39)]
		Use one relevant logarithm property correctly	**M1**		[… such as $p\ln a = \ln a^p$]
		Use second relevant logarithm property correctly	**M1**		[… such as $\ln a + \ln b = \ln ab$]
		Confirm $\frac{1}{2}\ln 104$	**A1**	5	[**AG**; necessary detail required]
	(ii)	Refer, in some form, to fact that tops of trapezia are below the curve	**B1**	1	[allow for any number of trapezia]

6	**(i)**	State Fig 3 or equiv	***B1**		
		Refer to *y* not defined uniquely for some *x*	**B1**	2	[dep ***B**; or 'vertical' line meets curve twice or equiv]
	(ii)	State Fig 1 or equiv	***B1**		
		Refer to *y* = *k* not having unique *x* for some *k*	**B1**	2	[dep ***B**; or horizontal line does not meet graph only once or equiv]
	(iii)	State Figs 2 and 5 or equiv	**B1**	1	
	(iv)	Show sketch with attempt at reflection in *x*-axis of the part of the curve for which $y < 0$	**M1**		[needs result to exist for $-a \le x \le a$]
		Show (more or less) correct sketch	**A1**	2	[with correct curvature]

7	**(i)**	State or imply that $a = 4$	**B1**		
		State or imply that $b = -2$	**B1**	2	
	(ii)	State or imply that volume is $\int \pi(ay+b)^{10}\ \mathrm{d}y$	**B1**√		[… following their *a*, *b* from **i**]
		Integrate $k(ay+b)^n$ to obtain $\dfrac{k_1(ay+b)^{n+1}}{(n+1)}$	**M1**		[… any *k* and k_1, any positive integer *n*]
		Integrate $k(4y-2)^{10}$ to obtain $\frac{1}{44}k(4y-2)^{11}$	**A1**√		[… any *k*, following their *a*, *b*]
		Obtain $\frac{512}{11}\pi$ or exact equiv	**A1**	4	

8	**(i)**	Refer to angle *ODA* being 60° or $\frac{1}{3}\pi$ and deduce $\frac{2}{3}\pi$	**B1**	1	[**AG**]
	(ii)	Use formula $r\theta$ or equiv at least once	**M1**		[needs angle in radians]
		Obtain either $12\times\frac{1}{3}\pi$ for *BC* or $6\times\frac{2}{3}\pi$ for *AC*	**A1**		
		Obtain $4\pi + 4\pi + 6$ and hence $8\pi + 6$	**A1**	3	[**AG**; necessary detail required]
	(iii)	Use formula $\frac{1}{2}r^2\theta$ or equiv at least once	**M1**		[needs angle in radians]
		Obtain $\frac{1}{2}\times 12^2\times\frac{\pi}{3}$ for *OBC* and $\frac{1}{2}\times 6^2\times\frac{2\pi}{3}$ for *ADC*	**A1**		
		Attempt correct process to find shaded area with valid attempt at area of triangle	**M1**		
		Obtain $24\pi - 12\pi - 9\sqrt{3}$ and hence $12\pi - 9\sqrt{3}$	**A1**	4	[or exact equiv]

9	**(i)**	Obtain either 37 or 1.728	**B1**		
		Confirm 38.728	**B1**	**2**	[**AG**; necessary detail required]
	(ii)	Recognise arithmetic progression with $a = 7$, $d = 15$	**B1**		
		Substitute into attempt at sum formula	**M1**		[formula quadratic in n, linear in a, d]
		Obtain $\frac{1}{2}\times 70(14 + 69\times 15)$ or equiv and hence 36715	**A1**	**3**	[**AG**]
	(iii)	Attempt use of logarithms to solve	**M1**		[… inequality or equation]
		Obtain $p\ln 1.2 < \ln 36715$ or equiv	**A1**		[… or equation]
		Obtain 57.65… and conclude 57	**A1**	**3**	[one value only clearly identified] [scheme for trial & improvement approach: M1 A2]
	(iv)	Relate attempt at S_q to attempt at v_{70}	***M1**		[attempt at term, not sum, of GP]
		Obtain $\frac{15}{2}q^2 - \frac{1}{2}q - 1.2^{70} < 0$ or equiv	**A1**		[… or equation]
		Attempt solution of quadratic equation for q	**M1**		[dep ***M**; or equiv such as T&I]
		Obtain 215.71… and conclude 215	**A1**	**4**	[one value only clearly identified]

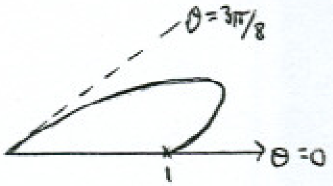
2633 Pure Mathematics 3

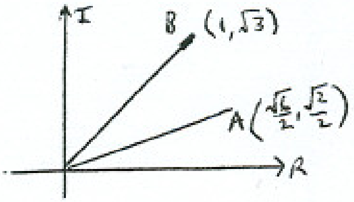
1		$1+(-2)(2x)+\frac{(-2)(-3)}{2!}(2x)^2+\frac{(-2)(-3)(-4)}{3!}(2x)^3$ $= 1 - 4x + 12x^2 - 32x^3$	M1 B1 A1 A1	4 **4**	For binomial coefficient in x^2/x^3 term For first two terms (simplified to) $1-4x$ For term $+12x^2$ correctly obtained Fir term $-32x^3$ correctly obtained
2		$\frac{dx}{d\theta} = \cos\theta - \theta\sin\theta$ $\frac{dy}{d\theta} = \cos\theta$ Hence $\frac{dy}{dx} = \frac{\cos\theta}{\cos\theta - \theta\sin\theta} = 1$ when $\theta = \pi$	M1 A1 B1 M1 A1 ft	5 **5**	For relevant use of the product rule For correct differentiation of x For correct differentiation of y For use of $\frac{dy}{dx} = \frac{dy}{d\theta} / \frac{dx}{d\theta}$ For correct substitution in their $\frac{dy}{dx}$
3	(i)	$R\cos\alpha = 3$, $R\sin\alpha = 1$ Hence $R = \sqrt{10}$ and $\tan\alpha = \frac{1}{3}$	M1 A1 A1	3	For stating these results, or by implication from correct/plausible values of R, α, or $\tan\alpha$ For correct (exact) value for R For correct (exact) value for $\tan\alpha$
	(ii)	$\theta - \tan^{-1}\left(\frac{1}{3}\right) = \cos^{-1}\left(\frac{2}{\sqrt{10}}\right)$ $\theta - 0.3217\ldots = 0.8860\ldots$ or $5.397\ldots$ Hence $\theta = 1.21$ or 5.72	M1 M1 A1 ft A1 ft	4 **7**	For correct process for one value of θ For quadrant RHS value For one correct answer For 2nd correct answer and no others in range [Allow values in degrees: 69.2 and 327.7]
4	(i)	$x - \frac{x}{x^2+1} = \frac{x^3+x=x}{x^2+1} = \frac{x^3}{x^2+1}$	B1	1	For correctly showing the given result
	(ii)	$\int\left(x - \frac{x}{x^2+1}\right)dx = \frac{1}{2}x^2 - \frac{1}{2}\ln\left(x^2+1\right)$	B1 M1 A1	3	For $\frac{1}{2}x^2 + \ldots$ For recognition of $f'(x)/f(x)$ type of integral For correct terms $\frac{1}{2}x^2 - \frac{1}{2}\ln\left(x^2+1\right)$
	(iii)	Integral is $\frac{1}{2}x^2 \ln\left(x^2+1\right) - \int \frac{1}{2}x^2 . \frac{2x}{x^2+1}\,dx$ i.e. $\frac{1}{2}x^2 \ln\left(x^2+1\right) - \frac{1}{2}x^2 + \frac{1}{2}\ln\left(x^2+1\right)$	B1 B1 B1 ft	3 **7**	For correct first term $\frac{1}{2}x^2 \ln\left(x^2+1\right)$ For correct simplified integrand $x^3/(x^2+1)$ For correct answer

5	(i)	L is $\mathbf{r} = \begin{pmatrix}3\\1\\2\end{pmatrix} + t\begin{pmatrix}3\\2\\-2\end{pmatrix}$ or equivalent	M1		For subtracting given vectors to find direction
			A1	2	For a correct (vector) equation
	(ii)	The point S corresponds to $t = 2$	B1		For correctly showing the given result
		Q corresponds to $t = -1$, so ratio is 2:3	M1		For relevant use of parameters
			A1	3	For correct ratio in any (exact) form [Alternatively by direct calculation of the two magnitudes which are $\sqrt{68}$ and $\sqrt{153}$]
	(iii)	Scalar product of the direction vectors is $1 \times 3 + 4 \times 2 + 2 \times (-2) = 7$	M1		For calculation of relevant scalar product
		$\cos\theta = \frac{7}{\sqrt{21}\times\sqrt{17}} = 0.3704\ldots$	M1		For use of correct formula with any vectors
		Hence angle is 68° to the nearest degree	A1	3 **8**	For correct answer (allow 1.19 radians)
6	(i)	Separating gives $\frac{1}{y^2}\frac{dy}{dx} = \frac{1}{x^2}$	M1		For dividing both sides by y^2
		Hence $-\frac{1}{y} = -\frac{1}{x} + c$	A1		For correct integration on both sides
			B1		For inclusion of (one) constant integration
		Required solution is $y = \frac{x}{1-cx}$	M1		For valid algebraic process to make y explicit
			A1	5	For correct answer (a.c.f.)
	(ii)	$(2,1) \Rightarrow y = \frac{x}{1+\frac{1}{2}x}$, so $y = \frac{8}{5}$ when x=8	M1		For using $x = 2$, $y = 1$ to evaluate c
			A1		For correct c
			A1 ft	3 **8**	For correct value of y
7	(i)	$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$ or $2y\frac{dy}{dx} = 1$ or equivalent	B1		For correct differentiation, in any form
		So integral is $\int \frac{1}{y^2(1+y)}.2y \; dy = \int \frac{2}{y(1+y)} \; dy$	M1		For substitution for x throughout
			A1	3	For showing the given result correctly
	(ii)	$\frac{2}{y(1+y)} = \frac{2}{y} - \frac{2}{1+y}$	M1		For correct form $\frac{A}{y} + \frac{B}{1+y}$
		Hence integral = $[2\ln y - 2\ln(1+y)]_2^3$	A1		For correct partial fractions
			A1 ft		For correct integration of their pfs
			B1		For correct limits for y
		$= 2\ln 3 - 2\ln 2 - 2\ln 4 + 2\ln 3 = 2\ln\frac{9}{8}$	M1		For use of limits and simplification
			A1	6 **9**	For correct answer $2\ln\frac{9}{8}$ or $\ln\frac{81}{64}$
8	(i)	$(x-8)^2 + (y-1)^2 = 4^2$	M1		For correct LHS or $x^2 + y^2 - 16x - 2y$
			A1	2	For correct equation, in any form
	(ii)	(a) Intersect where $(x-8)^2 + (mx-1)^2 = 16$	M1		For substitution
		i.e. $x^2(1 + m^2) - 2x(m + 8) + 49 = 0$	A1	2	For correctly obtaining the given equation
		(b) Real roots if $(m + 8)^2 \geq 49(1 + m^2)$	M1		For considering discriminant
		i.e. $48m^2 - 16m - 15 \leq 0$	A1		For correct quadratic inequality in m
		Hence $(12m + 5)(4m - 3) \leq 0$	A1		For solutions for m
		i.e. $-\frac{5}{12} \leq m \leq \frac{3}{4}$	A1	4	For correct set of values for m
	(iii)	EITHER $AOB = \alpha + \beta$ where $\tan\alpha = \frac{3}{4}$ and $\tan\beta = \frac{5}{12}$	M1		For identifying gradient with tan of angle
			A1		For handling negative sign correctly
			M1		For relevant use of addition formula

Hence tanAOB $= \dfrac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}} = \frac{56}{33}$	A1	4	For showing given answer correctly
OR $AOB = 2\theta$ where $\sin\theta = \frac{4}{\sqrt{65}}$	M1		For using relevant circle geometry and trig
Hence $\tan\theta = \frac{4}{7}$	A1		For correct conversion to tan
Hence $\tan AOB = \dfrac{2\times\frac{4}{7}}{1-\frac{16}{49}} = \frac{56}{33}$	M1		For relevant use of double-angle formula
	A1	4 **12**	For showing given answer correctly

MARK SCHEME FOR P4, JANUARY 2005

1. Attempt at I.F. from $\int -1\,dx$	M1	Allow $^{+}/.$
Write LHS as d/dx(y .their I.F.)	A1√	May be implied
Integrate their RHS as e^{3x} .their I.F.	A1√	$+c$ not needed
Obtain $y = {}^{1}/_{2}e^{3x} + ce^{x}$	A1	cao as $y =$; AEF
2. $e^{-x} \approx 1 - x + {}^{1}/_{2}x^{2}$	M1	Attempt at Maclaurin as 3 term quadratic
$\sin 2x \approx 2x - (2x)^{3}/3!$	M1	Attempt at Maclaurin in x and x^{3}
	A1	At least one correct
Multiply out	M1√	
Obtain $2x - 2x^{2} - x^{3}/3$	A1	cao - simplified
3. n=1; LHS = (1x4) = 4; RHS = ${}^{1}/_{3}$ x1x2x6	B1	Clear evidence of 4 on both sides
Assume $\Sigma r(r+3) = {}^{1}/_{3}k(k+1)(k+5)$	M1	May be implied
Add $(k+1)(k+4)$ to both sides	M1	
Obtain ${}^{1}/_{3}(k+1)(k+2)(k+6)$	A1	**Clear** algebraic evidence; allow multiplying out
Clearly bring together the two parts	A1	
4(i) $\frac{dy}{dx} = \frac{-2}{\sqrt{(1-4x^{2})}}$	M1	For $a / \sqrt{(1 - kx^{2})}$ $a \neq 0$, k $\neq 0$; by
	A1	cao Chain Rule or direct
	(S.R.	M1 For a / (sin($\cos^{-1}kx$) as above A1 For $a = -2$, $k = 2$)
(ii) Obtain $dx = {}^{1}/_{2}\cos\theta\, d\theta$	M1	
Substitute to $\int k\, d\theta$	M1	
Obtain ${}^{1}/_{2}\sin^{-1} 2x + c$	A1	c must be used
(iii) Equate $-{}^{1}/_{2}\cos^{-1} 2x\ (+ c_{1})$ to ${}^{1}/_{2}\sin^{-1} 2x + c$	M1	c_{1} need not be seen; allow their expressions
Obtain A.G.	A1	
Choose any value of x e.g. $x = 0$ for $a = {}^{1}/_{2}\pi$	B1	cao
	(S.R.	M1 d/dx($\cos^{-1}2x+\sin^{-1}2x$) = 0 A1 Obtain A.G.)
5.(i) Show $\sin {}^{3\pi}/_{4} + \cos {}^{3\pi}/_{4} = {}^{\sqrt{2}}/_{2} - {}^{\sqrt{2}}/_{2} = 0$	B1	A.G. – numeric evidence seen Or solve $r = 0$
	B1	Correct shape
	B1	Evidence of scale
(ii) Formula with correct r^{2}	B1	
Multiply r out to $\sin^{2} 2\theta + \cos^{2} 2\theta + 2\sin 2\theta\cos 2\theta$	M1	Does not need ½
Simplify to $1 + \sin 4\theta$	M1	Allow sub. e.g. $y = \sin 2\theta$; allow $a + b\sin 4\theta$, $b \neq 0$; allow recognition or other reasonable method
Integrate with correct limits	A1√	
Obtain ${}^{1}/_{16}(3\pi + 2)$	A1	cao AEEF

6.(i) Obtain $\frac{2}{r+1} - \frac{2}{r+3}$	M1 Attempt at P.F.
	A1 cao
(ii) Expand the first few terms	M1
$(^2/_2 - ^2/_4) + (^2/_3 - ^2/_5) + (^2/_4 - ^2/_6) + \dots$	
$+ (^2/_{(n-1)} - ^2/_{(n+1)}) + (^2/_n - ^2/_{(n+2)}) + (^2/_{(n+1)} - ^2/_{(n+3)})$	M1 Sufficient for difference method
Cancel	M1
Obtain $^5/_3 - ^2/_{(n+2)} - ^2/_{(n+3)}$	A1
(iii) State idea of limit e.g. $\text{Lt}_{N\to\infty} \frac{1}{N} = 0$	B1
State sum = 5/3	B1√ √ from (ii)
7.(i) Square a + ib	B1 (Allow De Moivre
Equate real and imaginary parts	M1 (Equate r and θ
Eliminate a or b to quadratic	M1 (Obtain r and (one) θ
Solve for correct a and b	M1 (Attempt correct form
Obtain $z = \pm((\sqrt{6} + i\sqrt{2})/2)$	A1 cao
(ii) 	B1 Two "correct" points in first quadrant
	B1 Correct "coordinates" seen (some idea of correct scale)
(iii) Rotate through $^\pi/_6$ (about O)	B1,B1 Allow degrees
Enlarge factor √2 (about O)	B1,B1
8.(i) Obtain $x = -2a$, $x = -a$, $y = 1$	B1,B1,B1 May be quoted
(ii) Write as $x^2(y-1) + 3axy + 2a^2y = 0$	M1 Attempt quadratic
Use $b^2 - 4ac \geq 0$ for real x	M1
Obtain quad. inequality $y(y+8) \geq 0$	M1
Solve the inequality	M1
Explain and obtain correct y	A1
	(S.R. M1 Diff'ate M1 Attempt to solve grad. = 0 A1 Correct t.p. M1 Attempt to justify inequality e.g.graph A1 Fully justified)
(iii) Select correct asymptote $y = 1$	B1
Solve asymptote with curve	M1
Obtain $(-2a/3, 1)$	A1

1	(i) Either $(1 + y)^4 = y^4 + 4y^3 + 6y^2 + 4y + 1$ or $(1 + y)^3 = y^3 + 3y^2 + 3y + 1$		*B1		
	$y^4 - 5y^2 + 6$ (= 0) WWW	**AG**	dep*B1		
	(ii) Attempt to solve for y^2 & then y	s.o.i	M1		
	$1 + \sqrt{2}$, $1 + \sqrt{3}$	both	A1		
	$1 - \sqrt{2}$, $1 - \sqrt{3}$	both	A1	5	**5**
2	Attempt to solve $\ln x = \frac{3}{x}$ AEF		M1		
	State or use formula $x - \frac{\ln x - \frac{3}{x}}{\frac{1}{x} + \frac{3}{x^2}}$ or $x - \frac{x \ln x - 3}{\ln x + 1}$	(denom reasonable attempt)	M1		
	1st iter correct		A1		
	Final answer (not dep on prev A1) 2.86	cao	*A1		
	2 d.p. justification	(either 2.855 & 2.865 signs or suff iterations)	dep*A1	5	**5**
3	$1 + 4x^2 = \cosh^2 u$	s.o.i.	B1		
	$dx = \frac{1}{2} \cosh u$ (du)	used	B1		
	$\frac{1}{2} \int \cosh^2 u$ (du)		B1		
	For $\int \cosh^2 u$ (du), use of $\cosh 2u = +/- 2 \cosh^2 u +/- 1$ or parts once + use of $\cosh^2 u/\sinh^2 u$ connection		M1		
	$k [u + \frac{1}{2} \sinh 2u]$ or $k[u + \sinh u \cosh u]$		A1		
	Final answer = $\frac{1}{4} \left[\sinh^{-1} 2x + 2x \sqrt{(1 + 4x^2)}\right] + c$ [Terms such as $\sinh(2 \sinh^{-1} 2x)$ not acceptable in final answer]		A1	6	**6**
4	(i) Tangent (to curve) (at A); AC and curve have same gradient		B1	1	
	(ii)(a) $y_0 + hf(x_0, y_0)$, not $y_0 + h\frac{dy}{dx}$; clear indic that $\frac{dy}{dx}$ is eval at (x_0, y_0)		B1	1	
	(b) $f(x_0 + h$, √ ans to (ii)(a))		B1	1	
	(iii) $y_{1.1} = 1 + 0.1$ x $\sqrt{(1^3 + 1^3)}$	(= 1.1414214)	M1		
	$y_{1.2}$ = their $y_{1.1}$ + $0.1\sqrt{(1.1^3 + \text{their } y_{1.1}^3)}$		M1		
	= 1.309 or better (1.3092935)		A1	3	**6**

5	(i) $\Sigma \alpha^2 = (\Sigma \alpha)^2 - 2\,\Sigma \alpha\beta$	AEF	M1		
	$\alpha\beta + \beta\gamma + \gamma\alpha = -5$		A1	2	
	(ii) $x^3 - 3x^2 + (\text{their } \Sigma\alpha\beta)x - 1 = 0$		B2		
	[S.R. Totally f.t. correct except for 1 incorrect sign → B1]			2	
	(iii) Using $x^3 - 3x^2 - 5x - 1 = 0$ & spotting $x = -1$ is a root or $(x + 1)$ is a factor		B1		
	Using $x + 1$ with factorisation or long division		M1		
	$x^2 - 4x - 1$		A1		
	$-1,\ 2 \pm \sqrt{5}$		A1	4	**8**
6	(i) Attempt to use parts with $u = x^n$, $dv = (1 - x)^{\frac{1}{2}}$		M1		
	$\left[-\frac{2}{3}x^n(1-x)^{\frac{3}{2}}\right] + \frac{2}{3}n\int x^{n-1}(1-x)^{\frac{3}{2}}(dx)$	Ignore limits here but note below	*A1		
	$= \frac{2}{3}n\int_0^1 x^{n-1}(1-x)^{\frac{3}{2}}(dx)$ **AG**	Limits present in sq. brackets	dep *A1	3	
	(ii) Multiplying out $x^{n-1}(1-x)$ & producing 2 separate integrals		M1		
	$I_n = \frac{2}{3}\,n\,I_{n-1} - \frac{2}{3}\,n\,I_n$		A1		
	Combining I_n terms and producing **AG**	WWW	A1	3	
	(iii) $I_0 = \frac{2}{3}$ or $I_1 = \frac{4}{15}$		B1		
	Attempt to use reduction formula at least once $(7I_2 = 4I_1, 5I_1 = 2I_0)$		M1		
	$\frac{16}{105}$; √ $\frac{4}{7}I_1$ or $\frac{8}{35}I_0$	(Ignore any decimals)	A1	3	**9**

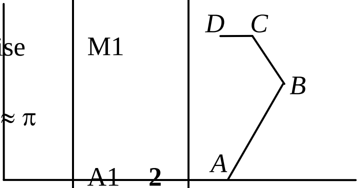
[If (iii) worked with a different method, then

M2 for complete method (M1 for partial, e.g. subst without changing limits)

A1 for answer]

7	(i) Sketch (*y*-axis symmetry, *x*-axis asymptote, curvature)		B1	1	
	(ii) Attempt to change d*x* into f(*u*) d*u*		*M1		
	$dx = \frac{1}{u}\ du$		A1		
	Integrand $\frac{2}{u^2+1}$		A1		
	$2\tan^{-1}u$		A1		
	Resubstitution or attempt to change limits		dep* M1		
	$2\tan^{-1}e - \frac{1}{2}\pi$	**AG**	A1	6	
	(iii) $k\int \text{sech}^2 x\ (dx)$ (where k is a constant, including 1)		M1		
	k tanh *x*		A1		
	$\pi\left(\frac{e^2-1}{e^2+1}\right)$ with at least 1 intermediate stage	**AG**	A1	3	**10**
8	(i) State/imply AE is $m^2 + 6m + 10$ (=0)		M1		
	$m = -3 \pm i$		A1		
	$e^{-3x}(A\cos x + B\sin x)$ or $Ae^{-3x}\sin(x+\alpha)$ or $Ae^{-3x}\cos(x-\beta)$ √complex m		√A1		
	State/imply PI is 1		B1		
	(GS is) (*y* =) their CF + their PI		√A1	5	
	(ii) Provided prev mark was not A0, substitute (0,0) into their equation		M1		
	Differentiate their equation, using suitable techniques (as necessary)		*M1		
	Substitute $x = 0$		*M1		
	Equate y_0' to 1		dep*M1		
	Produce $y = -e^{-3x}(\cos x + 2\sin x) + 1$ or exact equivalent		A2	6	**11**
	[S.R. Allow A1 for one minor arith/sign error]				

1 **(i)** Rotation (about the origin) anticlockwise, through $\frac{2}{3}\pi$ *OR* 120°	M1 A1 A1 **3**	Anticlockwise may be shown by an arrow, or as +ve **SR** 2 different transformations stated earn at most M1 (for any rotation) A0 A0
(ii) $(n=)\ 3$	B1 √ **1**	answer must f.t. from angle of rotation
		4
2 **(i)** Position vector of a point on plane *P*	B1 **1**	
(ii) Vector normal to the plane *P*	B1 **1**	
(iii) A (straight) line, through point with position vector **a**, in a direction normal to plane	B1 B1 B1 **3**	may be implied by points parallel to normal
		5
3 **(i)** Multiply by a^{-1} on left, or b^{-1} on right Obtain $(x=)\ a^{-1}cb^{-1}$	M1 A1 **2**	
(ii) *EITHER*: Expand RHS to give $r^2s^2 = rsrs$ Use multiplication by r^{-1} and s^{-1} to obtain $rs = sr$	M1 A1	**SR** done in reverse without ⇔ earns M1 A0 **AG**
OR: Multiply both sides by $(rs)^{-1}$ and use $(rs)^{-1} = s^{-1}r^{-1}$	M1	left or right multiplication
Use multiplication by r^{-1} or s^{-1} to obtain $rs = sr$	A1 **2**	**AG**
		4
4 **(i)** State $z^n = \cos n\theta + \mathrm{i}\sin n\theta$	B1	
Subtract $z^{-n} = \cos n\theta - \mathrm{i}\sin n\theta$ to obtain $2\mathrm{i}\sin n\theta$	B1 **2**	**AG**
(ii) $32\mathrm{i}\sin^5\theta$	B1	may be seen anywhere. Allow $2^5\mathrm{i}^5\sin^5\theta$
$\left(z - z^{-1}\right)^5 =$	M1	Attempt at binomial expansion
$z^5 - 5z^3 + 10z - 10z^{-1} + 5z^{-3} - z^5$	A1	
$= 2\mathrm{i}\sin 5\theta - 10\mathrm{i}\sin 3\theta + 20\mathrm{i}\sin\theta$	M1	Use result of part **(i)**
$\Rightarrow \left(\sin^5\theta =\right) \frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$	A1	**AEF**
LONGER ALTERNATIVE METHOD:		
$\mathrm{Im}\,(c+\mathrm{i}s)^5 \Rightarrow \sin 5\theta = 5c^4s - 10c^2s^3 + s^5$	M1	Expand at least imaginary terms of $(c+\mathrm{i}s)^5$
$\mathrm{Im}\,(c+\mathrm{i}s)^3 \Rightarrow \sin 3\theta = 3c^2s - s^3$	A1	Obtain identity for $\sin 5\theta$ *OR* $\sin 3\theta$
Put $c^2 = 1 - s^2$ in identities for both $\sin 5\theta$ and $\sin 3\theta$ and eliminate s^3 between them	M1	
$\Rightarrow \left(\sin^5\theta =\right) \frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$	A2 **5**	**AEF** **SR** A1 for 2 terms correct
		7

5 (i) Put $\Delta = \begin{vmatrix}7 & k & 1\\ k & 3 & 1\\ 1 & 7 & 3\end{vmatrix} = 0$	M1	
$\Delta = 7\times 2 - k(3k-1) + (7k-3)$	M1	Use correct expansion method
$= -3k^2 + 8k + 11$		
$k = -1,\ \frac{11}{3}$	A1 A1 **4**	**SR** If M0 M0, allow B1 for $k = -1$ stated
(ii) e.g. $8x - 4y = 12,\quad -4x + 2y = -6$ *OR* $10y + 4z = -6,\quad 20y + 8z = -12$ *OR* $20x + 4z = 24,\quad 50x + 10z = 60$	M1	Eliminate one variable between two equations
Put $x = t$, *OR* $y = u$, *OR* $z = v$	M1	Solve in terms of 1 parameter
EITHER $(x, y, z) = (t, 2t-3, -5t+6)$	A1	for 2 of x, y, z correct
OR $(x, y, z) = \left(\frac{1}{2}(u+3), u, -\frac{1}{2}(5u+3)\right)$	A1	for the 3rd correct
OR $(x, y, z) = \left(\frac{1}{5}(-v+6), -\frac{1}{5}(2v+3), v\right)$		**AEF** Allow $\frac{x}{1} = \frac{y+3}{2} = \frac{z-6}{-5} = t$ etc.
ALTERNATIVE METHOD:		
Find vector product of two of $[7, -1, 1],\ [-1, 3, 1]$ and $[1, 7, 3]$	M1	For finding the direction of the line of intersection of the 3 planes
Obtain $\pm[1, 2, -5]$	A1	or any multiple
Find a point on the common line	M1	e.g. by putting x, y or $z = 0$
Obtain $(x, y, z) = (t, 2t-3, -5t+6)$	A1 **4**	**AEF** or other forms as above
		8
6 (i) $(z^2 =)\ \frac{1}{4}\left(\cos\frac{2}{3}\pi + \mathrm{i}\sin\frac{2}{3}\pi\right)$ *OR* $\frac{1}{4}\mathrm{e}^{\frac{2}{3}\pi\mathrm{i}}$	B1	
$(z^3 =)\ \frac{1}{8}(\cos\pi + \mathrm{i}\sin\pi)$ *OR* $\frac{1}{8}\mathrm{e}^{\mathrm{i}\pi}$	B1 **2**	allow $-\frac{1}{8}$
(ii) 4 points shown, starting from the +ve real axis and going anticlockwise	M1	
arg AB $\approx \frac{1}{3}\pi$, arg BC $\approx \frac{2}{3}\pi$, arg CD $\approx \pi$		
and $\lvert AB\rvert > \lvert BC\rvert > \lvert CD\rvert$	A1 **2**	
(iii) Use G.P. sum $\frac{1-z^m}{1-z}$	M1*	for m = any function of n
$z^{6n} = k(\cos 2n\pi + \mathrm{i}\sin 2n\pi)$	B1	seen or implied, any $k \neq 0$
$z = \frac{1}{2}\left(\frac{1}{2} + \mathrm{i}\frac{\sqrt{3}}{2}\right)$	B1	**AEFcartesian** seen or implied
Attempt to make denominator real	M1 (dep*)	
Obtain $\frac{1}{3}\left(1 - \frac{1}{2^{6n}}\right)\left(3 + \mathrm{i}\sqrt{3}\right)$	A1 **5**	**AG**
(iv) State $\frac{1}{3}\left(3 + \mathrm{i}\sqrt{3}\right)$	B1 **1**	**AEF**
		10

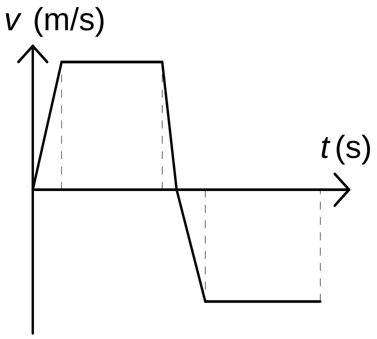
7 **(i)** $\mathbf{r}=\left[\frac{3}{2}, 3, 2\right]+$ $+\lambda[2, 1, 2]+\mu[-2, 1, 0]$	B1 B1 **2**	
(ii) Use $[2,1,2]\times[-2,1,0]$ $\Rightarrow (\mathbf{n}=)\ \pm[-2,-4,4]$	M1 A1	Use vector product of **b** and **c** or any multiple
Then *EITHER*: Use $\left(\frac{3}{2},3,2\right)$ to obtain $2x+4y-4z=7$	M1 A1	**AEF**
OR: Write 3 equations in x, y, z from **(i)** and eliminate λ and μ Obtain $2x+4y-4z=7$	M1 A1 **4**	**AEF**
(iii) Find $\overrightarrow{PQ}$ and attempt to normalise Obtain $(\hat{\mathbf{n}}_1=)\ \frac{1}{3}[\ -1,\ -2,\ 2]$	M1 A1 **2**	**AEF** but not $-\hat{\mathbf{n}}_1$ or multiples of $\hat{\mathbf{n}}_1$
(iv) Attempt to find $\lvert\overrightarrow{PQ}\rvert$ Obtain $\left(\sqrt{1^2+2^2+2^2}=\right)3$ $\overrightarrow{PQ}$ is the common perpendicular to l_1 and l_2	M1 A1 B1 **3**	Allow use of $\lvert\hat{\mathbf{n}}\,.\,\overrightarrow{PQ}\rvert$ to obtain $\left(\text{e.g. } \frac{1}{6}[-2,-4,4]\,.\,[-1,-2,2]=\right)3$
		11
8 **(i)** $\mathrm{ff}(x)=\dfrac{1}{1-\frac{1}{1-x}}=\dfrac{1-x}{-x}=1-\dfrac{1}{x}$	M1 A1 **2**	Putting f into itself **AG** An intermediate step is required
(ii) $\mathrm{f}^3(x)=\dfrac{1}{1-\left(1-\frac{1}{x}\right)}$ *OR* $1-\dfrac{1}{\frac{1}{1-x}}$ $=x\ \Rightarrow$ f has order 3	M1 A1 A1 **3**	Putting ff into f *OR* f into ff **AG** Simplification to x is required
(iii) {e, h}	M1 A1 **2**	M1 for 2 elements including e Allow fh, ffh, hf, hff in place of h
(iv) $\mathrm{fh}(x)=\dfrac{1}{1-\frac{1}{x}}$ *OR* $\mathrm{h(ff)}(x)=\dfrac{1}{1-\frac{1}{x}}$	M1	Attempt to find fh or h(ff)
$\mathrm{hf}(x)=\dfrac{1}{\frac{1}{1-x}}$ *OR* $\mathrm{(ff)h}(x)=1-\dfrac{1}{\frac{1}{x}}$	M1	Attempt to find hf or (ff)h
$\mathrm{fh}(x)$ *OR* $\mathrm{h(ff)}(x)=\dfrac{x}{x-1}$	A1	
$\mathrm{hf}(x)$ *OR* $\mathrm{(ff)h}(x)=1-x$	A1 **4**	**SR** Award A1 A0 if both elements are correct but the working for their derivation contains an error
		11

1	**(i)**	Momentum before collision = $0.3\times 1.5 - 0.2\times 2$ Momentum after collision = $0.2\times b - 0.3\times a$ $0.45 - 0.4 = 0.2b - 0.3a$ $b = 1.5a + 0.25$ **A.G.**	 B1 B1 M1 A1 4	Alternatively: Momentum lost by *A* $= 0.3\times 1.5 + 0.3\times a$ B1 Momentum gained by *B* $= 0.2\times b + 0.2\times 2$ B1 For using the principle of conservation of momentum
	(ii)	$a = 2/4 = 0.5$ $b = 1.5\times 0.5 + 0.25 = 1$	B1 B1ft 2	

2	**(i)**	$F = 14.7$ and $R = 3g$ $14.7 = 3\times 9.8\,\mu$ Coefficient is 0.5	B1 M1 A1 3	For using $F = \mu R$
	(ii)	$F = P\cos 30^{\circ}$ $R = 3g + P\sin 30^{\circ}$ $0.866P = 0.5(29.4 + 0.5P) \rightarrow$ $0.616P = 14.7$ $P = 23.9$	B1 B1 M1 A1 4	For using $F = \mu R$ and attempting to solve for *P*

3	**(i)**	$10\cos x = 5$ $x = 60$ $P = 10\sin x$ or $P^2 = 10^2 - 5^2$ $P = 8.66$ or $5\sqrt{3}$	M1 A1 M1 A1 4	For resolving in **i** direction or using trigonometry to find *x* in triangle of forces For resolving in **j** direction or using trigonometry or Pythagoras to find *P* in triangle of forces SR scale drawing (max 3 out of 4) Correct triangle of forces drawn to scale M1, then by measurement, magnitude of *P* = 8.6 or 8.7 (2sf) A1 *x* = 60 (2sf) A1
	(ii)	$H = 10\cos 45^{\circ} - 5$ $V = 12 - 10\sin 45^{\circ}$ $R^2 = 2.071^2 + 4.929^2$ Magnitude is 5.35 N	B1 B1 M1 A1 4	 For using $R^2 = H^2 + V^2$ Alternatively for the above 4 marks: If combining two forces initially then combining this resultant the third force M1 for a complete method, A1 for the magnitude of the two forces, A1 for angle from those forces, A1 for 5.35. SR scale drawing (max 2 out of 4) Correct polygon of forces drawn to scale M1, then by measurement, magnitude is 5.3 or 5.4 (2sf) A1

4	**(i)**		M1		For using $v = \int a\,dt$
		$v = 2t^{1.5}$ (+ C)	A1		
		$2(9)^{1.5} + C = 60 \Rightarrow C = 6$	M1		For using $v(9) = 60$
		Initial velocity is 6 ms^{-1}	A1	4	
	(ii)		M1*		For using $s = \int v\,dt$
		$s = 0.8t^{2.5} + 6t$	A1ft		ft incorrect non zero v_0
		$OP = (0.8(9)^{2.5} + 6\times 9) - (0 + 0)$ (= 194.4 + 54)	M1 dep*		For correct use of limits or equivalent
		Distance OP is 248(.4) m	A1	4	

5	**(i)**		M1		For an attempt at sketching the graph for the outward stage; v must be continuous, ≥ 0 and single valued throughout, and the graph must consist of 3 straight line segments
			A1		1st line segment must start at the origin and have +ve slope, 2nd line segment must have zero slope, 3rd line segment must have –ve slope and terminate on t axis. Values of v and t need not be shown.
			B1	3	Correct sketch of the graph for the return stage; values of v and t need not be shown.
	(ii)	OA = ½ 20×9 + 82×9 + ½ 8×9 = 90 + 738 + 36	M1		For using the idea that the distance is represented by the area of the relevant region
		Distance OA is 864 m	A1	2	
	(iii)	$\Delta t = 16$	B1		For time of acceleration stage on return journey
		Distance at constant speed = 864 – ½ 16×8	M1		For correct method of finding a distance at constant speed
		110 + 16 + 800/8	M1		For correct method for finding total time
		Total time is 226 s	A1ft	4	ft for 118 + ans(ii)/8

6	**(i)**	$h = 15\times 0.8 - \tfrac{1}{2}\,9.8\times (0.8)^2$	M1		For using $s = ut - \tfrac{1}{2}\,gt^2$
		$= 12 - 3.136$			
		Height is 8.86(4) m	A1	2	
	(ii)	$H - \tfrac{1}{2}\,9.8\times (0.8)^2 = 8.864$ or	M1		For using $H - \tfrac{1}{2}\,gt^2$ = ans(i) or
		$H = 15\times 0.8$			$H = ut$ (from $H - \tfrac{1}{2}\,gt^2 = ut - \tfrac{1}{2}\,gt^2$)
		$H = 12$	A1	2	
	(iii)	$0 = 15t - 4.9t^2,\ t \neq 0$ or	M1		For solving $0 = ut - \tfrac{1}{2}\,gt^2,\ t \neq 0$ or
		$0 = 15 - 9.8(t/2)$			for solving $0 = u - g(t/2)$ or
					equivalent
		$t_A = 3.06$ or $15/9.8$	A1		
		$12 = \tfrac{1}{2}\,9.8t^2$	M1		For solving $H = \tfrac{1}{2}\,gt^2$
		$t_B = 1.56$ or $\sqrt{24/9.8}$	A1ft		
		$t_A - t_B = 3.061 - 1.565$			
		Time interval is approx 1.5 s **A.G.**	B1	5	www

7	**(i)**		M1		For applying Newton's second law
					to either particle
		$0.32g - T = 0.32a$	A1		
		$T - 0.1g\sin\alpha - F = 0.1a$	A1		
		$R = 0.1g\cos\alpha$	B1		
		$F = \tfrac{1}{4}\,(0.08\text{g})$	M1		For using $F = \mu R$
		$0.32g - 0.06g - 0.02g = 0.42\text{a}$	A1		For a correct equation in a only
		$0.42\text{a} = 0.24\times 9.8$			
		Acceleration is 5.6 ms^{-2}	A1	7	
	(ii)	$2.8^2 = 2\times 5.6s$	M1		For using $v^2 = 2as$
		Distance is 0.7 m	A1ft	2	ft 3.92/ans(i)
	(iii)	$0.1a = -0.06g - 0.02g$	M1		For applying Newton's second law
					to A (continuing upwards)
		$a\ = -\,0.8g$	A1ft		ft incorrect magnitude of frictional
					force and/or weight component.
					Signs must be correct.
		$0 = 2.8^2 + 2(-7.84)s_2$	M1		For using $0 = u^2 + 2as$ to find s_2
		$s_1 + s_2 = 0.7 + 0.5$			
		Distance is 1.2 m	A1ft	4	ft incorrect ans(ii)

Mark scheme for M2 2638 (January 2005) (final)

1	(i)	$v = 2\frac{1}{2}$	B1	1	+ only	
	(ii)	$I = 0.2\text{x}2\frac{1}{2} - (-0.2\text{x}5)$	M1		Impulse = change of momentum	
		I = 1.5 Ns	A1		+ only	
		←	B1	3	to the left	**4**

2	(i)	c.of m. of cone 0.6 or 0.2	B1		clearly from vertex (0.6) or base(0.2)	
		moments	M1		moments about vertex or about base	
		1.2d = 0.5x0.6 + 0.7x1.2	A1		1.2e = 0.7x 0.4 + 0.5x1.0 (e = 0.65)	
		d = 0.95	A1	4	**AG**	
	(ii)	tanθ = 0.3/0.65	M1		geometry must be correct	
		θ = 24.8°	A1	2		**6**

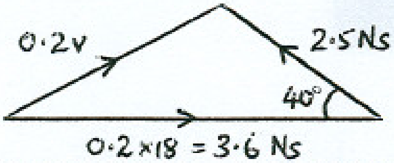
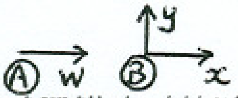
3	(i)	D = 500 N	B1			
		$500 = k \text{ x } 40^2$	M1			
		$k = 5/16$	A1	3	**AG** (or 0.3125)	
	(ii)	$20{,}000/30 - 5/16\text{x}30^2 = 350a$	M1		must have attempt at all 3 parts but	
		"	A1		not 500 for driving force or R	
		$a = 1.10 \text{ ms}^{-2}$	A1	3	or 1.1 or 185/168 or 1 + 17/168	**6**

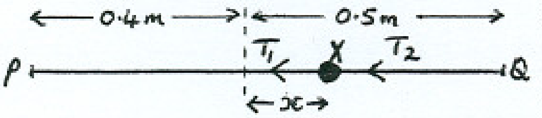
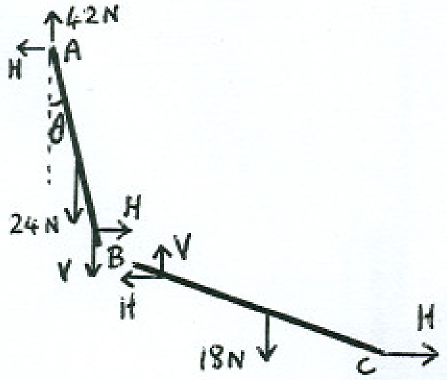
4	(i)	400cos30° x 25	M1			
		8660 Nm	A1	2	or 8.66 k…	
	(ii)	inc.in K.E. = 100	B1		$\frac{1}{2}.50.2^2$	
		inc in P.E. = 4190	B1		50x9.8x25sin20°	
		100+4190+25R = 8660	M1		must have 4 terms and 25R	
		"	A1			
		R = 175	A1	5		**7**
	or	a = 0.08	B1		no P.E. & K.E.	
		400cos30°-R-50gsin20°=50a	M1		3 marks available only if N II	
		R = 175	A1		used	

5	(i)(a)	Rcos15° = 500x9.8	M1		for resolving vertically	
		R = 5070 N	A1	2	components of R needed for M1s	
	(b)	$R\sin15° = 500v^2/60$	M1		N II and $a = v^2/r$ (not for R=500g)	
		"	A1		R can be x for this (value needed)	
		$v = 12.6 \text{ ms}^{-1}$	A1	3		
	(ii)(a)	Rcos15°=500.9.8+Fsin15°	M1		for resolving vertically (3 parts)	
		F = 3/7 R	B1		unknown F and R (M1 & B1)	
		R = 5730 N	A1	3		
	(b)	$R\sin15° + F\cos15° = 500.v^2/60$	M1		N II & $a = v^2/r$ (3 parts) (accept R=500g for M1 only)	
		"	A1		F & R can be wrong for 1st A1	
		$v = 21.5 \text{ ms}^{-1}$	A1	3		**11**

6	(i)	$0 = 14^2 \sin^2 60^\circ - 2.9.8h$	M1		or use of $v^2 \sin^2\theta / 2g$	
		h = 7.5 m	A1	2		
	(ii)	0 = 14sin60° - 9.8t	M1		use of v = u + at or similar	
		t = 1.24 s (t to max height)	B1		or $5/7\sqrt{3}$ or total time 2.47	
		d = 2x14cos60°x1.24	M1		must have 2	
		d = 17.3	A1	4	range=$u^2\sin 2\theta/g$ = 17.3 scores 4	
	(iii)	v = 4/7 x 7	M1			
		v = 4	A1		±	
		vert. comp v = 12.1	B1		or 14sin60° or $\sqrt{147}$	
		speed=$\sqrt{(4^2 + 12.1^2)}$	M1		may be implied	
		speed = 12.8 ms^{-1}	A1			
		$\theta = \tan^{-1}(12.1/4)$	M1		or equivalent (may be implied)	
		θ = 71.7° to horizontal	A1	7	or 18.3° to vertical	13

7	(i)	R = 196 N (not in terms of g)	B1		reaction force at ground	
		N.5sin60°=20.9.8.2.5cos60°	M1		moments about B or about T	
		“	A1		20.9.8.2½.½+F.5sin60°=R.2½	
		N = 56.6 (not in terms of g)	A1	4	reaction force at wall	
	(ii)	F = 56.6	B1√		√ their N (value required)	
		56.6 < 2/5 x 196 (78.4)	B1	2	or F/R = 0.289 < 0.4	
	(iii)	R' = 980 or 100g	B1			
		F' = 2/5R' = 392 or 40g	B1√		√ their R' (not 196)	
		N' = F'	B1		not 56.6 or 78.4	
		N'.5sin60° =	M1*		M(B) 3 parts (all moments)	
		20.9.8.2.5.½ +80.9.8.dcos60°	A1		any N if substituted	
		solve for d	M1*		depends on previous M1	
		d = 3.71 m	A1	7		13
	or	80gecos60°+20g.2.5cos60° +	M1*		M(T) last 5 marks above become	
		F'.5.sin60° = R'.5cos60°	A1		e = dist from T	
		e= 1.29	A1			
		d = 5 - e	M1*			
		d = 3.71	A1√			
	or	80gfcos60°+R'.2.5cos60° =	M1		M(G) last 4 marks become	
		N'.2.5sin60°+F'.2.5sin60°	A1		f = dist from G	
		f = 1.21	A1			
		d = 3.71	A1			

1	$\begin{pmatrix}2.5\cos140\\2.5\sin140\end{pmatrix}=0.2\begin{pmatrix}v_1\\v_2\end{pmatrix}-0.2\begin{pmatrix}18\\0\end{pmatrix}$ $v_1=8.424,\quad v_2=8.035$ Speed $=\sqrt{{v_1}^2+{v_2}^2}$ $=11.6\text{ ms}^{-1}$	M1 A1 M1 A1 **4**	Equation involving impulse and momentum (one component sufficient) Both component equations correct
	OR  $(0.2v)^2=3.6^2+2.5^2-2\times3.6\times2.5\cos40$ $v=11.6$	M1 A1 M1 A1	Impulse / momentum triangle Correct triangle
2 (i)	At highest point, $R+mg=m\dfrac{u^2}{a}$ $u^2=ag+\dfrac{Ra}{m}\geq ag$, so $u\geq\sqrt{ag}$	M1 A1 (ag) **2**	Or $mg=m\dfrac{u^2}{a}$ Or $u=\sqrt{ag}$
(ii)	$\frac{1}{2}mv^2-\frac{1}{2}mu^2=mg(2a)$ When $u^2=ag$, $v^2=5ag$ $R-mg=m\dfrac{v^2}{a}$ $R=6mg$	M1 A1 A1 M1 A1 (ag) **5**	Using conservation of energy from highest point to lowest point
3	Velocities after collision  $y=6\sin60\quad(=3\sqrt{3}\approx5.196)$ $mw+mx=m(17)-m(6\cos60)$ $(\;w+x=14\;)$ $x-w=0.6(17+6\cos60)$ $(\;x-w=12\;)$ $w=1,\quad x=13$ Speed of A is $w=1\text{ ms}^{-1}$ Speed of B is $\sqrt{x^2+y^2}=14\text{ ms}^{-1}$	B1 M1 A1 M1 A1 M1 A1 A1 **8**	Conservation of momentum Restitution equation Obtaining w or x

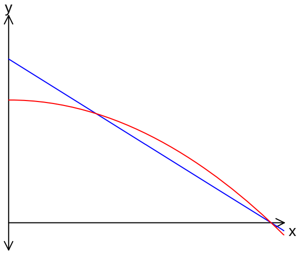
4 (i)			
	When displaced distance x from equilibrium, Tension in PX is $T_1=\frac{60}{0.4}x$ $(=150x)$	M1 A1	Using $\frac{\lambda x}{l}$
	Compression in XQ is $T_2=\frac{45}{0.5}x$ $(=90x)$	A1	
	$-T_1-T_2=m\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$	M1	
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=-\frac{240}{m}x$, hence motion is SHM	A1 5	
(ii)	Period is $2\pi\sqrt{\frac{m}{240}}$	B1 ft	ft from eqn of form $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=-\omega^2x+k$
	$2\pi\sqrt{\frac{m}{240}}=0.48 \Rightarrow m=1.40$	M1A1 3	
	OR $\omega=\frac{2\pi}{0.48}$ B1 $\frac{2\pi}{0.48}=\sqrt{\frac{240}{m}} \Rightarrow m=1.40$ M1A1		
5 (i)			
	Moments about B for BC $H(1.2\cos 60)-18(0.6\sin 60)=0$ $H=15.6$	M1 A1 A1 3	Moments equation for BC Correct moments equation
(ii)	Horizontal component $=H=15.6$ N (to right) Vertical component $=V=18$ N (downwards)	B1 ft B1 2	Directions must be clear If B0, give B1 for 15.6 (ft) and 18
(iii)	Moments about A for AB $H(1.6\cos\theta)-V(1.6\sin\theta)-24(0.8\sin\theta)=0$ $\theta=27.5°$	M1 A2 A1 4	Moments equation for AB (or ABC) Give A1 if one error

6 (i)	$0.1\times 9.8-0.02v=0.1\frac{\mathrm{d}v}{\mathrm{d}t}$	M1 A1	Using N2L to obtain a diff eqn
	$t=\int\frac{1}{9.8-0.2v}\,\mathrm{d}v$	M1	separation of variables
	$=-5\ln(9.8-0.2v)+C$	A1	$+C$ not required
	$v=6$ when $t=0 \Rightarrow C=5\ln 8.6$	M1	
	$t=-5\ln(9.8-0.2v)+5\ln 8.6$		
	$e^{0.2t}=\frac{8.6}{9.8-0.2v}$	M1	
	$9.8-0.2v=8.6e^{-0.2t}$		
	$v=49-43e^{-0.2t}$	A1 (ag) 7	Must be correctly obtained
(ii)	$s=\int_0^{2.5}v\,\mathrm{d}t=\left[49t+215e^{-0.2t}\right]_0^{2.5}$	M1 A1	Integrating v For $49t+215e^{-0.2t}$
	$=(122.5+130.4)-(215)$	M1	Using both limits (or finding constant of integration, then putting $t=2.5$)
	$=37.9$ m	A1 4	Accept 38 m
7 (i)	Length of each string is $\sqrt{1.2^2+0.35^2}=1.25$	B1	
	Tension $T=\frac{112\times 0.45}{0.8}$ $(=63)$	M1	
	$2T\cos\theta=mg$ where $\cos\theta=\frac{0.35}{1.25}$	M1	
	$m=\frac{2\times 63\times 0.28}{9.8}=3.6$	A1 (ag) 4	Must be correctly obtained
(ii)	Initial EE is $2\times\frac{112\times 0.45^2}{2\times 0.8}$ $(=28.35)$	M1 A1	Using $\frac{\lambda x^2}{2l}$ (Award A1A1 for elastic energies if only one string is considered)
	Final EE is $2\times\frac{112\times 0.4^2}{2\times 0.8}$ $(=22.4)$	A1	
	By conservation of energy $\frac{1}{2}\times 3.6(3^2-v^2)+(28.35-22.4)=3.6\times 9.8\times 0.35$	M1 A1	Equation involving EE, KE and PE
	$v=2.33\text{ ms}^{-1}$	A1 6	
(iii)	e.g. Brick is a particle No air resistance Strings are light Strings obey Hooke's law *or* No energy is lost in the strings	B1B1B1 3	For three assumptions

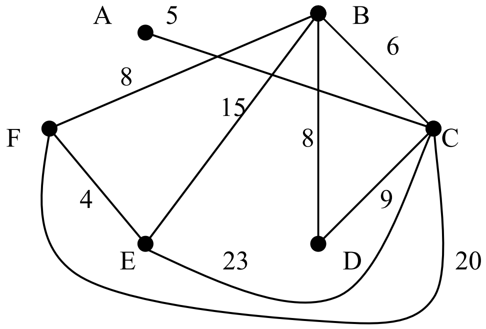
1. (i)	0.3x0.37	M1		Multiplying probs
	0.111	A1		
			2	
(ii)	0.7x0.23+0.3x0.63	M1		Both HM and MH
	0.35	A1		
			2	
2. (i)	$^{22}C_{11}$ seen or attempt with factorials	M1		
	705432	A1		
			2	
(ii)	$^{11}C_6$ and $^{11}C_5$ seen	M1		
	Multiplied	M1		
	213444/705432	M1		(Their $^{11}C_6x^{11}C_5$)/their (i)
	0.3025...			
	0.3025...	A1		allow 2541k/8398k
			4	
	ALITER 2(ii)			
	$\frac{11}{6!5!}\times\left(\frac{11}{22}\times\frac{10}{21}\times\frac{9}{20}\times\frac{8}{19}\times\frac{7}{18}\times\frac{6}{17}\right)\times\left(\frac{11}{16}\times\frac{10}{15}\times\frac{9}{14}\times\frac{8}{13}\times\frac{7}{12}\right)$			
		M1		Both 5 and 6 term probabilities seen.
		M1		Both 5 and 6 term probabilities multiplied
		M1		Fully correct method including 11!/(6!5!)
		A1		0.303 or equiv.
3. (i)	Use B(18,0.35) table	M1		Or probs,at least one correct.
	0.9788-0.3550	M1		fully correct method
	0.6238, 0.624	A1		
			3	
(ii)	$^{22}C_{10}(0.35)\ ^{10}(0.65)^{13}$	M1		allow p,q muddle
		M1		fully correct method
	0.11668,0.117	A1		
			3	
(iii)	e.g. May be biased sample,			
	Residences may not be indep.	B1		any relevant reason
			1	
4. (i)	Student 1 2 3 4 5 6			
	Rank mean2 3 5 4 6 1			
	Rank grade 2 4 3 5 6 1	B1		correct ranks (or reversed)
	$\sum d^2 = 6$	M1		from ranked data.
	r =1-(6x6)/(6x(36-1))	M1		fully correct method (their sum of d^2)
	29/35 or 0.829	A1		
			4	
(ii)	Use Mathematics grade	B1ft		
	Greater correlation	B1		
			2	
(iii)	e.g.Sample too small to generalise	B1		any relevant reason
			1	
5.(i)	t 0.5 1.5 3 5 8	B1		
	f 8 20 50 24 18	B1		
	mean=448/120	M1		from their f,t
	3.73(33....)	A1		
			4	

(ii)	Evaluate 2249/120	M1		from their f,t
	subtract mean^2 and sq.rt.	M1		
	2.19	A1		
			3	
	ALITER 5(ii)			
	e.g. $(0.5-3.733)^2$x8	M1		any correct term
				from their f,t
	Fully correct method	M1		from their f,t
	2.19	A1		
6.(i)	P(HH)matchings=$(1/4)^2$,			
	P(HT)matchings=$(1/2)^2$,			
	P(TT)matchings=$(1/4)^2$	B1		
	(1/4)^2+(1/2)^2+$(1/4)^2$	M1		allow even if errors in HT matchings
	3/8	A1		
	A.G.			
			3	
(ii)	Geometric distribution stated or implied.	M1		
	q^2 or 1-p-qp	M1		
	with q=5/8 (and p=3/8, if used)	M1		fully correct method
	25/64 or 0.391	A1		
			4	
(iii)	8/3 aef	B1		allow 2.67
	Refer to average number of matchings.	B1		
			2	
7(i)	median=(4.8+5.2)/2=5.0	B1		
	Find LQ(3.75),UQ(5.65)subtract	M1		allow slight variation
	1.9	A1ft		
			3	
(ii)	show 1.6,6.1,median and quartiles.	M1		recognisable attempt at box-plot
		A1ft		allow one error on diagram
		A1ft		fully correct (f.t.)
	NB Graph paper not used	M1(max)		
			3	
(iii)	Shivani's is positive skew			
	Emma's is negative skew	B1ft		for either
	Emma's IQR=1.5	B1		can be implied
	Shivani's is more variable	B1ft		
			3	
8(i)	Lines cross at mean x,mean y	M1		stated or implied
	e.g. y=18.5+0.1(16.6+0.4y)	M1		reasonable attempt at sim. equns.(to find both)
	mean y=21 , mean x=25	A1		
			3	
(ii)	5x25=125 and 5x21=105	B1		from correctly obtained means.
			1	
(iii)	$9/\sqrt{22.5\times 90}$	M1		any of Sxx,Sxy,Syy correct.
		M1		fully correct method
	0.2	A1		
			3	
(iv)	use y on x because x given, y needed.	B1		
	y=18.5+0.1x26	M1		
	y=21(.1)	A1		allow 2sf answer.
			3	
(v)	unreliable,r small	B1		any relevant reason
			1	

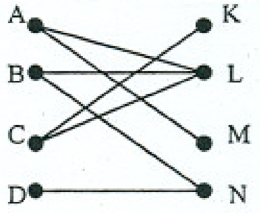
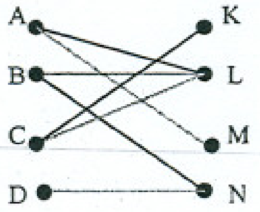
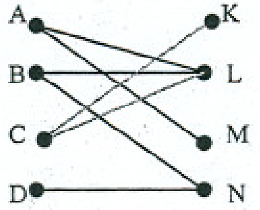
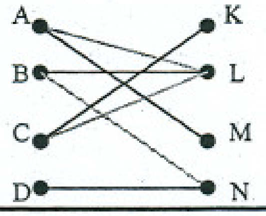
1	(i)	1 – 0.8571 0.143	M1 A1	**2**	0.8571 seen, 1 – 0.6767, or formula, ± 1 term Answer, a.r.t. 0.143 [0.1429]
	(ii)	$\mu = 4$ 0.785	M1 A1	**2**	$\mu = 4$ stated or implied Answer, a.r.t. 0.785 [0.7851]
2	(i)	Those with strong feelings are more likely to respond Only readers will respond	B1 B1	**2**	One valid relevant reason, e.g. “not everyone will respond” A different valid relevant reason
	(ii)	Obtain a list of the population Given each member a number Select using random numbers Contact those selected	B1 B1 B1	**3**	*Any 3 out of:* List population [*not* readers] Allocate numbers to list [*not* random numbers] Random numbers mentioned (not “hat”) Contact those selected [SR: Census: B1B0B1]
3	(i)	$\frac{80-\mu}{\sigma} = \Phi^{-1}(0.95) = 1.645$ $\frac{70-\mu}{\sigma} = \Phi^{-1}(0.75) = 0.674$ σ = 10.3 μ = 63.1	M1 A1 B1 M1 A1 A1	**6**	Standardise one and equate to Φ^{-1} [*not* Φ] Both standardised = Φ^{-1}, signs correct Both [1.64, 1.65] and [0.674, 0.675] Correct method to solve for one unknown σ correct, a.r.t. 10.3 μ correct, a.r.t. 63.1 [63.06] [σ^2: M1A0B1M1A0A1] [cc: M1A1B1M1A0A0]
	(ii)	Distribution bimodal/not symmetrical so normal distribution not appropriate	B1	**1**	Any relevant valid comment: “not symmetric” is enough
4		H_0: $p = 0.75$ H_1: $p < 0.75$ B(16, 0.75) α: 0.1897, compare with 0.10 β: Compare 10 with 9; P($\le$ 9) = 0.0796 Do not reject H_0 Insufficient evidence that spokesman is wrong	B1 B1 M1 M1 A1 M1 A1√	**7**	Hypotheses correctly stated in terms of p or π [wrong/no symbol: B1B0, but x or $\bar{x}$: B0] B (16, 0.75) stated or implied Correct comparison Correct p value, or probability for critical region Correct method & comparison Outcome interpreted in context [SR: N(12, 3): B1B1 M1M0A0 M0A0] [SR: 2-tail: B1B0 M1M0A1 M1A1]
5	(i)	$e^{-20}\frac{20^{22}}{22!}$ = 0.0769	M1 A1	**2**	Formula correct Answer, a.r.t. 0.0769
	(ii)	N(20, 20) $\Phi\left(\frac{22.5-20}{\sqrt{20}}\right) - \Phi\left(\frac{21.5-20}{\sqrt{20}}\right)$ = 0.7119 – 0.6311 = 0.0808	B1 M1 A1 A1	**4**	N(20, 20) seen or implied, allow 20^2 etc $\frac{x-\lambda}{\sqrt{\lambda}}$ twice, allow wrong or no cc, or no √ Both, ccs correct, $\sqrt{\lambda}$ Answer, in range 0.08055 to 0.08085 [SR: PDF: $\frac{1}{\sqrt{20\times 2\pi}}\exp -\frac{1}{2}\left(\frac{22-20}{\sqrt{20}}\right)^2$ M2A2]
	(iii)	$\frac{0.0808-0.07691}{0.07691}\times 100\%$ = 5.06%	M1 A1	**2**	±[(ii) – (i)]÷(i) seen or implied, a.e.f. Answer in range 4.77 to 5.14

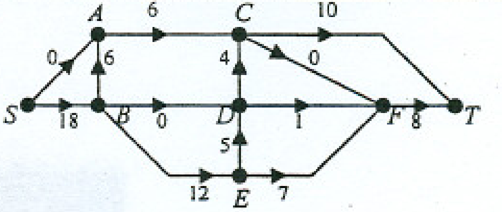
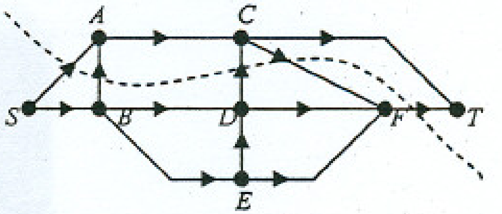
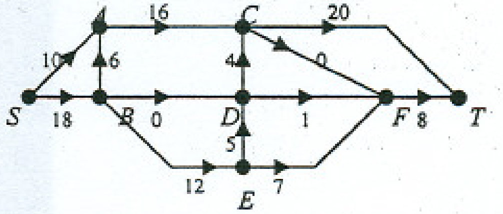
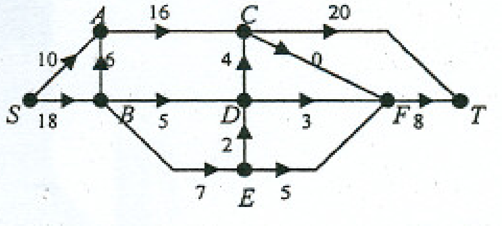
6	(i)	Reject H_0 when it is correct	B1	**1**	Correct statement [*not* "probability"]
	(ii)	B(300, 0.05) N(15;) 14.25 or 3.77(5) $\frac{19.5-15}{\sqrt{14.25}}$ [= 1.192] 0.117	M1 A1 A1 M1 A1 A1	**6**	B(300, 0.05) stated or implied Normal, correct value of μ, stated Correct value of σ^2 or σ Standardise with *np* and √*npq*, *npq*, *np* or √*np* Correct cc and √*npq* Answer, a.r.t. 0.117
	(iii)	Yes, required by binomial	B1	**1**	"Yes" stated with "binomial" mentioned
7	(i)	$\int_0^q (1-\tfrac{1}{2}x)dx = 0.75$ $q-\frac{q^2}{4}=\frac{3}{4}$ $q^2-4q+3=0$; $(q-3)(q-1)=0$ $q = 1$	M1 A1 M1 A1	**4**	Equate integral to 0.75 Correct equation, a.e.f., allow *x* etc throughout Solve relevant quadratic *q* = 1 only, cwd
	(ii)	 Total area must be 1 also Hence curve must cross line.	M1 B1 A1 B1	**4**	Curve concave-downwards, positive in $0 \le x \le 2$ Intersects *x*-axis at same place as line, curve must be positive in $0 \le x \le 2$ Decreasing curve, f(0) < 1, clearly crossing straight line Area mentioned, argument not wrong
	(iii)	Upper quartile of model 2 is higher as graph of f_2 is above that of f_1	B1 B1	**2**	Correct statement, from reasonable graph Correct justification, from reasonable graph
8	(i)	$N(\mu, \sigma^2/5)$	B1 B1	**2**	Normal, mean μ Variance $\sigma^2/5$ [*not* $\sigma/\sqrt{5}$]
	(ii)	H_0: μ = 42 H_1: μ < 42	B1 B1		One hypothesis correct, μ used Second hypothesis correct, μ used [wrong/no symbol: B1B0, but *x* or $\bar{x}$: B0]
	α:	$z = \frac{36.6-42}{8/\sqrt{5}}$ $z = -1.51$ $-1.51 < -1.28$ *or* $\Phi(z)$ [= 0.9345] > 0.90	M1 A1 B1√		α: Correct form for *z* [allow 42 – 36.6 unless *explicitly* μ = 36.6] Correct *z*, a.r.t. (–)1.51 –1.28 seen, allow + if consistent, *or* explicit comparison of 0.9345√ with 0.90/0.0655√ with 0.10, *p* correct to 3 SF
or	β:	$CV = 42 - k.8/\sqrt{5}$ *k* = 1.28 CV = 37.4 > 36.6	(M1) (B1) (A1)		β: Correct form for CV, must use 42 *k* = 1.28 seen CV = 37.4 and compare 36.6
		Reject H_0 Significant evidence that mean path length is less than 42	M1 A1√	**7**	Correct overall method/comparison (√5 needed) and "reject"/"do not reject" Correct contextualised conclusion
	(iii)	Lower significance level means decreased probability of rejecting H_0 when true, i.e. of claiming it is an ocron when it is not, so scientists want to be more cautious about accepting particles as ocrons.	M1 A1	**2**	Any valid contextualised comment, e.g. "More accurate" Comment equivalent to last clause on left [NB: P(claim it's an ocron when it's not) smaller P(not claim it's an ocron when it is) bigger]

1 (i)	Twice number of arcs = sum of orders of vertices	B1 **1**	$2 \times 16 = 3 + 3 + 6 + 4 + 4 + 4 + 4 + 4$	1C
(ii)	*AB*	B1 **1**		1E
(iii)	8	B1 **1**	**Total = 3**	1E
2 (i)	Using algorithm to get as far as *A-B-C-E-D* Route: *A-B-C-E-D-A* Length = 31 miles	M1 A1 B1 **3**	Allow M mark if they have used algorithm correctly starting at another node 31 cao	3E
(ii)	Minimum connector of remaining vertices *AB*, *AD*, *DE* = 16 (miles) Add two shortest arcs from *C* 16 + 6 + 7 = 29 (miles)	M1 A1 **2**	can be implied from 3+5+8 NOT 16 + 12 **Total = 5**	2C
3 (i)	Decreasing order: 8 7 6 5 5 5 4 2 From 1st roll: 8 2 From 2nd roll: 7 From 3rd roll: 6 4 From 4th roll: 5 5 From 5th roll: 5	M1 A1 **2**	Cutting from 5 rolls cao for first-fit decreasing	2E
(ii)	From 1st roll: 7.5 1.5 From 2nd roll: 6.5 3.5 From 3rd roll: 5.5 4.5 From 4th roll: 4.5 4.5	M1 A1 **2**	Attempt to show a suitable cutting from 4 rolls not in excess of 10 In any order	2C
(iii)	Sum of lengths = 38 metres, so rolls must be at least 13 metres long. From 1st roll: 7.5 5.5 From 2nd roll: 6.5 4.5 1.5 From 3rd roll: 4.5 4.5 3.5	B1 M1 A1 **3**	13 Cutting from 3 rolls cao in any order **Total = 7**	3C

4 (i)		M1 A1 **2**	Any correct graph with vertices *A* to *F* labelled, arcs may cross. Check *CF* in particular. All weights correct, be kind on ambiguous placings.	2E
(ii)	*A* 1 0 *B* 3 11 11 *C* 2 5 5 *D* 4 14 14 *E* 6 23 28 26 23 *F* 5 19 25 19	 M1 A1 B1	Follow through reasonable diagrams If boxes not used in this way, working must be clear and unambiguous Attempt at updating labels (Either E or F correct) All temporary labels correct (Ignore any extras) All permanent labels correct and a valid order of labelling.	
	Shortest route = *A – C – B – F - E*	B1 **4**	Accept *C – B – F* 23 is not sufficient	4C
(iii)	$AC = 5$ $AE = 23$ $AF = 19$ $EF = \underline{4}$ $CF = \underline{14}$ $CE = \underline{18}$ 9 37 37	M1	Identifying *A*, *C*, *E*, *F* and length (9)	
	Total weight of all arcs = 98	B1	98 seen or implied	
	Length of shortest route = 107 km	A1 **3**	cao Adding their 9 to 98 **Total = 9**	3C
5 (i)	$x \geq 0, y \geq 0$ $4x + 3y \leq 12$ $y - 2x \leq 1$	B1 B1 B1 **3**	Both Or equivalent Or equivalent Condone the use of < for each non trivial inequality	3C
(ii)	(0, 0), (3, 0), (0, 1) (0.9, 2.8) $(0, 0) \rightarrow P = 0$; $(0, 1) \rightarrow P= 3$; $(0.9, 2.8) \rightarrow P = 12.9$; $(3, 0) \rightarrow P = 15$	B2 B1 M1	Any two correct ⇒ B1 Exact Calculating $P = 5x + 3y$ for at least one of their vertices or clear evidence of using an appropriate line of constant profit	
	$x = 3$ and $y = 0$ $P = 15$	A1 A1 **6**	cao Must state P=15 or max is 15	6E
(iii)	Either consider the gradient of profit line ($-\frac{1}{3}a$) and the gradients of the boundary lines (2 and $-1\frac{1}{3}$) or calculate *Q* at vertices ⇒ 3, 0.9*a*+8.4, 3*a* Hence require $a \leq -6$ (or written using set notation)	M1 M1 A1 **3**	Any two calculations of *Q* with any *a* at any vertices (apart from *a* = 5) ⇒ M1 M1 for 0.9*a*+8.4, M1 for 3*a* Any specific value ≤ -6 or valid subset ⇒ B1 Condone *a* < -6 **Total = 12**	3A

<table>
<tr>
<td>6 (i)</td>
<td>
<table>
<tr><th><i>N</i></th><th><i>D</i></th><th>Output</th></tr>
<tr><td>30</td><td></td><td>2</td></tr>
<tr><td>31</td><td>3 5 7</td><td>31</td></tr>
<tr><td>32</td><td></td><td>2</td></tr>
<tr><td>33</td><td>3</td><td>3</td></tr>
<tr><td>34</td><td></td><td>2</td></tr>
<tr><td>35</td><td>3 5</td><td>5</td></tr>
<tr><td>36</td><td></td><td>2</td></tr>
<tr><td>37</td><td>3 5 7</td><td>37</td></tr>
</table>
</td>
<td>M1
A1
B1
B1
B1
B1
6</td>
<td>Output 2 from any one of 30, 32, 34, 36
Output 2 from all of 30, 32, 34, 36
Output 31 from 31
Output 3 from 33
Output 5 from 35
Output 37 from 37</td>
<td>6E</td>
</tr>
<tr>
<td>(ii)</td>
<td>Output cannot be 6 because
it can only be 2 or an odd number.
Output cannot be 9 because
if <i>N</i> is divisible by 9 then it was also divisible by 3 so the output would have been 3 (or possibly 2).</td>
<td>B1
B1
2</td>
<td>May combine both parts, eg ‘6 and 9 are not prime’ $\Rightarrow$ B2
May talk about factors of 6 and/or 9</td>
<td>2C</td>
</tr>
<tr>
<td>(iii)</td>
<td>Output is the least factor of <i>N</i>
(other than 1) that is a prime.</td>
<td>B1
B1
2</td>
<td></td>
<td>2A</td>
</tr>
<tr>
<td>(iv)</td>
<td>In any iteration, the algorithm will either jump to step 4 or step 5 and will stop
or in step 3 the value of <i>D</i> will increase by 2, and eventually D^2 will exceed <i>N</i>.</td>
<td>B1
B1
2</td>
<td>Reference to hitting stopping conditions
Reference to <i>D</i> increasing until $D^2 > N$
Total = 12</td>
<td>2C</td>
</tr>
<tr>
<td>7 (i)</td>
<td>$2x + 3y + z + s = 12$
$x + 2y + 4z + t = 8$</td>
<td>B1
1</td>
<td>Both</td>
<td>1E</td>
</tr>
<tr>
<td>(ii)</td>
<td>
<table>
<tr><th><i>P</i></th><th><i>x</i></th><th><i>y</i></th><th><i>z</i></th><th><i>s</i></th><th><i>t</i></th><th>-</th></tr>
<tr><td>1</td><td>-1</td><td>4</td><td>2</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>2</td><td>3</td><td>1</td><td>1</td><td>0</td><td>12</td></tr>
<tr><td>0</td><td>1</td><td>2</td><td>4</td><td>0</td><td>1</td><td>8</td></tr>
</table>
Pivot on 2 in <i>x</i> column
<table>
<tr><th><i>P</i></th><th><i>x</i></th><th><i>y</i></th><th><i>z</i></th><th><i>s</i></th><th><i>t</i></th><th>-</th></tr>
<tr><td>1</td><td>0</td><td>5½</td><td>2½</td><td>½</td><td>0</td><td>6</td></tr>
<tr><td>0</td><td>1</td><td>1½</td><td>½</td><td>½</td><td>0</td><td>6</td></tr>
<tr><td>0</td><td>0</td><td>½</td><td>3½</td><td>-½</td><td>1</td><td>2</td></tr>
</table>
$x = 6, y = 0, z = 0$
$P = 6$
</td>
<td>B1
B1
B1
M1
M1
A1
B1
B1
8</td>
<td>± (-1 4 2 0 0 0)
2 3 1 1 0 12 and 1 2 4 0 1 8
ft plausible tableau for B and M marks
Correct pivot choice
<i>P</i> has increased
Two basis columns (apart from <i>P</i>)
Correct tableau cao
ft their final tableau
ft their final tableau or their $x - 4y - 2z$</td>
<td>2C
6A</td>
</tr>
<tr>
<td>(iii)</td>
<td>
<table>
<tr><th><i>P</i></th><th><i>x</i></th><th><i>y</i></th><th><i>z</i></th><th><i>s</i></th><th><i>t</i></th><th>-</th></tr>
<tr><td>1</td><td>0</td><td>6</td><td>6</td><td>0</td><td>1</td><td><i>c</i></td></tr>
<tr><td>0</td><td>0</td><td>-1</td><td>-7</td><td>1</td><td>-2</td><td>12-2<i>c</i></td></tr>
<tr><td>0</td><td>1</td><td>2</td><td>4</td><td>0</td><td>1</td><td><i>c</i></td></tr>
</table>
$x = c, y = 0, z = 0$ or $(c, 0, 0)$
P = <i>c</i> (Optimal value of <i>P</i> decreases to <i>c</i>)
</td>
<td>M1
A1
A1
3</td>
<td>Correct tableau (with algebraic entries or a numerical value for c<6)
condone P < 6, $x < 6$, $y = 0$, $z = 0$
Total = 12</td>
<td>3A</td>
</tr>
</table>

1 (i)		B1 1	Each part requires a separate diagram
(ii)		B1 1	
(iii)		B1 1	Alternating path M A L B N D clearly shown on a diagram OR Dijkstra method used and alternating path listed
(iv)		B1 1	Matching must be shown on a diagram, not just listed **Total = 4**

2 (i)	A valid flow of 18 with *BA*, *BE* and *DC* all saturated	M1 A1 2	
(ii)	Cut shown correctly 28 cao	B1 B1 2	
(iii)	A valid flow of 28 with *SA*, *BA*, *DC*, *FT* all saturated	B1 1	(add 10 around SACT from a correct answer to (i)) or follow through from a valid flow in (i) 
(iv)	Any valid flow with *BD*, *DF* saturated and *SA*, *BA*, *DC*, *FT* saturated (flow = 28)	M1 A1 2	 **Total = 7**

3 (i)	Dummy is needed to make a square matrix.	B1 1	To give a 'person' for the fifth task.
(ii)	$\begin{array}{cccccc} & \mathbf{V} & \mathbf{W} & \mathbf{X} & \mathbf{Y} & \mathbf{Z} \\ \mathbf{Q} & 25 & 25 & 25 & 25 & 25 \\ \mathbf{R} & 21 & 17 & 11 & 10 & 20 \\ \mathbf{S} & 10 & 12 & 7 & 12 & 11 \\ \mathbf{T} & 18 & 10 & 6 & 6 & 12 \\ \mathbf{U} & 8 & 12 & 5 & 10 & 11 \end{array}$		
	Reduce rows $\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 11 & 7 & 1 & 0 & 10 \\ 3 & 5 & 0 & 5 & 4 \\ 12 & 4 & 0 & 0 & 6 \\ 3 & 7 & 0 & 5 & 6 \end{array}$	M1	Correct method for reducing matrix If columns are reduced first, but otherwise correct, award method mark only here then follow through (ie M1, A0)
	Columns are then reduced too Covering shows this is an incomplete matching $\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 11 & 7 & 1 & 0 & 10 \\ 3 & 5 & 0 & 5 & 4 \\ 12 & 4 & 0 & 0 & 6 \\ 3 & 7 & 0 & 5 & 6 \end{array}$	A1	Correct values in matrix after reducing both ways
	Augment by 3 $\begin{array}{ccccc} 0 & 0 & 3 & 3 & 0 \\ 8 & 4 & 1 & 0 & 7 \\ 0 & 2 & 0 & 5 & 1 \\ 9 & 1 & 0 & 0 & 3 \\ 0 & 4 & 0 & 5 & 3 \end{array}$	M1	Follow through from their reduced matrix Correct method for augmenting by 3 in one step
	Covering shows this is still an incomplete matching $\begin{array}{ccccc} 0 & 0 & 3 & 3 & 0 \\ 8 & 4 & 1 & 0 & 7 \\ 0 & 2 & 0 & 5 & 1 \\ 9 & 1 & 0 & 0 & 3 \\ 0 & 4 & 0 & 5 & 3 \end{array}$	A1	Correct values in matrix after one augmentation
	Augment by 1 $\begin{array}{ccccc} 1 & 0 & 4 & 4 & 0 \\ 8 & 3 & 1 & 0 & 6 \\ 0 & 1 & 0 & 5 & 0 \\ 9 & 0 & 0 & 0 & 2 \\ 0 & 3 & 0 & 5 & 2 \end{array}$	M1	Follow through from their matrix after first augmentation Correct final matrix eventually reached
	Complete matching $\begin{array}{ccccc} 1 & \boxed{0} & 4 & 4 & 0 \\ 8 & 3 & 1 & \boxed{0} & 6 \\ 0 & 1 & 0 & 5 & \boxed{0} \\ 9 & 0 & \boxed{0} & 0 & 2 \\ \boxed{0} & 3 & 0 & 5 & 2 \end{array}$	A1	Correct matching for their final matrix $\begin{array}{ccccc} 1 & 0 & 4 & 4 & \boxed{0} \\ 8 & 3 & 1 & \boxed{0} & 6 \\ 0 & 1 & \boxed{0} & 5 & 0 \\ 9 & \boxed{0} & 0 & 0 & 2 \\ \boxed{0} & 3 & 0 & 5 & 2 \end{array}$ or $\begin{array}{ccccc} 1 & 0 & 4 & 4 & \boxed{0} \\ 8 & 3 & 1 & \boxed{0} & 6 \\ \boxed{0} & 1 & 0 & 5 & 0 \\ 9 & \boxed{0} & 0 & 0 & 2 \\ 0 & 3 & \boxed{0} & 5 & 2 \end{array}$
	R–Y, S–Z, T–X, U–V stated cao (with or without dummy)	B1	R–Y, S–X, T–W, U–V or R-Y,S-V,T-W,U-X
	Cost = 35 stated cao	B1 8	(accept £35 or equivalent) **Total = 9**

4 (i)	The minimum weight on any route will determine the widest possible load for that route.	B1	Min on a route = widest possible load for route
	The maximum of these minima will give the maximum width possible	B1	Route with maximum widest possible load
		2	

(ii)

Stage	State	Action	Minimum	Maximin
1	0	0	4	4
	1	0	6	6
	2	0	5	5
2	0	0	Min(6, 4) = 4	
		1	Min(7, 6) = 6	6
		2	Min(6, 5) = 5	
	1	0	Min(7,4) = 4	4
		1	Min(3,6) = 3	
		2	Min(4,5) = 4	4
	2	0	Min(5,4) = 4	
		2	Min(7,5) = 5	5
3	0	0	Min(5,6) = 5	5
		1	Min(6,4) = 4	
		2	Min(4,5) = 4	

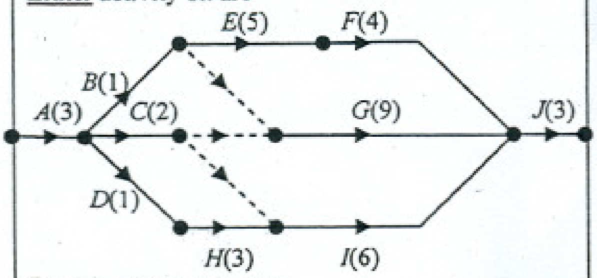
	Structure of table
M1	Stage column completed correctly
M1	State column completed correctly
A1	Action column completed correctly (check (2; 2))
B1	Stage 1 correct
	Stage 2
M1	Correct method for min values (4, 6, 5, 4, 3, 4, 4, 5)
A1	Suboptimal maximin values correct
	Stage 3
M1	Correct method for min values (5, 4, 4)
A1	Suboptimal maximin value correct (5)
	No follow through within table

Route (3;0) – (2;0) – (1;1) – (0;0) or *A* – *B* – *F* – *H* Accept *A* – *B* – *G* - *H*	B1	Or trace back their table
Widest load = 5 metres	B1	Or from their table, with units
	10	

Total = 12

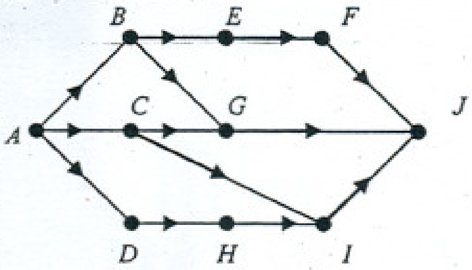
5 (i)

Either activity on arc



Durations not necessary

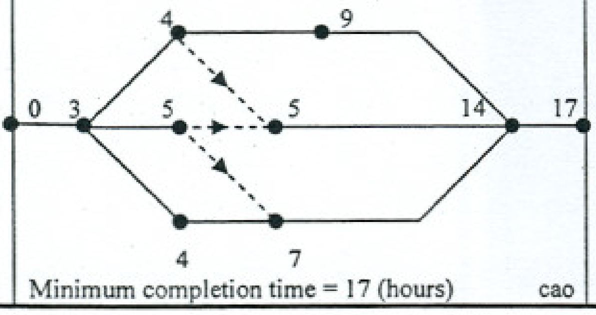
Or activity on node

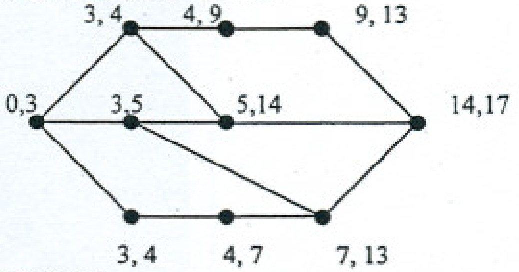


B1 All precedences correct

Forward pass (follow through network if possible)

M1 Forward pass (follow through network if possible) (start or end, consistently)

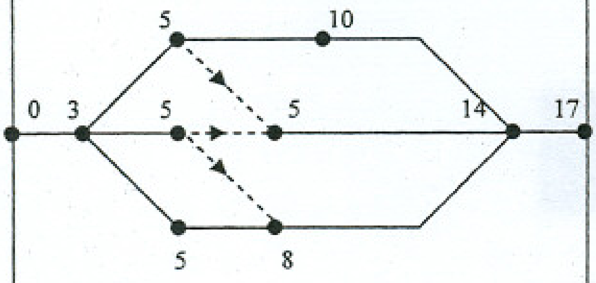
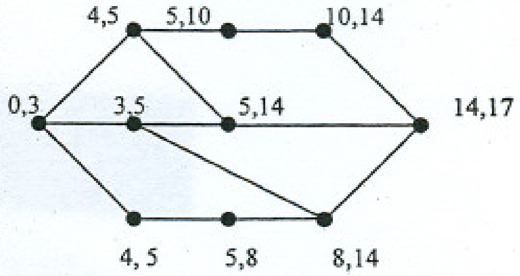
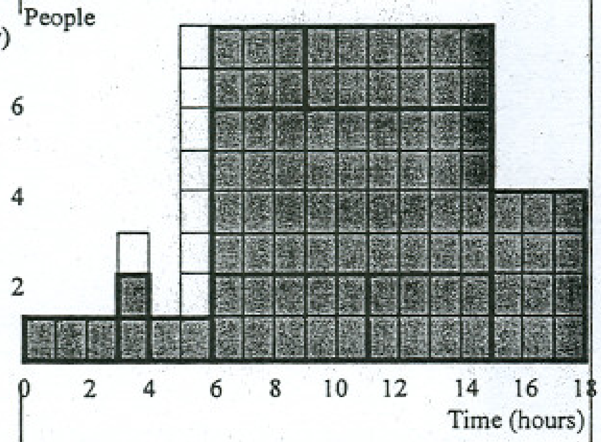


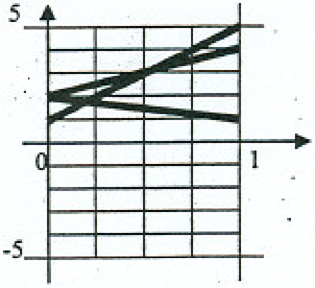


A1

Minimum completion time = 17 (hours) cao

B1 17 stated

	Backwards pass (follow through if possible, to start = 0) 	M1	Backwards pass (start or end, consistently) (follow through if possible, to start = 0)
		A1	
	Critical activities *A*, *C*, *G*, *J*	B1 7	*A*, *C*, *G*, *J* listed (and no others)
(ii)	Session one *B* *A* *A* *A* *D* *C* *C*		Diagram, list or any appropriate form
	Session two *E* *E* *E* *E* *E* *G* *G* *G* *G* *G* *G* *H* *H* *H*	M1	Critical activities (*A*, *C*, *G*, *J*) correctly placed with *G* split across two sessions only (*A* and *C* in first session, *G* across second and third sessions and *J* in fourth session)
	Session three *F* *F* *F* *F* *G* *G* *G* *I* *I* *I* *I* *I* *I*	A1	Valid timings and precedences, completed in four six hour sessions.
	Session four *J* *J* *J*	2	Other possibilities, eg using earliest start times
(iii)	Start *I* in session two, immediately after *H* and run Into session three. Follow *I* by *J* in session three.	B1	Describing moving activity *I* forwards
	Activity *F* needs to finish earlier, to do this we need to split *F* so that it starts immediately after *E*.	B1 2	Activity *F* needs to be split
(iv)		M1	Resource histogram on graph paper follow through from reasonable (iii) if possible Vertical axis showing number of people Horizontal axis showing time
		A1	Any valis histogram with no 'hanging' blocks (no need to see individual activities) Variations are possible in the first six hour session
	Sessions need 2, 8 and 8 people respectively	B1 3	Either 2, 8, 8 or 3, 8, 8 stated **Total = 14**

6 (i)	Row minima are -3, -8, 0 $\Rightarrow$ maximin =0 Column maxima are 0, 4, 5 $\Rightarrow$ minimax = 0 Row maximin = column minimax $\Rightarrow$ stable	M1 A1	Identifying row R and column W (correct method, even with small arithmetic errors) Explaining stability, or argued in words
	Farmer should leave field resting and make no profit	B1 3	'Resting' and 'no profit' in context
(ii)	$1 > -1$, $4 > 2$ and $5 > -1$	B1	Or differences 2, 2, 6 all positive (calculations must be correct)
	Farmer can always get a bigger profit by growing corn than by growing flax.	B1 2	Farmer should not grow flax whatever the weather Farmer should never grow flax.
(iii)	W: $1(p) + 2(1-p) = 2-p$ D: $4(p) + 2(1-p) = 2+2p$ M: $5(p) + 1(1-p) = 1+4p$	B1 B1 B1 3	Or all multiplied by 1000
(iv)	Lines plotted accurately on an appropriately labelled graph.  	B2	Follow through from (iii) 3 correct lines (2 lines correct $\Rightarrow$ B1)
	$2-p = 1+4p \Rightarrow p = 0.2$	M1	Stating correct p value for their graph or equations ($0 \le p \le 1$)
	W: 1.8 (£1800) D: 2.4 (£2400) M: 1.8 (£1800)	A1 4	Calculating expected profit for each type of weather
(v)	$0.1(2-p) + 0.3(2+2p) + 0.6(1+4p)$ Maximum when $p = 1$	M1 A1 2	$1.4 + 2.9p$ follow through from (iii) cao, from correct expression
			Total = 14

1 (i)	$11^{-2}=\frac{1}{121}$	B1 1	$\frac{1}{121}$ ($\frac{1}{11^2}=$ B0)
(ii)	$100^{\frac{3}{2}}=1000$	M1 A1 2	Square rooting or cubing soi 1000
(iii)	$\sqrt{50}+\frac{6}{\sqrt{3}}$	B1	$5\sqrt{2}$ (allow $\pm$)
	$=5\sqrt{2}+\frac{6\sqrt{3}}{3}$	M1	Attempt to rationalise $\frac{6}{\sqrt{3}}$
	$=5\sqrt{2}+2\sqrt{3}$	A1 3	cao
		<u>6</u>	
2	q=2	B1	(allow embedded values)
	r=3	B1	
		M1	$qr^2+10=p$ or other correct method
	p=28	A1$\sqrt{\ }$ 4	
		<u>4</u>	
3(i)	$y=5\sqrt{2x}$	M1	$\sqrt{2x}$ or $\sqrt{\frac{x}{2}}$ seen
		A1 2	$y=5\sqrt{2x}$
(ii)	Translation $\begin{pmatrix}0\\-3\end{pmatrix}$	B1	Translation
		B1 2	$\begin{pmatrix}0\\-3\end{pmatrix}$ o.e.
		<u>4</u>	

4	Either $y = 2x + 1$ or $y = \frac{x^2 + 11}{3}$	M1	Substitute for x/y or attempt to get an equation in 1 variable only
	$x^2 - 6x + 8 = 0$	A1	Obtain correct 3 term quadratic
	$(x-2)(x-4) = 0$	M1	Correct method to solve 3 term quadratic
	$x = 2 \quad x = 4$	A1	or one correct pair of values B1
	$y = 5 \quad y = 9$	A1	second correct pair of values B1 c.a.o
	OR $x = \frac{y-1}{2}$ $\frac{(y-1)^2}{4} - 3y + 11 = 0$ $y^2 - 14y + 45 = 0$ $(y-5)(y-9) = 0$ $y = 5 \quad y = 9$ $x = 2 \quad x = 4$		
			SR If solution by graphical methods: setting out to draw a parabola and a line M1 both correct A1 reading off of coordinates at intersection point(s) M1 one correct pair A1 second correct pair A1 OR No working shown: one correct pair B1 second correct pair B1 full justification that these are the only solutions B3
		5	

5 (i)		B1	Correct curve in +ve quadrant
		B1 2	in –ve quadrant
(ii)		M1	Positive cubic with clearly seen max and min points
		A1	(-1,0) (0,0) (1,0) Any one point stated or marked on sketch
	(-1,0) (0,0) (1,0)	A1 3	Curve passes through all 3 points and no extras stated or marked on sketch
(iii)		B1	Graph only in bottom right hand quadrant
		B1 2	Correct graph, passing through origin
		7	

6 (i)	$49-4\times-2\times3=73$	M1	Uses b^2-4ac
	2 real roots	A1	73
		B1$\sqrt{}$ 3	2 real roots (ft from their value)
(ii)	$(p+1)^2-64=0$ or $2[(x+\frac{p+1}{4})^2-\frac{(p+1)^2}{16}+4]=0$	M1	Attempts b^2-4ac = 0 (involving p) or attempts to complete square (involving p)
		A1	$(p+1)^2-64=0$ aef
	p= -9,7	B1	*p*= -9
		B1 4	p= 7
		7	

7 (i)	$\frac{dy}{dx}=2x^3-3$	B1	1 term correct
		B1 2	Completely correct (+c is an error, but only penalise once)
(ii)	$y=2x^3+2x^2+3x+3$	M1	Attempt to expand brackets
	$\frac{dy}{dx}=6x^2+4x+3$	A1	$2x^3+2x^2+3x+3$
		A1	2 terms correct
		A1 4	Completely correct SR Recognisable attempt at product rule M1 one part correct A1 second part correct A1 final simplified answer A1
(iii)	$y=x^{\frac{1}{5}}$	B1	$x^{\frac{1}{5}}$ soi
	$\frac{dy}{dx}=\frac{1}{5}x^{-\frac{4}{5}}$	B1	$\frac{1}{5}x^c$
		B1 3	$kx^{-\frac{4}{5}}$
		9	
8(i)	$2[10+x+x]>64$	B1 1	$20+4x>64$ o.e.
(ii)	$x(x+10)<299$	B1	$x(x+10)<299$
	$x^2+10x-299<0$ $(x-13)(x+23)<0$	B1 2	Correctly shows $(x-13)(x+23)<0$ **AG** SR Complete proof worked backward B2
(iii)	$x>11$	B1 $\sqrt{}$	$x>11$ ft from their (i)
	$(x-13)(x+23)<0$	M2	Correct method to solve $(x-13)(x+23)<0$ eg graph
	$-23<x<13$	A1	$-23<x<13$ seen in this form or as number line SR if seen with no working B1
	$\therefore 11<x<13$	B1 5	
		8	

9(i)	$\frac{dy}{dx} = 4x$	B1	$4x$
	At x=3 , $\frac{dy}{dx} = 12$	B1 2	12
(ii)	Gradient of tangent = - 8	M1	$\frac{dy}{dx} = -8$
	$4x = -8$ $x = -2$	A1	x=-2
	$y = 8$	A1 3	y=8
(iii)	Gradient = 6	B1 1	Gradient = or approaches 6
(iv)	$\frac{dy}{dx} = 2kx$	M1	$\frac{dy}{dx} = 2kx$
	$x = 1$	M1	$\frac{dy}{dx} = 2k$
	$\frac{dy}{dx} = 2k$ $k = 3$	A1$\sqrt{\ }$ 3	$k = 3$ CWO
		<u>9</u>	

10(i)	Gradient DE = $-\frac{1}{2}$	B1 1	$-\frac{1}{2}$ (any working seen must be correct)
(ii)	$y-3=-\frac{1}{2}(x-2)$	M1	Correct equation for straight line, any gradient, passing through F
		A1	$y-3=-\frac{1}{2}(x-2)$ aef
	$x+2y-8=0$	A1 3	$x+2y-8=0$ (this form but can have fractional coefficients e.g. ½x + y – 4 = 0
(iii)	Gradient EF = $\frac{4}{2}$ =2	B1	Correct supporting working must be seen
	$-\frac{1}{2}\times 2=-1$	B1 2	Attempt to show that product of their gradients = - 1 o.e.
(iv)	DF=$\sqrt{4^2+3^2}=5$	M1	$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ used
		A1 2	5
(v)	DF is a diameter as angle DEF is a right angle.	B1	Justification that DF is a diameter
	Mid-point of DF <u>or</u> centre of circle is $(0,1\frac{1}{2})$	B1	Mid-point of DF <u>or</u> centre of circle is $(0,1\frac{1}{2})$
	Radius = 2.5	B1	Radius = 2.5
	$x^2+(y-\left(\frac{3}{2}\right)^2)=\left(\frac{5}{2}\right)^2$ $x^2+y^2-3y+\frac{9}{4}=\frac{25}{4}$	B1$\sqrt{}$	$x^2+(y-\left(\frac{3}{2}\right)^2)=\left(\frac{5}{2}\right)^2$
	$x^2+y^2-3y-4=0$	B1 5	$x^2+y^2-3y-4=0$ obtained correctly with at least one line of intermediate working. <u>SR</u> For working that only shows $x^2+y^2-3y-4=0$ is equation for a circle with centre $(0,1\frac{1}{2})$ B1 radius 2.5 B1
		<u>13</u>	

1		$(3+2x)^3 = 27+54x+36x^2+8x^3$	M1		For recognisable binomial expansion attempt
			A1		For any two terms correct, possibly unsimplified
			A1		For all four terms correct and simplified
		$(3-2x)^3 = 27-54x+36x^2-8x^3$	B1√		For changing the appropriate signs
		Hence $(3+2x)^3-(3-2x)^3 = 108x+16x^3$	A1	**5**	For answer $108x+16x^3$ or $4x(27+4x^2)$
				$\boxed{5}$	
2	**(i)**	$u_2=-1,\ u_3=\frac{1}{2},\ u_4=2,\ u_5=-1$	B1		For correct value -1 for u_2
			B1√		For correct u_3 from their u_2
			B1√	**3**	For correct u_4 and u_5 from their u_3 and u_4
	(ii)	u_1, u_4, u_7, etc all have the value 2	B1		For recognising the repeating property
		Hence $u_{199}=2$, giving $u_{200}=-1$	M1		For division by 3, or equivalent
			A1		For correctly linking relevant term to a term already found
			A1	**4**	For the correct answer -1
		(SR - Answer only is B1)			
				$\boxed{7}$	
3	**(i)**	$\frac{LB}{\sin 65°}=\frac{200}{\sin 35°}$ OR $\frac{LA}{\sin 80°}=\frac{200}{\sin 35°}$	M1		For correct use of the sine rule in ΔLAB (could be in **ii**)
		$\Rightarrow LB=316.0198\ldots$ $\Rightarrow LA=343.39...$	A1		For correct value of (or explicit expression for) LB or LA
		Hence $p=LB\sin 80°=311\text{ m}$ $p=LA\sin 65 = 311\text{m}$	M1		For calculation of perpendicular distance
			A1	**4**	For correct distance (rounding to) 311
	(ii)	$LC^2=200^2+316^2-2\times200\times316\times\cos100°$	M1		For use of cosine rule in ΔLBC or LAC
		$\left(or\ LC^2=400^2+343^2-2\times400\times343\times\cos 65°\right)$	A1√		For correct unsimplified numerical expression for LC^2 following their LA or LB
		Hence $LC=402$ m	A1	**3**	For correct distance (rounding to) 402
				$\boxed{7}$	
4	**(i)**	$\frac{16}{1^2}=16$ and $16=17-1^2$ stated			
		$1=\frac{16}{4^2}$ and $1=17-4^2$ stated	B1	**1**	For complete verification for both points
	(ii)	Area is $\int_1^4\left(17-x^2-\frac{16}{x^2}\right)\mathrm{d}x$	M1		For appropriate subtraction (at any stage) – correct order
		$=\left[17x-\frac{1}{3}x^3+\frac{16}{x}\right]_1^4$	*M1		For integration attempt with any one term OK
			A1		For $17x-\frac{1}{3}x^3$ completely correct
			M1		For correct form kx^{-1} for third term
			A1		For correct k, for their stage of working
		$=68-\frac{64}{3}+4-17+\frac{1}{3}-16=18$	M1dep*M		For use of limits – correct order
			A1	**7**	For correct answer 18
				$\boxed{8}$	

5	**(i)**	$\sin\theta\tan\theta=\sin\theta\times\frac{\sin\theta}{\cos\theta}=\frac{1-\cos^2\theta}{\cos\theta}$	M1		For use of $\tan\theta=\frac{\sin\theta}{\cos\theta}$
			M1		For use of $\cos^2\theta+\sin^2\theta=1$
		Hence $1-\cos^2\theta=\cos\theta(\cos\theta+1)$,			
		i.e. $2\cos^2\theta+\cos\theta-1=0$, or equiv	A1	**3**	For showing given equation correctly
	(ii)	$(2\cos\theta-1)(\cos\theta+1)=0$	M1		For solution of quadratic equation in $\cos\theta$
		Hence $\cos\theta=\frac{1}{2}$ or -1	A1		For both values of $\cos\theta$ correct
		So $\theta=60°,\ 300°,\ 180°$	A1		For correct answer $60°$
			A1		For correct answer $180°$
			A1√	**5**	For a correct non-principal-value answer, following their value of $\cos\theta$ (excluding $\cos\theta$ = -1, 0, 1) **and** no other values for θ.
				$\boxed{8}$	
6	**(a)**	$\int(x^3+2x)\,\mathrm{d}x=\frac{1}{4}x^4+x^2+c$	M1		For expanding and integration attempt
			A1		For $\frac{1}{4}x^4+x^2$ correct
			B1	**3**	For addition of an arbitrary constant (this mark can be given in **(b)(i)** if not earned here), and no dx in either
	(b) (i)	$\int x^{-\frac{1}{2}}\,\mathrm{d}x=2x^{\frac{1}{2}}+c$	B1		For use of $\frac{1}{\sqrt{x}}=x^{-\frac{1}{2}}$
			M1		For integral of the form $kx^{\frac{1}{2}}$
			A1	**3**	For correct term $2x^{\frac{1}{2}}$
	(ii)	$0=2\sqrt{4}+c\Rightarrow c=-4$	M1		For use of $x=4, y=0$ to evaluate c
			A1t		For correct c from their answer in **(b)(i)**
		Hence curve is $y=2x^{\frac{1}{2}}-4$	A1t	**3**	For equation of the curve correctly stated
				$\boxed{9}$	
7	**(i)**	Length of OD is 6 cm	B1		For stating or using the correct value of r
		Angle DOE is $\frac{1}{3}\pi$ / 1.047^c / $60°$ / $^1/_6$ of circle	B1		For stating or using the correct angle
		Hence arc length DE is 2π cm (allow 6.28 cm)	B1		For correct use of $s=r\theta$ or equiv in degrees
		Area is $\frac{1}{2}\times6^2\times\frac{1}{3}\pi=6\pi$ cm^2 $\left(\text{or } \frac{60}{360}\times\pi\times6^2\right)$	B1	**4**	For obtaining the given answer 6π correctly
	(ii)	Area of small triangle is $\frac{1}{2}\times6^2\times\frac{1}{2}\sqrt{3}=9\sqrt{3}$	*M1		For use of $\Delta=\frac{1}{2}ab\sin C$, or equivalent
			A1		For correct value $9\sqrt{3}$, or equiv
		Area of segment is $6\pi-9\sqrt{3}$	M1dep*M		For relevant use of (sector – triangle)
		Hence shaded area is $(18\sqrt{3}-6\pi)$ cm^2	A1	**4**	For correct answer $18\sqrt{3}-6\pi$, or exact equiv
					Scheme for alternative approaches:
			*M1		Attempt area of big triangle / rhombus / segment, using $\Delta=\frac{1}{2}ab\sin C$, or equivalent
			A1		Correct area
			M1dep*M		Relevant subtraction
			A1		For correct answer $18\sqrt{3}-6\pi$
				$\boxed{8}$	

8	**(i) (a)**	Sketch showing exponential growth	M1		For correct shape in at least 1st quadrant
		Intersection with y-axis is $(0,1)$	A1	**2**	For 1st and 2nd quadrants, and y-coordinate 1 stated
	(b)	Sketch showing exponential decay	M1		For correct shape in at least 1st quadrant
		Intersection with y-axis is $(0,2)$	A1	**2**	For 1st and 2nd quadrants, and y-coordinate 2 stated
	(ii)	$a^x = 2b^x$	B1		For stating the equation in x
		Hence $x\log_2 a = \log_2 2 + x\log_2 b$	M1		For taking logs (any base)
			M1		For use of one log law
			M1		For use of a second log law
		i.e. $x = \dfrac{1}{\log_2 a - \log_2 b}$	A1	**5**	For showing the given answer correctly
				$\boxed{9}$	
9	**(i)**	$ar^3 - a = 4(ar^2 - ar)$	M1		For using ar^{n-1} to form an equation
		Hence $r^3 - 4r^2 + 4r - 1 = 0$	A1	**2**	For showing the given equation correctly
	(ii)	$1 - 4 + 4 - 1 = 0$	B1		For correct substitution of $r = 1$, or state no remainder
		Factors are $(r-1)(r^2 - 3r + 1)$	M1		For attempted division, or equivalent
			A1	**3**	For correct factor $r^2 - 3r + 1$
	(iii)	$r = \dfrac{3 \pm \sqrt{5}}{2}$	M1		For solving the relevant quadratic equation
			A1	**2**	For correct roots in exact form
	(iv)	The relevant value of r is $\dfrac{3-\sqrt{5}}{2}$ (or decimal equiv)	B1		For selecting the appropriate value of r
		Hence $S_\infty = \dfrac{a}{1 - \frac{1}{2}\left(3-\sqrt{5}\right)}$	M1		For relevant use of $\dfrac{a}{1-r}$
		$= \dfrac{2a}{-1+\sqrt{5}} = \dfrac{2a\left(-1-\sqrt{5}\right)}{\left(-1+\sqrt{5}\right)\left(-1-\sqrt{5}\right)}$	M1		For correct process for rationalising, using two term surd expression
		$= \frac{1}{2}a\left(1+\sqrt{5}\right)$	A1	**4**	For showing the given answer correctly
				$\boxed{11}$	

Final mark scheme

Nikki Adams 16/01/2005

1	**(i)**	$R = W\cos\alpha$	M1		For resolving forces perpendicular to the plane
		Magnitude is 96 N	A1	2	
	(ii)	Magnitude is 24 N	B1	1	AG From correct work.
	(iii)	$P = 100\times0.28 - 24$ $P = 100\times0.28 + 24$	M1		For resolving 3 forces parallel to the plane (either case)
		(a) $P = 4$	A1		
		(b) $P = 52$	A1	3	

2	**(i)**	Momentum of A and B before collision = $0.4\times6 - 1.2\times2$	B1		Alternatively: Momentum lost by $A = 0.4\times(6 - v)$ B1
		Momentum of A and B after collision = $0.4v + 1.2\times1$	B1		Momentum gained by $B = 1.2\times(1 + 2)$ B1
		$0.4\times6 - 1.2\times2 = 0.4v + 1.2\times1$ ($v = -3$)	M1		For using the principle of conservation of momentum
		Speed is 3 ms^{-1}	A1		Positive answer only
		Direction is away from B	A1 ft	5	ft from v
	(ii)	$1.2\times1 - 4m = -1.2\times0.5 + 2m$ or $1.2\times1 + 1.2\times0.5 = 4m + 2m$			For momentum equation :-
			B1		with lhs correct
			B1		with rhs correct
		$m = 0.3$	B1	3	
					SR If mgv used for momentum instead of mv, then (i) Speed is 3 ms^{-1} B1 Direction is away from B B1 ft (ii) m = 0.3 B1

3	**(i)(a)**		M1		For resolving 3 forces parallel to the x-axis
		$X = 2\times8\cos30^\circ - 5\sin40^\circ$	A1		
		Component is 10.6 N	A1 ft		ft for 4.17 from sin/cos mix only
	(i)(b)	$Y = 5\cos40^\circ$	B1		
		Component is 3.83 N	B1 ft	5	ft for 3.21 from sin/cos mix only
	(ii)	$R^2 = 10.64^2 + 3.83^2$	M1		For using $R^2 = X^2 + Y^2$
		Magnitude is 11.3 N	A1 ft		
		$\tan\theta = 3.83/10.64$	M1		For using $\tan\theta = Y/X$
		Direction is 19.8° anticlockwise from +ve x-axis	A1 ft	4	

4	**(i)**		M1	For using $a = \dot{v}(t)$
		Acceleration is $1 + 0.2t$	A1 2	
	(ii)		M1	For solving $a(t) = 2.8$ for t
		$t = 9$	A1	
			M1*	For integrating $v(t)$ to find $s(t)$
			A1	For $t^2 \div 2$ correct in $s(t)$
			A1	For $t^3 \div 30$ correct in $s(t)$
		$s(9) = 9^2 \div 2 + 9^3 \div 30 - (0 + 0)$	dep*M1	For correct use of limits or equivalent
		(= 40.5 + 24.3)		
		Distance is 64.8 m	A1 ft 7	ft their $a = \dot{v}(t)$ from (i)

5	**(i)**	Heights are $7t - ½ gt^2$ and $10.5t - ½ gt^2$	B1 1	
	(ii)	Expression is $3.5t$	B1 1	From correct (i)
	(iii)	$0 = 7 - 9.8t$	M1	For using $v = u - gt$ with $v = 0$
		$t = 5/7$ or 0.714	A1	
		Difference is 2.5 m	A1 ft 3	ft value of t
	(iv)	$t = 1$	B1 ft	For using ans(ii) = 3.5 correctly
		Greater than 5/7 (may be implied) or $7 - g \times 1$ is -ve	M1	For comparing this t with the time to greatest height or considering the sign of v_A for this t
		Direction is downwards	A1 3	
	(v)	$h_A = 7 \times 1 - ½\ 9.8 \times 1^2$	M1	For using $h = ut - ½ gt^2$ with relevant t
		Height is 2.1 m	A1 2	

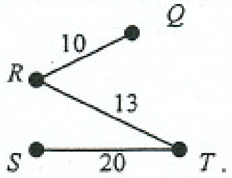
6	**(i)**		M1		For using the idea that the gradient represents acceleration or for using $v = u + at$
		Accelerating for 4 s	A1	2	
	(ii)		M1		For using the idea that the distance is represented by the area of the trapezium or using suitable formulae for the two stages of the journey
		AB = ½ (16 + 20)8	A1ft		
		Distance is 144 m	A1	3	
	(iii)		B1		Graph is single valued and continuous and consists of two straight line segments with one segment from the origin and the other parallel to the t axis
			B1	2	Graph for Q is the reflection of the graph for P in the t axis
	(iv)		B1		Graph is single valued and continuous and consists of two parts, one of which is a straight line segment,with x increasing from 0 for the interval $0 < t < 20$
			B1		$x_P(20)$ appears to be equal to $x_Q(0)$
			B1	3	Graph for P appears to be the reflection in x = ans(ii) $\div$ 2 of graph for Q
	(v)	t = 20 – (½ 144 $\div$ 8) or 16 + 8(t-4) = 128 – 8(t-4) or equivalent	M1		For complete method of finding the required time
		Value of t is 11	A2	3	
					SR Allow B1 for t = 11 without explanation

7	**(i)**		M1		For applying Newton's second law to either particle
		$T - F = 0.3a$	A1		
		$0.2g\sin 70^{\circ} - T = 0.2a$	A1		
		$R = 0.3g$	B1		
		$F = 0.4(0.3g)$	M1		For using $F = \mu R$
		$0.2g\sin 70^{\circ} - 0.4(0.3g) = 0.5a$	M1		For eliminating F and T or a
		Acceleration is 1.33 ms^{-2}	A1		
		Tension is 1.58 N	A1	8	
	(ii)	$a = -0.4g$	B1		May be scored in (iii)
		$0 = 1.5^2 - 2 \times 3.92s$	M1		For using $v^2 = u^2 + 2as$ with $v = 0$
		Distance is 0.287 m	A1	3	
	(iii)		M1		For using $v = u + at$ or equivalent with $v = 0$ for A
		$0 = 1.5 - 3.92t$	A1ft		ft value of a from (ii)
		$t = 0.383$ (may be implied)	A1		
		$a = g\sin 70^{\circ}$	B1		For acceleration of B
		$s = 1.5(0.383) + ½9.8\sin 70^{\circ}(0.383)^2$ (= 0.574 + 0.674)	M1		For using $s = ut + ½ at^2$ or equivalent with $u \neq 0$
		Distance is 1.25 m	A1	6	

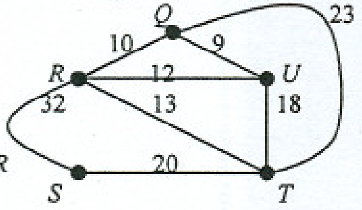
1	(i)	*A* Points lie close to straight line	B1 B1	2	Valid reason, eg “linear”. Not “strong correlation”
	(ii)	*C* Non-linear relationship	B1 B1	2	eg curve or quadratic
2	(i)	Median 8 Quartiles 6, 24	B1 B2	3	B1 for each Allow IQR = 24 - 6
	(ii)	Extreme values/skew distort mean or 35 mentioned	B1	1	Accept just “data skewed”. Not “anomaly”
	(iii)	Advantage: retains data values Disadv: harder to read (eg) median harder to compare distr’s visual comparison harder	B1 B1	2	Not “Can be shown on same diag”
3	(i)	2 3 4 1 6 5 7 6 5 4 7 2 3 1 1 2 3 4 5 6 7 7 6 5 4 3 2 1 $\Sigma d^2 = 14$ $r_s = 1 - \frac{6\Sigma d^2}{7(7^2 - 1)}$ r_s = ¾	M1 M1 A1 M1 A1	5	Rank both sets consistently Find Σd^2, dep ranks attempted. Allow arith errors $\Sigma d^2 = 14$ Use formula correctly, dep 2nd M1 Answer ¾ or a.r.t. 0.750
	(ii)	Rankings generally agree dep $r_s > 0.5$	B1f	1	Must have “agree” or “similar” etc, Not ‘rankings well correlated’ If $r_s < 0.5$, “generally don’t agree”: B1
4	(i)	$k = 1 - \left(\frac{1}{4} + \frac{1}{5} + \frac{2}{5} + \frac{1}{10}\right)$ $^1/_{20}$	M1 A1	2	Use $\Sigma p = 1$ or 0.05
	(ii)	$\text{E}(X) = \Sigma x\text{p}(x)$ $= -1/10$ $\Sigma x^2\text{p}(x) = 2$ $\Sigma x^2\text{p}(x) - \mu^2$ $= 1.99$	M1 A1 M1 M1 A1	5	Use $\Sigma x\text{p}(x)$ with a value for *k* and correct signs –1/10 or –0.1 only Attempt $\Sigma x^2\text{p}(x)$ } or $\Sigma(x-\mu)^2\text{p}(x)$: M2 Subtract their μ^2 } Answer, 1.99 or 1 99/100
5	(i)	(a) Geo(0.05) $(19/20)^5(1/20)$ = 0.0387	M1 M1 A1	3	Geo(0.05) or 0.95 stated or implied q^5p attempted Answer, a.r.t. 0.0387 ISW
		(b) $(19/20)^{10}$ = 0.599	M1 M1 A1	3	q^{10} or $1 - p - pq \ldots - pq^9$ [q^9 or q^{11}, or one wrong term: M1M0] Answer, a.r.t. 0.599 $1 - (^{19}/_{20})^{10}$: M0M0A0
	(ii)	Mean = $1/p$ = 20	M1 A1	2	20, cao
6	(i)	B(5, 3/8) $^5C_2(3/8)^2(5/8)^3$ = 5625/16384 or 0.343	M1 M1 A1	3	B(5, 3/8) stated or $^3/_8$, $^5/_8$ seen and sum of powers = 5 Correct expression Answer, a.r.t. 0.343 ISW
	(ii)	½ $p_1 = {}^3/_8$ p_1 = ¾ **AG**	M1 A1	2	or $^3/_8$ / $^1/_2$ or $^3/_8$ x 2 ¾ correctly obtained. Must see explicit step. Verification eg $^1/_2$ x $^3/_4 = {}^3/_8$ or $^3/_8/^3/_4 = {}^1/_2$: M1A1
	(iii)	½ $p_2 = {}^1/_3$ $p_2 = {}^2/_3$	M1 A1	2	or $^1/_3$ / $^1/_2$ or $^1/_3$ x 2 Answer 2/3 or a.r.t. 0.667

7 (i) Boxes are independent	B1	Both must be in context
Probability same for each box	B1 2	
(ii) (a) B(8, 0.1)	M1	B(8, 0.1) stated or 0.1, 0.9 seen and sum of powers =8
0.4305	A1	0.43[05] correct
(b) 1 – P(≤ 1)	M1	$1 - 0.8131$ or $1 - (0.9^8 + 8\text{x}0.9^7\text{x } 0.1)$ correct
0.1869	A1 4	Answer, a.r.t. 0.187
(iii) 2 × 0.4305 × 0.1869	M1	(a) x (b) }
	M1	2 x (a) × (b) }
0.16092	A1 3	Answer, a.r.t. 0.161
8 (i) $\frac{2\times 7!}{8!}$	M1	7! and 8! used or $^7P_7$ and $^8P_8$
	M1	Correct formula, with "2 ×"
= ¼	A1 3	Answer, $^1/_4$ or 0.25 only
(ii) $^1/_4$ or 4! × 4! or 3! × 3! or $^{3!}/_{4!}$	M1	
$\left(\frac{1}{4}\right)^2$ or $\frac{3!\times 3!}{4!\times 4!}$	M1	Correct expression
= $^1/_{16}$	A1 3	or 0.0625
(iii) Attempt subdivide, allow one error.	M1	By description or listing or implied by probs, eg 1 – (ii) – P(sep by 1)
Correct subdivision into 3 or 13 cases	M1	All 3 or all 13 cases clearly present
Correct expression	M1	
= $\frac{13}{16}$	A1 4	or 0.8125 or a.r.t. 0.813 only
Eg correct: $1 - 3\times\frac{1}{16}$; $1 - \text{(ii)} - 2\times\frac{3!\times 3!}{4!\times 4!}$ $\frac{3!\text{ x }3!\text{ x }13}{(4!\text{ x }4!)}$; $(^3/_4)^2 + 2\text{ x }{}^1/_4\text{ x }{}^2/_4$		Eg incorrect: $1 - \frac{3!\text{ x }3!\text{ x }3}{8!}$: M1M1M0A0 $1 - {}^1/_{16} - \frac{3!\text{ x }3!}{4!\text{ x }4!}$: M1M0M0A0
9 (i) $\dfrac{264 - \frac{90\times 15}{5}}{1720 - \frac{90^2}{5}}$ or $\dfrac{264-5\times 18\times 3}{1720-5\times 18^2}$	M1	Formula correctly used
= –0.06 AG	A1	–0.06 correctly obtained
$y - {}^{15}/_5 = -0.06(x - {}^{90}/_5)$	M1	or $a = {}^{15}/_5 - (-0.06)\text{ x }{}^{90}/_5$
$y = 4.08 - 0.06x$	A1 4	Complete equation correct
(ii) Substitute $x = 20.5$ ($y = 2.85$)	M1	Allow 20 ($y = 2.88$) or 20.49
Substitute $x = 19.5$ ($y = 2.91$)	M1	
	A1 3	Answer 0.06 or –0.06, c.w.d
2.91 – 2.85 = 0.06		
(iii) –0.6, 0.5	B1	–0.6 correct
	B1 2	0.5 correct
(iv) 1.5	B1	
Calculated equation minimises this quantity	B1 2	Not "Low value for Σe^2 means points near line"
(v) $\bar{e} = \Sigma e_i/5$	M1	$\Sigma e_i/5$ used
= 0	A1	Answer 0, cwd, cao
$\Sigma e_i^2/5$ (– her $\bar{e}$)^2	M1	$\Sigma e_i^2/5$
= 0.3	A1 4	0.3 only, must see -0^2 or – 0 in variance. ie: No working: $\bar{e} = 0$: M1A1; Var = 0.3: M1A0

1		Original list 6 5 9 4 5 2		Decreasing order can score method marks only
		After 1st pass $\boxed{5\ \ 6}$ 9 4 5 2	M1	For 1st pass correct with shuttle sort
		After 2nd pass $\boxed{5\ \ 6\ \ 9}$ 4 5 2	M1	For 2nd pass correct with shuttle sort or follow through from previous list
		After 3rd pass $\boxed{4\ \ 5\ \ 6\ \ 9}$ 5 2	M1	For 3rd pass correct with shuttle sort or follow through from previous list
		After 4th pass $\boxed{4\ \ 5\ \ 5\ \ 6\ \ 9}$ 2	M1	For 4th pass correct with shuttle sort or follow through from previous list
		After 5th pass 2 4 5 5 6 9	A1	For final list from correct shuttle sort, with results at end of each pass clearly shown
		May be shown vertically	$\boxed{5}$	
2	(i)	Number of arcs × 2 = sum of orders of vertices	M1	For a general method
		⇒ (3+3+4+4+4+4) ÷ 2 = 11 arcs	A1	For 11 calculated
				Drawing a specific case to get 11 scores B1 only
	(ii)	Semi-Eulerian, it has exactly two odd vertices	B1	For semi-Eulerian with avalid reason Accept 'two odd nodes'or 'two nodes of order 3' as minimal reasons
	(iii)	Complete graph on five vertices has only 10 arcs, so 11 arcs means that all six vertices are connected.		For a good explanation of the general case by considering orders of vertices A weak explanation may score B1
		Or, a vertex of order 4 must join to four others so five vertices are connected. The sixth vertex has order at least three and cannot connect to itself so it must join to the other five.		A diagram of a specific case is not sufficient
		Or any equivalent reasoning.	B2 $\boxed{5}$	
3	(i)	Minimum spanning tree with *U* removed	M1	For 43 or arcs *QR*, *RT*, *TS* or a convincing attempt to find minimum spanning tree for {*Q*, *R*, *S*, *T*}



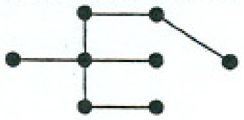
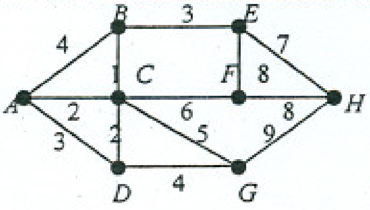
		QR + *RT* + *TS* = 43 miles		
		Join *U* back in using two shortest arcs 43 + 9 + 12	M1	For their 43 + 9 + 12
		= 64 miles	A1	cao (miles may be implied)
	(ii)	Trying to apply nearest neighbour method	M1	For a correct start to an application of nearest neighbour with any start vertex, ie at least: *QURTS*, *STRQU*, *TRQU* or *UQRTS*



		Start from *R*	M1	For *R* as start vertex (may be implied from cycle)
		to give *RQUTSR*	A1	For *RQUTSR*
		= 89 miles	A1 $\boxed{7}$	For 89 (miles may be implied) from valid method

4 **(i)**

	1st	3rd	2nd	4th	5th	7th	6th	8th
	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
A	-	4	2	3	-	-	-	-
B	4	-	$\boxed{1}$	-	3	-	-	-
C	$\boxed{2}$	1	-	2	-	6	5	-
D	3	-	$\boxed{2}$	-	-	-	4	-
E	-	$\boxed{3}$	-	-	-	8	-	7
F	-	-	$\boxed{6}$	-	8	-	-	8
G	-	-	5	$\boxed{4}$	-	-	-	9
H	-	-	-	-	$\boxed{7}$	8	9	-

4	**(i)**		Answer should be on insert
		M1	For starting by choosing row *C* in column *A*
		M1	For choosing more than one entry from column *C*
		A1	For a correct order (*A*), *C*, *B*, *D*, *E*, *G*, *F*, *H*
		B1	For correct entries chosen or a correct tree drawn 
	Quickest time is at least 25 hours	B1	For 25 Accept 'more than 25'
(ii)		M1 A1	For a correct graph drawn For correct weights shown
(iii)			Follow through graph, if possible, provided same conclusion is valid
	If *AC* is used then either *B* or *D* is excluded. <u>Or</u> must pass through *C* in getting between *B* and *D*, so *AC* is impossible.	B1	For explaining what happens if *AC* is used or why *AC* cannot be included.
(iv)			Follow through graph, if possible, provided same conclusion is valid
	If *EF* is not used then passing through either *E* or *F* will take the team to *H*, the team will not be able to visit both *E* and *F*.	B1	For stating the effect of not using arc *EF* or for considering all possible routes into *H*
(v)			Follow through graph, if possible
	A B E F C D G H	M1	For this route
	A D G C B E F H	M1	For this route
	The second route is quicker (32 hours compared with 36 hours)	A1 $\boxed{12}$	For identifying *A D G C B E F H* as the quicker <u>or</u> for calculating 32

5	(i)	$x \geq 0, y \geq 0$	B1	For both trivial constraints; allow >
		$y \leq 2x + 1$	B1	For this inequality, or equivalent; allow <
		$4x + 3y \leq 12$	B1	For this inequality, or equivalent; allow <
	(ii)	(0, 0), (3, 0), (0, 1)	B2	For these three vertices, any two correct ⇒ B1
		(0.9, 2.8)	B1	For this vertex exact, in decimals or fractions
		$(0, 0) \rightarrow P = 0$; $(0, 1) \rightarrow P = 3$; $(0.9, 2.8) \rightarrow P = 12.9$; $(3, 0) \rightarrow P = 15$	M1	For calculating $P = 5x + 3y$ for at least one of their vertices or clear evidence of using an appropriate line of constant profit
		$x = 3$ and $y = 0$	A1	For the correct values of x and y clearly identified
		$P = 15$	A1	For 15 clearly identified as the optimum value
	(iii)	Either consider the gradient of the profit line ($-\frac{1}{3}a$) and the gradients of the boundary lines (2 and $-1\frac{1}{3}$)	M1 M1 M1	One method mark for each appropriate gradient calculated correctly or for each appropriate value of Q calculated correctly
		or calculate Q at vertices ⇒ 3, $0.9a+8.4$, $3a$		
		Hence require $a \leq -6$	A1	For the correct set of values identified [$a = -6$ or any valid proper subset of the correct answer with no method shown ⇒ B1 only]
			$\boxed{13}$	
6	(i)	$5x + 3y - 5z + s = 15$ $2x + 6y + 8z + t = 24$	B1	For both equations correctly stated
	(ii)	$\begin{array}{ccccccc} P & x & y & z & s & t & - \\ \hline 1 & -2 & 5 & 1 & 0 & 0 & 0 \\ \hline 0 & 5 & 3 & -5 & 1 & 0 & 15 \\ 0 & 2 & 6 & 8 & 0 & 1 & 24 \end{array}$	B1 B1	For $\pm$(-2 5 1) in objective row Follow through from part (i) For 5 3 -5 1 0 15 and 2 6 8 0 1 24 or equivalent in constraint rows
		Pivot on 5 in x column	B1	For correct pivot choice for their tableau
		$\begin{array}{ccccccc} P & x & y & z & s & t & - \\ \hline 1 & 0 & 6.2 & -1 & 0.4 & 0 & 6 \\ \hline 0 & 1 & 0.6 & -1 & 0.2 & 0 & 3 \\ 0 & 0 & 4.8 & 10 & -0.4 & 1 & 18 \end{array}$	M1 M1 A1	For a correct method for their table and their pivot choice For increasing P For correct tableau or equivalent, cao
	(iii)			ft their tableau provided not yet optimal
		Pivot on 10 in z column	M1	For correct pivot choice
		$\begin{array}{ccccccc} P & x & y & z & s & t & - \\ \hline 1 & 0 & 6.68 & 0 & 0.36 & 0.1 & 7.8 \\ \hline 0 & 1 & 1.08 & 0 & 0.16 & 0.1 & 4.8 \\ 0 & 0 & 0.48 & 1 & -0.04 & 0.1 & 1.8 \end{array}$	A1	For correct tableau or equivalent, cao
		$x = 4.8, y = 0, z = 1.8$	B1	For all three correct values for their final tableau
		$P = 7.8$	B1	For correct value for their final tableau
	(iv)	We must now pivot on the 2 in the x column, this gives		
		$\begin{array}{ccccccc} P & x & y & z & s & t & - \\ \hline 1 & 0 & 11 & 9 & 0 & 1 & 24 \\ \hline 0 & 0 & -12 & -25 & 1 & -2.5 & k-60 \\ 0 & 1 & 3 & 4 & 0 & 0.5 & 12 \end{array}$	M1	For showing what happens to tableau, only need to show enough to be able to deduce answer (eg top row: 0 11 9 0 1 or y column)
		Hence $y = 0$ Accept 'no change to y'	A1	For correctly deducing $y = 0$ in general case. Only using a specific value of k (eg $k = 60$) with no general argument ⇒ M1, A0 Do not imply method mark from statement '$y = 0$' with no method seen.
			$\boxed{13}$	

7 (a) (i)	*A*: 1 0 *B*: 2 2; 2 *C*: 3 4; 4 *D*: 4 7; 8 7 *E*: 6 9; 12 11 9 *F*: 7 14; 15 14 *G*: 5 8; 8 *H*: 8 18; 20 18 *J*: 9 38; 39 38		Answer should be on insert
		M1	For correct temporary labels at *D* and *E* (condone extras here)
		A1	For all temporary labels correct **(with no extras)**
		M1	For value 38 at *J*
		A1	For all permanent labels correct
		B1	For the correct order of assigning permanent labels: *A, B, C, D, G, E, F, H, J*
	Shortest route: $A-B-G-E$ Length = 900	B1	For correct route and length. Accept route reversed and accept length = 9
	Shortest route: $A-C-D-F-H-J$ Length = 3800	B1	For correct route and length. Accept route reversed and accept length = 38
(ii)			Follow through from (i), if possible
	Length: 4700 metres	B1	For 47 or 4700
		M1	For $E-G-B-A$, or reversed, as part of a longer route
	$E-G-B-A-C-D-F-H-J$	M1	For $A-C-D-F-H-J$, or reversed, as part of a longer route
		A1	For whole route correct
(iii)	Explanation:		May be implied
	$G-B-A-C-D-F-H$	M1	For identifying that route will not visit every
	E and *J* will be left out (either is sufficient)	A1	vertex.
(b)	Odd nodes are *A, C, D, E, F, G* Need to pair *C, D, F, G* in the shortest way	M1	For trying to pair *C, D, F, G* (and no others)
	$CD = 3$ and $FG = 7 \Rightarrow 10$ ($CF = 10$, $DG = 11$ and $CG = 8$, $DF = 7$)	A1	For *CD*, *FG* or 10 (or 1000)
	Sum of all weights = 147	M1	For 147 (or 14700) or a good attempt seen or implied
	Length = <u>15700 metres</u>	A1	For 15700 metres (or 15700 m or 157 hundred metres or 15.7 km). But 157 ⇒ M1, A0
		$\boxed{17}$	

Pure Mathematics

Chief Examiner's Report

Two Pure Mathematics units for the new specification were examined for the first time in this session – Core Mathematics 1 and Core Mathematics 2. The numbers of candidates were boosted by candidates from Year 13 taking the opportunity to switch from the legacy specification to the new specification. Both papers evidently proved accessible to the majority of candidates and much excellent work was seen.

A significant feature of Core Mathematics 1 is that candidates are not permitted the use of a calculator and it is pleasing to note that this did not, in general, seem to cause a problem to candidates. There is no intention to set questions containing awkward numerical calculations but candidates who can calculate efficiently and accurately will obviously be at an advantage.

Centres should note that there is a new booklet – *Mathematical Formulae and Statistical Tables* (MF1) – for use with units of the new specification. The new booklet contains several of the formulae which candidates might need in answering questions in Core Mathematics 2.

4721: Core Mathematics 1

General Comments

The response to this first Core Mathematics 1 paper was encouraging. The majority of candidates made a reasonable attempt at every question and there was little evidence of problems caused by the lack of a calculator. Very many candidates scored well, with some scoring full marks although, in contrast, a sizeable minority appeared unable to cope with any question requiring knowledge beyond that of GCSE Mathematics.

Most candidates were able to demonstrate their algebraic skills at an appropriate level but even the more able candidates sometimes found it difficult to apply these techniques to the problem solving questions.

Candidates appeared to have had enough time to finish the paper.

Comments on Individual Questions

1) Most candidates were able to make a good attempt at this question, although marks were lost in a variety of ways.

    (i) The majority of candidates knew that 11^{-2} meant $\frac{1}{11^2}$ but they failed to score full marks because either they did not evaluate the denominator or they could not square 11 correctly.

(ii) Some candidates thought that they needed to square and cube root, rather than square root and cube. The wrong answer of 30 was seen rather too frequently.

(iii) Most candidates dealt with $\sqrt{50}$ correctly but found it more difficult to rationalise the denominator of the fraction. Some candidates multiplied the whole expression by $\sqrt{3}$ or by 3.

2) There were very few fully correct solutions to this question. Most candidates successfully found $q = 2$ but fewer realised that $r = 3$, with $r = 6$ being the most common error. Having correctly written $2(x-3)^2$ many candidates later stated that $r = -3$. Only the most able candidates successfully solved for p. The incorrect expression $2[(x-3)^2] - 9 + p$ was commonly seen. Some candidates tried to expand the right hand side of the identity, usually making at least one error, although a few had success with this method. A significant number of candidates failed to complete this question.

3) Most candidates lost marks on this question.

(i) Many candidates were unaware that the stretch factor needed to be applied to the x variable before square rooting, $10\sqrt{x}$ being the most common answer.

(ii) Although many candidates clearly recognised the transformation and correctly stated its direction and magnitude, a mark was needlessly lost because the transformation was not defined as a 'translation'. The words 'move' and 'shift' were often seen instead.

4) There were a pleasing number of perfect solutions to this question. The need to eliminate one variable was widely understood and, although some candidates chose the more complicated route of rearranging the quadratic equation to make y the subject, they were still generally successful. It was disappointing to see candidates making sign errors when expanding the expression $x^2 - 3(2x + 1)$ and also to note that a few candidates seemed unable to solve a quadratic equation.

5) Although the majority of candidates understood the meaning of the word 'sketch', they still felt that they had to use graph paper! A significant minority plotted a finite number of points and these candidates scored low marks on this question.

(i) Candidates who knew the general shape of the graph occasionally lost marks by allowing the ends of the graph to curve away from the asymptotes.

(ii) Only the ablest candidates found all 3 roots, most identifying $x = 0$ and $x = 1$ as the only x intercepts. Of those who did realise that there were 3 roots, many drew an x^3 curve with a horizontal portion incorporating all 3 intercepts. It was disappointing to see so many parabolas for this part of the question.

(iii) There were some good curves drawn, although many candidates repeated their curve in a second quadrant.

6) This question was answered well by many candidates.

(i) Some candidates failed to deal correctly with the double negative, ending up with 25, from which they followed on correctly for the number of real roots.

(i) Weaker candidates struggled with this part. Although many candidates knew that the discriminant must be zero for equal roots, having got as far as $(p + 1)^2 - 64 = 0$, they failed to solve the quadratic equation correctly. Some abandoned the question. A different approach of trying to work out p by inspection of the original equation rarely gained more than the 1 mark for $p = 7$.

7) Even the weakest candidates scored well on this question, demonstrating a good command of differentiation. Very many candidates scored all 9 marks.

(ii) It was widely understood that the brackets had to be expanded first and only a very few errors were seen here.

(iii) This part was more demanding but the only candidates who failed to score any marks were those who were unable to recognise that $\sqrt[5]{x}$ can be written as $x^{\frac{1}{5}}$.

8) Very few candidates scored full marks on this question. In a few cases, candidates confused perimeter and area. A few papers also showed failed attempts to multiply 13 by 23 correctly.

(i) This part was understood by most candidates although a few seemed unable to interpret the wording.

(ii) Some concise proofs were seen but a large number of candidates started with the given inequality and worked backwards. They often stopped at $x^2 + 10x < 299$, presumably thinking it self-evident that the area was $x^2 + 10x$. However, this was unacceptable in a question requiring a proof.

(iii) Most candidates solved the linear inequality correctly but appeared to struggle with the quadratic inequality. Many candidates failed to demonstrate a clear method and simply stated that $x < 13$ and/or $x < -23$. Others stated that, as a length could not be negative, $x < 13$. The solutions scoring well almost always involved a sketch graph. Candidates who tried to use an algebraic method usually made an error in their working.

9) (i) A sound understanding of differentiation was again in evidence in this part of the question, although weaker candidates often evaluated y and then used gradient = $\frac{y}{x}$.

(ii) A significant minority used a gradient of $\frac{1}{8}$, instead of -8.

Successful solutions to parts (iii) and (iv) were strongly linked. Few candidates understood the meaning of part (iii) and many just left out the last two parts of this question. Of those that did try part (iii), 6.003 or ‘less than 6.03’ were common wrong answers.

(iv) This part was usually done well by candidates who answered part (iii) correctly. Others tried the differentiation approach but were hampered by their lack of a value for the gradient. Some candidates started with one of the given chords, attempted to express the y coordinates in terms of k and hence calculated the value for k. However, this method proved successful only for the stronger candidates.

10) The first four parts of this question were well done by most, with many perfect scores.

The proof in the final part proved more demanding, although most of those who attempted it scored some marks.

(i) The negative numbers involved again caused a problem for weaker candidates. There was a sizeable minority whose working showed $\frac{-1}{-2} = \frac{1}{2}$, as they attempted to correct their original gradient after doing part (iii).

(ii) This part was well done but the final mark was too often lost because of mistakes when rearranging, usually omitting to multiply the 4 by 2.

(iii) Most candidates showed clearly how they knew that the lines were perpendicular, although a few simply stated that they were. A few candidates chose to find the lengths of all 3 sides of the triangle and then apply Pythagoras' theorem to show that the triangle was right angled, which was acceptable.

(iv) This was competently done with only a very few candidates adding the coordinates instead of subtracting them.

(v) There were many alternative methods in evidence in this part and some interesting and unexpected solutions seen. Many candidates failed to gain the first mark by assuming that DF was the diameter without justifying this in any way. Candidates who first wrote down the radius and centre and then used $(x - a)^2 + (y - b)^2 = r^2$ were generally successful. However, many candidates started with the given equation and worked backwards. This method tended to lose marks if the candidate did not link their rearranged equation to the points D, E and F in any way. Others, having used the given equation and identified the centre of the circle, went on to show that D, E and F were equidistant from it. A few candidates, mostly from the same small number of centres, used the formula for a circle linking the ends of the diameter. This gave a neat and concise proof. Weaker candidates merely showed that the coordinates of the 3 points fitted the given equation without any reference to a circle. Only a very small number of candidates appeared to run out of time during this last part.

4722: Core Mathematics 2

General Comments

This paper was accessible to the majority of candidates and the standard was generally good. There were very few candidates scoring low marks overall. There were a number of straightforward questions where candidates who had mastered routine concepts could pick up a number of marks. These included questions on the binomial theorem, the sine and cosine rules and simple integration. Some questions had more challenging aspects, which only the most able candidates were successful at answering. This was particularly true of parts of Qs 3, 8 and 9. Many candidates were reluctant to work with exact values, including surds, and this was obvious in Q7 in particular. Whilst some scripts contained clear and explicit methods, on others the presentation was poor making it difficult to follow methods used and to decipher answers given. On questions where the answer has been given, candidates must ensure that they provide enough detail to be convincing.

Comments on Individual Questions

1) This question was generally well answered, with the majority of candidates able to make a reasonable attempt at the question. Some candidates did use the binomial theorem, generally accurately, though there were a few instances of 3^3 becoming 9 not 27. Another common error was lack of brackets leading to only the *x* and not the 2 being squared and/or cubed. Some centres had obviously stressed the importance of clear presentation, including brackets, and their candidates generally produced accurate answers. A surprising number of candidates elected to expand the brackets rather than using the binomial theorem. There were some errors, but this method was often successful. Less successful were the candidates who attempted to expand all three brackets simultaneously – this was rarely, if ever, accurate and often resulted in a quadratic not a cubic. Only the most able candidates realised the connection between the two brackets and could change the relevant signs. The vast majority embarked on a second expansion, often with the same errors as before. This could still earn candidates the B1 mark.

2) (i) This part of the question was generally done well, with few errors. However, a small number of students did not understand the meaning of a recursive sequence and instead used $u_{n+1} = {}^1 / {}_{(1-n),}$ possibly misled by u_2 being given by ${}^1/{}_{(1-2)}$.

(ii) A number of candidates could state the correct value for u_{200}, but struggled to express their reasoning clearly. For examiners to award full marks there had to be a convincing and detailed explanation. Many candidates could identify the repeating sequence, but a surprising number thought that the period was 4 or even 5. A number of candidates attempted division by 3, but some still struggled to use this idea to decide which of the three terms u_{200} would be.

3) (i) The vast majority of candidates could use the sine rule accurately to find one, or usually both, of *LA* and *LB*. However, most then simply compared their answers and stated that *LB* was the shorter distance. In order to be certain that this was the shortest side, some candidates even found the length of *LC* at this stage as well. Only the most able candidates could identify the perpendicular from *L* to *AB* as being the shortest distance, but this was subsequently calculated accurately in virtually all solutions.

(ii) This part of the question was generally done well, with candidates being familiar with the cosine rule and able to apply it accurately. The majority considered triangle *LBC*, though some made sign errors when evaluating the expression because of $\cos 100^{\circ}$ giving a negative value. Some of the more able candidates, having found the shortest distance of 311m in part (i), then successfully used Pythagoras' Theorem in the relevant right-angled triangle. Full marks were awarded to those candidates who had used the cosine rule in part (i), and candidates who hadn't felt the need to use the sine rule in part (i) also gained credit if it then appeared as part of the solution in part (ii).

4) (i) To gain this mark, candidates had to verify both points in both equations, which a number failed to do. The alternative approach, which was seen fairly frequently, was to equate the two equations and then solve. For those who eliminated y, this resulted in a quartic equation and it was pleasing to see this being solved successfully on a number of occasions.

(ii) Virtually all candidates appreciated the need for integration and could make a reasonable attempt at doing so. Both expressions were usually integrated correctly, though there were a few slips on integrating $16x^{-2}$. Limits were usually used correctly, though a few candidates used 1 and 16, or substituted the wrong way round. The main problem for some candidates was deciding in which order to subtract the two curves. There were often errors if this was done as a first step, but it was always correct if done at the end of the question with two numerical areas. Candidates who did find a negative solution should have been able to appreciate from the question that this was impossible. A few candidates attempted an inappropriate manipulation of the two equations before integrating, usually resulting in a product or the quartic equation from part (i). Subsequent work with such expressions gained no credit.

5) (i) The majority of candidates could quote the appropriate identities, and usually attempted substitution at a relevant point. A surprising number of candidates found the algebraic manipulation difficult, especially when multiplying through by the $\cos\theta$ introduced as the denominator of $\tan\theta$. However, a large number of concise and accurate solutions were seen.

(ii) This part of the question was also usually done well. The majority of candidates could identify that a quadratic technique would be appropriate and could attempt the solution of the equation, usually correctly. They could then identify the two principal values, and many had no difficulty in finding the third value of θ as well. A few gave 330° as the final angle, and another error was to give further, incorrect, values for θ often as additional solutions to $\cos\theta = -1$.

6) (a) This question was generally very well done. Virtually all candidates appreciated the need to expand the brackets and could then attempt the subsequent integration. This was nearly always done successfully, though a few candidates failed to gain the final mark by omitting +c, or by leaving $\int$ or dx as part of their final answer.

(b)(i) This was also usually well done, though weaker candidates struggled to rewrite the expression using the correct power. If this was done successfully, most could then integrate to get an expression of the correct form, but the coefficient sometimes caused problems.

(ii) The more able candidates appreciated the relevance of the previous integration, simply substituted the point (4, 0) and provided an elegant and concise solution. A few candidates failed to express their answer as an equation and hence lost the final mark. However, many candidates saw this as a question on the equation of a straight line and proceeded accordingly, using a gradient of ½, coming from $^{dy}/_{dx}$. For candidates who made an error in part (i), full marks were available if they subsequently used their answer correctly.

7) (i) This part of the question was generally done well, with candidates able to identify the radius and angle in the sector, and then apply relevant formulae. The length of the arc was often given as a decimal, but many candidates could then provide convincing working for the exact value of the area of the sector. A few found the decimal area and then tried to argue that this was equal to 6π, which gained no credit. Some candidates worked in degrees, either successfully by using fractions of a circle or unsuccessfully by attempting to apply radian formulae.

(ii) It was pleasing to see that most candidates could attempt an appropriate method to find the required area, but the majority of candidates did not appreciate the need for an exact answer, or were not confident in attempting to do so, and gained only two marks of the four available. Candidates who found the area of a triangle by using Pythagoras were more likely to provide exact values, whereas those who used $\frac{1}{2}ab \sin C$ rarely converted $\sin {}^{\pi}/_{3}$ to an exact value and went straight to a decimal equivalent. A minority of candidates could not even make a reasonable attempt at the area of a triangle or the rhombus.

8) (i) There was a disappointing response to this question, with only the most able candidates gaining full marks. Often the graphs seen were not of the correct shape, and many existed only in the first quadrant. Even those candidates who drew correct graphs often omitted to state the coordinates of the points of intersection, gave answers involving a or b, or drew both graphs through (0, 1). On some occasions, the correct graphs appeared to come from selecting particular values for a and b, an approach that was condoned by examiners.

(ii) Whilst some excellent solutions were seen to this question they were unfortunately in the minority. Many could state the initial equation of $a^x = 2b^x$, and realised the need to take logs. Most then tried to do too many steps at once, resulting in a second line of $x\log a = x\log 2b$. A common error was for $\log 2b$ to then become $\log_2 b$, which meant that the method mark available for use of $\log a + \log b = \log ab$ was rarely awarded. Most candidates gained at least one mark from this question, but this was obviously an area of weakness for many candidates.

9) (i) This question produced a very varied response, with a number of candidates not even attempting the question or struggling to form the initial equation. However, there were also a number of candidates who could make a correct statement and hence show the required equation.

(ii) This was generally very well done, with the majority of candidates able to make a good attempt at finding the quadratic factor, a number doing so simply by inspection. Of the other methods used, equating coefficients was generally more successful than algebraic division, where sign errors were common. Most candidates showed that $(r - 1)$ was a factor by finding f(1), but a number of candidates did not attempt this. Those who used division, had to make it explicit that there was no remainder in order to gain the mark.

(iii) This part was well done, with the majority of candidates successfully finding the roots of their quadratic factor from part (ii). Subsequent manipulation of a correct answer betrayed insecurity when dealing with surds.

(iv) Most candidates gained one mark by substituting one of their values into the correct formula for S_∞, though some could not quote the correct formula despite it being given in the formula book. Very few candidates could identify the required value of r, and only the more able candidates appreciated the need to rationalise the denominator and could do so appropriately.

2631: Pure Mathematics 1

General Comments

Most candidates produced a reasonable overall attempt at the paper, with no individual question proving unduly difficult. However, untidy work, poor setting out and many careless mistakes resulted in a substantial loss of marks in certain cases. Although a 'follow through' approach is adopted in certain situations when producing the mark scheme, candidates should be aware that most questions have at least half their marks allocated to accuracy in some form or other. Hence consistently making errors, even if the correct method is adopted, can result in a considerable reduction in a candidate's final mark.

Comments on Individual Questions

1) (i) Common answers were 11/2, 2/11, $1/11^2$ and $11^{1/2}$.

(ii) Many candidates did not know how to rationalise $\frac{6}{\sqrt{3}}$ so -6√3 was a frequent answer, as was 50√3 + 6 following multiplication by √3.

2) This question was done very badly. Answers of $p = -9$, $p = 19$ and $p = -19$ were frequently seen, as was $r = -3$. Of those who obtained the correct answer many used unorthodox methods. Most of the errors arose due to the failure to deal with the sign of $2r^2$ when completing the square, usually adding the term onto p instead of subtracting it. Many omitted the factor 2, whilst others tried to include p within the completion of the square by writing $p/2$ inside the bracket, but then failed to return it to p when expanding the bracket.

3) This question was not answered well by most candidates. Only a few candidates obtained $\sqrt{2x}$, many having either $10\sqrt{x}$ or $2.5\sqrt{x}$. In addition, the constant 3 was added as many times as it was correctly subtracted. A few candidates even included the constant 3 within the square root sign.

4) (a) In many instances only one solution was given. A very common error was to change $\tan(2x) = 1$ to $\tan(x) = 1/2$. Others believed that the answers were 67.5 and 157.5, although usually with no working as to how these answers were obtained.

(b) Many reached $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$ but proceeded no further. Some decided that $\sin^2\theta + \cos^2\theta = 1$ must enter somewhere and tried to force it into the equation as opposed to allowing it to enter naturally in the numerator following use of a common denominator.

5) The question was generally done well, although attempts to solve for x with y still in the equation were often seen. Some took a very round about route but succeeded in the end. The factorisation and the use of the quadratic formula were undertaken much better than in previous years. However, careless algebraic errors were still present, namely $-3(2x+1)$ becoming $-6x + 3$ or $-6x - 1$, etc. The values of y were sometimes omitted.

6) (i) This was usually well done; some, however, used $\sqrt{b^2 - 4ac}$ or $b^2 + 4ac$.

(ii) The $p = -9$ solution was often lost and many gave the answers as inequalities, namely $p > -9$ and $p > 7$ or $-9 < p < 7$, despite having started with the condition $b^2 - 4ac = 0$. Several candidates tried to solve for x.

7) (i), (ii) There were a number of candidates who had problems with signs when finding the gradients. Some used $\frac{x - x_1}{y - y_1}$ whilst others altered the rule for perpendicular lines to fit their answers!

(iii) Most candidates found this straightforward but the equation was often not arranged in the correct form.

8) (i) This was usually correct although some found the value of *y*.

(ii) Candidates often failed to find the gradient of the tangent correctly, using 1/8 or 8 instead of -8, possibly not reading the question properly.

(iii) Most had no idea what was required. Many used ‘nearly’ or ‘close to’ 6.

(iv) Those who had the correct answer for part (iii) usually obtained $k = 3$. However, some avoided the need for the answer to part (iii) and instead validly employed the given gradients of the chords by using either $(y_3 – y_1)/(1.1 – 1) = 6.3$ or $(y_2 – y_1)/(1.01 – 1) = 6.03$.

9) (i) Most had no idea what a cubic graph looked like, often drawing quadratics or even quartics. They frequently failed to give the full coordinates and some had the *x* and *y* transposed.

(ii) Limits were often incorrect – probably because of incorrect graphs. Candidates found areas formed with the positive *x*-axis. Often results were fixed to give a positive area. However, the integration of $x^3 – x$ was usually undertaken correctly.

10) (i) This statement was usually correct, but some only found half the perimeter or used 10*x* for the length. Too many used = or $\geq$ instead of >.

(ii) Many tried to work backwards but failed to complete the process.

(iii) The majority failed to show a method, such as a sketch, for the solution of the quadratic inequality. Many also failed to show $-23<x<13$ as they simply disregarded the negative value with no reason. In spite of this the majority found the correct final inequality.

2632: Pure Mathematics 2

General Comments

This was a paper which enabled candidates to demonstrate their understanding and mathematical competence. No question presented widespread difficulty to candidates and there was particularly pleasing work seen on the binomial theorem, differentiation and sequences. Less convincing in general were attempts at questions involving integration and functions. Candidates seem to have had sufficient time to complete the paper and a considerable number of candidates showed confidence and ability in dealing with all the topics being assessed.

Several of the questions invited candidates to confirm given answers. Candidates usually find such given answers helpful. If their attempt seems to be correct, they can continue with confidence; if their attempt is wrong, they have the opportunity to look for their error. In exchange, candidates are expected to provide solutions which are sufficiently detailed and clear to leave no doubt that the answers have been genuinely obtained. Many candidates recognise this but others would benefit by addressing this aspect of their work.

Comments on Individual Questions

1) Most candidates realised that the answer to part (i) involved a natural logarithm although there were a few attempts involving x^{-2} or x^0. The majority of candidates provided answers to part (ii) involving $\mathrm{e}^{\frac{1}{2}x}$ but, in many cases, that answer was $2\mathrm{e}^{\frac{1}{2}x}$. Sometimes the cause was clearly inaccurate work with fractions but, in other instances, there was uncertainty about the process of integration. On this occasion, omission of the constant of integration was not penalised.

2) Part (i) was answered well and many candidates produced efficient, accurate work. A significant number of candidates proceeded beyond the correct answer $256 + 1024x + 1792x^2$ to produce $1 + 4x + 7x^2$, having divided each term by 256. As has been noted previously, candidates were possibly misunderstanding the request to simplify coefficients.

   Candidates did not answer part (ii) so well. A common error was 896, the result of failing to square $\frac{1}{2}$ in the term $1792(\frac{1}{2}y^2)^2$. Some candidates did not exploit the link with part (i) and started again with a new expansion.

3) Most candidates adopted a method for part (i) using the factor theorem to produce two simultaneous equations. There was a considerable amount of careless work in solving the equations with sign errors proliferating and 3^3 becoming 9 often enough to prompt comment from several examiners. A number of candidates equated f(–1) and f(3); this raised some doubt about understanding but, since it does lead to the correct values of *p* and *q*, full credit was allowed. An alternative approach involved consideration of coefficients in the identity $x^3 + px + q \equiv (x+1)(x-3)(Ax+B)$. Some candidates managed this successfully but careless work caused problems for others.

   The two marks available in part (ii) proved elusive for many candidates. In many cases there was no recognition that two factors of f(*x*) were already known and that therefore the third factor could be written down. Attempts to solve the equation $x^3 - 7x - 6 = 0$ involved fresh applications of the factor theorem or, in a few cases, use of the formula for the solution of a quadratic equation. There was considerable confusion between factors and roots with many candidates thinking they had concluded the solution when writing $(x+1)(x-3)(x+2)$.

4) It was pleasing to see many accurate solutions with appropriate use of the chain rule and correct evaluation. The differentiation of the expression in Model 1 was usually correct but there were more errors with the second expression and omission of the factor $4t^3$ was the common error. The values, correct to 3 significant figures, of the derivatives in Models 1 and 2 were 19.7 and 19.8 respectively. Some candidates decided that the implication of the question was that the values should be identical when given to their chosen degree of accuracy. This led them to state a value of 19.7 in the second case, whether their differentiation had been correct or not.

5) This question was designed to assess candidates' knowledge of two topics – the trapezium rule and the use of logarithm properties. The vast majority had no difficulty in gaining the two marks allocated to the former. However, in many cases, the working was carried out using decimals with a conclusion noting that $\frac{1}{2}\ln 104$ gave the same decimal value and this approach could earn no more than the two marks. Careful detailed work was needed to earn all five marks and, whilst many could demonstrate the use of relevant properties, a final step showing how $\frac{1}{4}\ln 10816$ became $\frac{1}{2}\ln 104$ was absent in many cases.

Many attempts at part (ii) were far from convincing and explanations were often poorly expressed. The best responses involved a sketch showing the curve and the two trapezia together with a comment pointing out the two regions accounting for the discrepancy in area.

6) Parts (i) and (ii) of this question revealed considerable uncertainty about the nature of functions. Figure 1 was often wrongly identified as the answer to part (i) perhaps with a comment about a modulus or a comment to the effect that functions never behave like this. On many scripts the answer given for part (ii) – Figure 3 because one *x* value gives more than one *y* value – was wrong but, if it had been given as the answer to part (i), would have earned the two marks.

The vast majority of candidates correctly gave Figures 2 and 5 as the answer to part (iii). Most candidates provided an acceptable sketch for part (iv) although some lost a mark because their sketch was not sufficiently convincing as far as the shape of the reflected part was concerned. In some cases where candidates had included their attempt on a copy of the given diagram, it was not always clear how much was intended as the required answer.

7) Most candidates answered part (i) correctly although some decided the value of *a* was $\frac{1}{4}$. It was very disappointing to see several attempts which involved $4y = 2 + \sqrt[5]{x}$ becoming $4y^5 = 2^5 + x$ followed by $x = 4y^5 - 2^5$.

The relevance of part (i) to part (ii) was missed by a number of candidates who proceeded to attempt $\int \pi y^2 \mathrm{d}x$. Most candidates did realise that $\int \pi((4y-2)^5)^2 \mathrm{d}y$ was required but this often led to $\int \pi(4y-2)^7 \mathrm{d}y$. Those progressing as far as $\int \pi(4y-2)^{10} \mathrm{d}y$ still had to integrate correctly and many did not do so. Some produced an integral involving $(4y-2)^9$ and another common error was an integral involving

$\frac{1}{11}(4y-2)^{11}$. The lower limit for the definite integral was occasionally incorrectly identified as 0. An exact answer was requested but a considerable number of candidates ignored this and gave the answer as 46.5π or 146.

8) Candidates generally provided sufficient justification for the result in part (i), usually starting with reference to angle *ADO* as one of the angles of an equilateral triangle. Having been alerted in the first part to the use of radians, most candidates were able to find the length of the arc *AC*. Finding the length of the arc *BC* proved slightly more troublesome because the size of angle *O* had to be established. With the required answer given, necessary detail was needed and this was generally in evidence.

The formula for the area of a sector was well known and applied as required in part (iii). Two aspects prevented many candidates from recording all four marks. Finding the area of the triangle was a problem for many; $\frac{1}{2}\times 6\times 6$ was a common error and other incorrect attempts involved $\sqrt{45}$. The second aspect concerned the requirement for the area to be exact; many candidates provided the approximate answer 22.1 cm^2. Others concluded with $12\pi - 18\sin\frac{1}{3}\pi$ but, for the award of the final mark, the replacement of $\sin\frac{1}{3}\pi$ by $\frac{1}{2}\sqrt{3}$ was needed.

9) The answer in part (i) was readily confirmed although some candidates did not provide sufficient detail. Merely identifying 37 and 1.728, but without showing how they had been obtained, was not enough to earn both marks. There were few problems with part (ii); candidates recognised that an arithmetic progression was involved and confirmed the result accurately.

Parts (iii) and (iv) presented more problems. One common error was to interpret v_p as the sum of *p* terms of the geometric progression. In part (iii), the expression $1.2\times 1.2^{p-1}$ for v_p was frequently simplified to 1.44^{p-1}. Although a few candidates adopted a trial and improvement method, the majority appreciated that the introduction of logarithms was an appropriate strategy. To earn the final mark in part (iii), candidates had to identify 57 as the answer but there were many answers such as $p <$ 57.65 and $p <$ 58. In part (iv) candidates were faced with a quadratic inequality and solving this was a problem for some. Using the formula for the solution of a quadratic equation did not seem to occur to them and various invalid ruses for finding *q* were tried. Again not all candidates appreciated the need for the answer to be a positive integer.

Questions on arithmetic and geometric progressions are usually answered well and, although this question had its challenging aspects, there were many very good solutions. Indeed, some candidates who had not produced particularly convincing work elsewhere in the paper did score well on this final question.

2633: Pure Mathematics 3

General Comments

Able candidates found few problems in gaining a very respectable mark on this paper, and only a minority failed to score at least half the available marks. Examiners were heartened by the degree of understanding and competence displayed by candidates; basic ideas and techniques were addressed with confidence and skill.

Only in the second question and the final two parts of the ultimate question did a large proportion of candidates find real difficulty, and in Q8(ii) (b) and (iii) most struggled to find appropriate methods of solution.

Work was invariably legible and well presented, and there was no evidence of candidates running out of time.

Comments on Individual Questions

1) Few candidates failed to score full marks. Occasionally incorrect forms were used for the coefficients of the term(s) in x^2 and/or x^3, or errors were made in simplifying coefficients.

2) Very many candidates used $\left(-\theta\sin\theta\right)$ as the derivative of the product $x=\theta\cos\theta$. Later, the value of $-\pi^{-1}\cot\pi$ was unusually then set equal to zero or to -5.8 (approximately), when it is actually non-finite.

3) (i) Most candidates scored all three marks, though often preferring to use degrees rather than radians. A few solutions featured an approximate value of *R*, or had $\tan\alpha=3$ or $-1/3$ instead of $+1/3$.

(ii) Although most candidates obtained the correct value for one solution, many did not seek a second solution for θ or simply subtracted the first one from 2π °.

4) (i) Several correct techniques were seen, but many candidates could not successfully address the proposition or even failed to attempt to do so.

(ii) Using the result from part (i), most candidates integrated successfully. A sizeable minority, however, integrated $x\left(x^2+1\right)^{-1}$ to obtain a constant multiple of $x\ln\left(x^2+1\right)$.

(iii) After a correct integration by parts, candidates generally used the result from part (ii), except for those whose key integrand was $x\left(x^2+1\right)^{-1}$ or $x^2\left(x^2+1\right)^{-1}$, instead of $x^3\left(x^2+1\right)^{-1}$.

5) (i) Only a few incorrect solutions were seen, and these were due to use of vector OP or OQ, instead of vector PQ, to specify the direction of the line.

(ii) Often the ratio $QS:PS$ or $PQ:QS$ was found. More seriously, a few candidates used $\left|\overrightarrow{PQ}\right| = \left|\overrightarrow{OP}\right|.\left|\overrightarrow{OQ}\right|$.

(iii) A minority used $\overrightarrow{OP}$ or $\overrightarrow{OQ}$, instead of $\overrightarrow{PQ}$, as one of the two vectors. Reference was also made to $|\underline{a}|.|\underline{b}| = (a+b)\cos\theta$, or $= ab\sin\theta$.

6) (i) Most candidates scored the first three marks, but the majority then re-wrote the preliminary result $y^{-1} = x^{-1} + C^{-1}$ as $y = x + C$. Weaker solutions involved $\int y^2 dy$ instead of $\int y^{-2} dy$ or lacked a constant of integration.

(ii) An appropriate solution was obtained by almost everyone, albeit usually on a follow-through basis.

7) (i) Candidates usually noted that $\mathrm{d}x = 2x^{1/2}\mathrm{d}y = 2y\mathrm{d}y$, but some solutions then foundered through use of $\mathrm{d}x = \frac{\mathrm{d}y}{\mathrm{d}x}.\mathrm{d}y$.

(ii) A minority of candidates wrongly attempted to express $\frac{1}{x\left(1+\sqrt{x}\right)}$ in the form $\frac{A}{x} + \frac{B}{1+\sqrt{x}}$, later being unable to integrate $\left(1+\sqrt{x}\right)^{-1}$. Most candidates, however, used the y integral from part (i), though some forgot to change the limits from $y = 4, 9$ to $y = 2, 3$. Occasionally errors were made in simplifying the correct four-term solution into a 1 or two-term format.

8) (i) A few candidates lost a mark by using an inappropriate value for the square of the radius.

(ii)(a) Some candidates lost an accuracy mark, using mx^2 for m^2x^2 or by making a sign error. As in part (i), almost everyone scored both marks, however.

(b) Few candidates scored any marks. It was required to use the discriminant, a function of *m*, for the quadratic equation in x from part (ii)(a), but instead attempts were made to find the two roots of the x-equation or to differentiate the equation, sometimes setting the derivative equal to zero. The key principle here was that, for the line to meet the circle, the discriminant, equal to $\left\{4(m+8)^2 - 4.49\left(1+m^2\right)\right\}$, must be greater than or equal to zero.

(iii) Only occasionally was a viable method seen. One option was to note that the values of *m* from part (ii)(b) were the tangents of the angles BOx and AOx. The required angle AOB being the sum of the two latter angles, its tangent then follows by the usual formula. The other option, essential for those unable to solve part (ii)(b), was to show that $OC = \sqrt{65}$, and hence $OA = 7$ and thus $\tan A\hat{O}C = 4/7$, where C is the centre of the circle.

2634: Pure Mathematics 4

General Comments

The majority of candidates found the examination accessible, answering questions in the order set. There appeared to be little evidence of problems with time. Candidates were better prepared and able to produce good solutions to the more standard types of question asked. The first three questions gave most candidates a sound start to the paper, and candidates were able to pick up marks in most questions. The result was fewer poor scripts. However, it appeared that very high marks were seen less frequently, often as a result of a lack of precision. In particular, candidates should know that an answer given in a question must be clearly derived and not merely written down as 'obvious'.

Examiners commented on the overlong answers produced by many candidates, when an absence of accuracy or simplification led to marks being lost. Candidates continue to omit "dx" or "dθ" in integrals, even when they have used a substitution in their integration. However, it was also pleasing that candidates were able to deal successfully with questions such as 6 or 7 which, although standard, were different from questions set in the past.

Comments on Individual Questions

1) This was well answered and gave candidates a good start. The majority recognised that an integrating factor was required, and e^{-x} was derived in most cases. Candidates who produced e^x instead were still able to gain 3 marks. Some candidates do not appear to appreciate the link with the product rule, and errors such as $-ye^{-x} = \int e^{2x}\,dx$ were common. Others failed to multiply the RHS by e^{-x}. The most common error was to either omit the constant of integration or to put it in at the end as an afterthought. Some candidates knew and could apply the C.F./P.I method.

2) This was found to be a very straightforward question, particularly with the hint given. The vast majority produced reasonable Maclaurin series for e^{-x} and $\sin 2x$ separately, and then multiplied these together. Errors were mainly algebraic, although a surprising number of candidates omitted the cubic term in the expansion of $\sin 2x$ at the multiplication stage. Other candidates who attempted Maclaurin on $2\sin x \cos x$ were also successful but wasted time. Those who attempted to derive Maclaurin by differentiating $e^{-x} \sin 2x$ repeatedly were less successful.

3) This was a standard induction question on summation, for which candidates were well prepared. Nevertheless, there was a general lack of appreciation of what constitutes a proof and a general lack of precision in presenting it. However, many candidates scored at least 4 marks. Examiners were looking for candidates to bring together the facts that it was true for n =1 and that **if** true for $n = k$, then it has been shown true for $n = k + 1$. There were too many statements such as "Assume $n = k$" or "Let $n(n + 3) = ⅓\, n(n + 1)(n + 5)$". The best presented solutions often came from candidates who showed it was true for n=1 **after** the assumption and proof for n=k leading to n=k + 1. A few candidates used standard results for $\sum r$ and $\sum r^2$ and gained no marks.

4) (i) It was surprising how many candidates omitted the –2 in the differential, and this obviously led to problems in part (iii). The Chain Rule did not appear to be well-known, with even the more successful candidates using a substitution to get the required result.

(ii) Many candidates had apparently not seen a substitution of this type before and did not use the ½ sin θ accurately to produce cos θ in the denominator. Other candidates lost marks by omitting the constant of integration (a possible lead-in to part (iii)), or by leaving their answer in terms of θ.

(iii) With the problems experienced in the first two parts, it was difficult for candidates to do the 'hence'. The best solutions involved differentiating the given expression with a as a variable and producing da/dx = 0, leading to the given answer. Candidates using an integration approach often produced a = 0 as either the constant of integration was omitted or $\int$ 0 dx was given as 0. Even so, many candidates produced $a = \frac{1}{2}\pi$ for one mark. Overall, few candidates scored more than 4 marks on this question.

5) (i) With the answer being given, candidates needed to give some numeric evidence that r = 0 for the given θ. Others solved the equation and produced the required θ as one of the solutions of the trigonometric equation. The sketch often lacked clarity, being too small and lacking the detail required, such as some idea of scale (r = 1 when θ = 0 or the tangent at $\theta = \frac{3}{8}\pi$). A rough copy off a calculator gained one mark without such detail.

(ii) There were many good solutions to this part, involving a variety of methods. Only a minority recognised $\sin^2 2\theta + \cos^2 2\theta = 1$, being more prepared to use the equivalent expressions in terms of cos 4θ for both $\sin^2 \theta$ and $\cos^2\theta$. Even so, although time was wasted, such candidates usually gained at least 4 marks. The term in 2sin 2θ cos 2θ produced solutions via sin 4θ, substitution, recognition and even parts! It was pleasing to see that the question did not throw candidates, and that they could answer such a question accurately.

6) (i) Most candidates picked up the marks, although the time difference between those who used the cover-up rule and those who solved simultaneous equations after equating coefficients must have been substantial.

(ii) Again, this was generally done well, with most candidates using sufficient terms at the start and end of the summation to ensure that they could spot the correct cancelling. As this occurred every other line, candidates dealt with this situation well. Candidates were not penalised for attempting to simplify a correct expression, but often this led to a more difficult part (iii).

(iii) There was a general lack of accuracy in determining the convergence of the series where some notion of 'as $N \rightarrow \infty$, $1/N \rightarrow 0$' was expected. Candidates getting part (ii) wrong could still gain the marks for their sum to infinity.

7) (i) Designed so that candidates with graphical calculators did not have too much of an advantage, this part of the question produced many good solutions. The majority of candidates used the a + ib approach, producing a quadratic in a^4 or b^4, whilst others used a form of De Moivre. A very small number produced correct answers with little or no working, and full credit was given. Marks were generally lost because of algebraic errors rather than a lack of knowledge.

(ii) Again, marks were lost when no evidence of scale was seen.

(iii) The word ‘transformation’ did not appear to be well-known. Even candidates who were aware of what was required too often transformed point *A* to point *B*, usually as a translation. Designed to test the ‘geometrical effects’ of multiplying two complex numbers, candidates who recognised ‘transformation’ rarely equated this to an enlargement plus rotation. Marks were awarded for any two transformations which worked, as long as they were expressed exactly. These included ‘reflection in $\theta = \frac{1}{4}\pi$’ and various shears. Those who knew the connection between the moduli and arguments of z and z^2 could gain some credit if they were related to some idea of transformation.

8) (i) Candidates who expressed y in partial fractions lost time. Minor algebraic errors were allowed if they led to the correct asymptotes.

(ii) With the answer to be gained given in the question, many candidates lost marks by going from $y(y + 8) \geq 0$ or $y(y+8)>0$ to the answer given without any detail at all. If it was not clear that they had correctly reached the required solution, two marks were usually lost. A sketch or table of values would have been useful at the end of this part.

(iii) Most candidates selected the one correct asymptote. Minor algebraic errors and not writing the answer as a coordinate as requested led to some marks being lost. Candidates obtaining a different $y = k$ asymptote could still score 2 marks here.

2635: Pure Mathematics 5

General Comments

There was a wide range of candidates with marks ranging from 60 to 11. The majority were well prepared and had a sound knowledge of the topics tested; the level of algebraic and other manipulative techniques was pleasingly high. However, as on previous occasions, presentation was frequently scrappy and many candidates obviously felt little pride in their work. It is, of course, noticeable that, in general, those who present their work well are likely to produce more accurate sets of solutions.

Comments on Individual Questions

1) (i) This was completed correctly by almost every candidate although there were a few who turned it into a convoluted form of manipulation.

   (ii) Many candidates obtaining $y^2 = 2$ or 3 lost concentration and failed to use the negative square roots.

2) There appeared to be over-reliance on graphic calculators and some failed to give the final answer with sufficient appropriate working.

3) The substitution technique was pleasingly accomplished and no-one just substituted d*u* for d*x*. The majority used the cosh 2*u* formula correctly and then substituted back – a few made mistakes at this final stage or left the answer in the unacceptable form of $\sinh(2\sinh^{-1}2x)$.

4) The final part of this question (use of Euler's method) was well done but parts (i) and (ii) were poorly answered. It appeared that candidates had rushed without thinking and just put down the first thing which came into their heads. The three-line statement at the beginning had often been only cursorily read and some relevant aspects were missing from their answers.

5) (i) A few wrote $(\alpha+\beta+\gamma)^2 = \alpha^2+\beta^2+\gamma^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$ and some tried to produce an instant equivalence for $\alpha\beta+\beta\gamma+\gamma\alpha$ but the majority did well.

   (ii) Occasionally a sign error intruded here.

   (iii) This was probably one of the weakest parts of the paper. A considerable number of candidates failed to realise that the equation, found in part (ii), should be used in conjunction with the factor theorem to find the roots. Many started again, using the symmetric functions and attempted eliminations although this rarely resulted in the equation $\alpha^3-3\alpha^2-5\alpha-1=0$ being obtained.

6) (i) This was well answered, almost everyone realising that integration by parts was involved. It was pleasing to note that, although scrappy careless presentation was often seen elsewhere in the paper, no candidate failed to show the limits clearly on his/her '*uv*' portion of the integration.

(ii) A few were unable to make use of the suggested method and did not realise that the integral should then be separated into two parts – but the majority did well.

(iii) Almost everyone evaluated I_2 satisfactorily; one or two made the expected error in finding I_0 or I_1 in that the limits of 0 and 1 in conjunction with a power of (1 – *x*) combined to produce a negative answer.

7) (i) Given the availability of graphical calculators, this part was surprisingly badly done.

(ii) Again, the substitution technique was well carried out and the given result obtained by most.

(iii) There were many cases of misreading here and the arc-length or the area of the surface of revolution were frequently attempted; those working in the correct direction generally obtained the given answer satisfactorily.

8) Part (i) was generally done well. Most candidates knew what to do in part (ii) but their work was sometimes spoiled by inaccuracies, particularly at differentiation.

2636: Pure Mathematics 6

General Comments

The entry for this session was small because the majority of centres use the whole of Year 13 to prepare their candidates for this unit. A pleasing number of candidates demonstrated their ability to answer all or most of the questions in a competent manner, perhaps having started work for the unit in Year 12. However, a sizeable number were unable to make much headway with many of the questions, although they seemed to have some familiarity with the topics tested. Some of the topics in the Specification are encountered for the first time in this unit and much practice is needed in order to be able to answer questions on them. Q6, involving the sum of a complex series, was the most demanding question, but many candidates tackled it in the manner expected. The paper appeared to be of the correct length, as there was no indication that candidates had either run out of time or had much time to spare.

Comments on Individual Questions

1) (i) The majority of candidates realised easily that the matrix **M** represented a rotation, and most stated correctly that it was in the anticlockwise sense. But a very common error was to give the angle as $\frac{1}{3}\pi$ rather than $\frac{2}{3}\pi$; and a few answers gave two different transformations, despite the indication in the question that only one transformation was involved.

   (ii) The answer to part (ii) depended on the angle given for the rotation in part (i), and this was usually answered correctly.

2) The majority of candidates answered parts (i) and (ii) correctly. Part (iii) gave a vector equation which would have been unfamiliar to many, and they were expected to use their knowledge of the vector product to understand that the points with position vector $(\mathbf{r}-\mathbf{a})$ must be parallel to the vector **n**. This clearly shows that the points lie on a straight line parallel to the normal to the plane. The third mark was obtained by stating that the line passes through the point with position vector **a** on the plane, but only very good candidates scored full marks on this question.

3) It is pleasing to report that the ability of many candidates to understand the procedures involved in group algebra has improved since the last time this aspect was tested.

   (i) Most answers showed that multiplication on the left and right by the inverses of *a* and *b* isolated the element *x* and gave the solution. There were few candidates who did not know how to start.

   (ii) Nearly all answers started by removing the brackets on the right hand side. As the answer was given it was necessary to demonstrate clearly that multiplication on both sides by inverse elements then gave the required result.

4) (i) Most candidates were able to demonstrate this standard result correctly.

(ii) This is an example of a question where practice in techniques is essential. In the Specification there are two types of result involving multiple angles and powers, but they require different approaches. In this part an expression for a power of sinθ in terms of sines of multiples of θ was requested, and the neatest method is to start by expanding $(z - z^{-1})^5$, as suggested by the result of part (i). Those answers which used this method were usually correct, with only the occasional algebraic error. However, some candidates used instead the expansion of $(\cos\theta + i\sin\theta)^5$, which is more suited to finding sines or cosines of multiples of θ in terms of powers of sinθ or cosθ. The required result can be found by this method, and a few such answers scored full credit, but the majority who tried this approach got no further than an identity for sin5θ, not realising that they had to find a similar expression for sin3θ as well, in order to complete the question.

5) (i) Candidates are well used to expanding a 3x3 matrix and solving the resulting equation in the case where the matrix is singular, and the first part of this question was done well.

(ii) However, the technique for solving simultaneous equations in the case when the answers have to be expressed in terms of a parameter appeared not to be well known. Frequently candidates correctly eliminated one variable between two of the equations, but were then unable to make further progress. Quite often several correct equations in two variables were found, but there seemed to be a lack of understanding that one of the variables should be changed to a parameter and the other two variables found in terms of that parameter. Some candidates clearly did appreciate the geometric aspect of the situation, as they gave their answers in the form of the cartesian equation of a straight line. Provided the resulting three fractions were put equal to a parameter, this was accepted, although the more explicit form where x, y and z are each found in terms of the parameter is to be preferred in this instance. The alternative method of using a vector product was not often seen.

6) (i) The expressions for z^2 and z^3 were usually found correctly, with the most common error being to omit to square and cube the modulus.

(ii) Those who answered this part correctly drew a diagram which indicated that the points were the first four points of a spiral type of locus. But in many cases there seemed to be little appreciation of the geometrical effect of adding successive powers of z, whose moduli decreased and whose arguments increased by equal amounts. A generous mark scheme allowed one mark to be given if there was some indication that the points lay on an anticlockwise path starting from the positive real axis, but some of the candidates who thus benefited had probably not appreciated the spiral aspect.

(iii) A good number of candidates realised that the series was geometric and the standard formula for the sum could be used. The simplification to the form given required some dexterity, but it was not difficult to apply the stages shown in the mark scheme.

(iv) It was pleasing to see that even those candidates who had been unable to do part (iii) realised that they could answer part (iv) correctly by letting $\frac{1}{2^{6n}}$ tend to zero.

7) This question tested various vector techniques, leading to an appreciation of the fact that the shortest distance between two skew lines is the length of their common perpendicular. The points *P* and *Q* were, in fact, the end points of the common perpendicular, but this was not obvious until part (iv) was reached.

(i) Although nearly all candidates wrote the correct vectors **b** and **c**, errors in the vector **a** were common, with many finding the vector joining *P* or *Q* to the mid-point of *PQ*.

(ii) The majority of candidates knew how to do this part and correct answers were often seen, although some arithmetical errors were made.

(iii) A common error here was to find a unit vector normal to the plane, rather than to use the position vectors of *P* and *Q*. As *P* and *Q* are on the normal, the answer thus obtained may have been correct, though it was more often in the wrong sense.

(iv) The intention of this part was that it could now be seen that, because *PQ* is in the direction of the normal, the shortest distance is simply the length of *PQ*, which had already been found in part (iii). Many candidates used instead the familiar formula for shortest distance and this was allowed, as it could be argued that they were still using a 'hence' approach. The last mark was awarded if reference was made to *PQ* being perpendicular to both lines.

8) This question tested the properties of a group of order 6 which may already have been familiar to some candidates, but no prior knowledge of it was assumed. Most parts of the question were done well, although there were a number of elementary algebraic errors which should not occur at this level; for example, 'simplification' of $\frac{1}{1-x}$ to $\frac{1}{1}-\frac{1}{x}$ was seen on several occasions.

(i) Most answers were correct, although the required intermediate step was not always evident.

(ii) Most candidates realised that they had to show that $\mathrm{fff}(x)$ was equal to x, and this was often done correctly. There was some incorrect identification of x with the identity, but this was not penalised.

(iii) Most answers stated the elements e and h, although some gave e, f and ff, which is the subgroup *F*.

(iv) There are various combinations of the given elements which lead to the last two elements of the group *H*, some requiring less manipulation than others, but any working which led legitimately to a correct answer was, of course, allowed. A few candidates wrote the answers down without showing any working and this was not penalised if the elements were correct; but it should be remembered that incorrect answers with no working score no marks.

4728: Mechanics 1

General Comments

The majority of candidates displayed a good understanding of the Mechanics concepts covered in this specification. There was evidence that candidates were well prepared for the examination, many of them gaining high marks. There were few poor scripts.

Comments on Individual Questions

1) (i) The correct answer was obtained by almost all candidates. Exceptions included those who had taken the weight of the box to be 100g N or 10g N.

   (ii) Most candidates were aware of the need to show their method when working towards a given answer.

   (iii) Inaccuracy was sometimes present as a result of unnecessarily finding the angle α in degrees and then rounding its value before using it in the equations for P. Answers in the form of inequalities did not score full marks.

2) While there were many completely correct answers, a large number of candidates had problems with signs. A diagram showing the positive direction for momentum would have been found useful. In part (i), some answers were left as v = - 3, with the speed not given as a positive quantity.

3) (i) This was generally well answered, although there was a significant number of errors made in finding complementary angles; for example, 60° instead of 50° being given as the complement of 40°. Problems also arose as a result of confusing sine and cosine when resolving, and inconsistent expressions such as X = 2 x 8sin30° - 5sin40° were occasionally seen. While some candidates considered only two of the forces, attempting to find their resultant using the cosine rule, others treated the resultant as a fourth force in equilibrium with the three given forces, leading to the negative answers X = - 10.6 and Y = -3.83.

   (ii) The magnitude of the resultant was usually found correctly but the angle made by the resultant with the positive x-axis was frequently given as 19.9° rather than the correct 19.8°. This arose as a result of using rounded values of X and Y from part (i). Some candidates had drawn a diagram in which the resultant was incorrectly represented by a line joining the ends of the X and Y components leading to an error when finding its direction. The angle made by the resultant with the x-axis was occasionally calculated erroneously as $\tan^{-1}$(X/Y).

4) This was the best attempted question with full marks being very common.

   (i) The majority of candidates differentiated to find the acceleration, the main error being the loss of the first term, i.e. giving the answer as just $a = 0.2t$.

(ii) The value of t when a = 2.8 was usually correctly found and although most candidates went on to integrate the given expression to find the distance, there were a few who wrongly attempted to use equations that apply only in the case of constant acceleration.

5) (i) There was a sizeable number of incorrect expressions given for the heights above ground of A and B after time t, such as $s_A = 7t + ½ gt^2$, $s_B = 10.5t + ½ gt^2$; $s_A = ½ (7+0)t$, $s_B = ½ (10.5+0)t$; or even just $s_A = 7t$ and $s_B = 10.5t$ without g being introduced at all.

(ii) There was a surprisingly large number of sign errors made when subtracting the expressions for the heights of A and B found in part (i), and the answer $3.5t - 9.8t^2$ was particularly common, leading to problems later in the question.

(iii) The simplest way to find the difference in heights was to find the time when A was at its maximum height and use this in the answer to part (ii). Most candidates obtained t = 0.71 or better but sometimes this was by a circuitous route, e.g. finding A's maximum height and then solving the equation $2.5 = 7t - ½ gt^2$. An alternative method of finding the difference between the heights was to find B's height at time t = 0.714 and subtract A's maximum height from it, thus obtaining 5 – 2.5 = 2.5. There were quite a number of errors which could arise from an attempt at this latter method, one of which was to find the difference between A's and B's maximum heights.

(iv) The best solutions to this part were those in which candidates solved $3.5t = 3.5$, using their answer to part (ii), to obtain t = 1. Comparison of this time with the time when A was at its highest point, or calculation of A's velocity and noting a negative value, then enabled many candidates to reach the correct conclusion that A was travelling downwards. Errors in answers to part (ii), however, caused significant problems here. Some candidates used a method that involved a comparison of the differences in the heights of A and B, e.g. comparing 3.5 with the value of $h_B - h_A$ of 2.5 when A was at its maximum height. An essential ingredient of the argument here should have been that the expression $3.5t$ for the height difference increases with time, but this was rarely stated.

(v) This part of the question was often successfully attempted.

6) There were very few errors made in the straightforward parts (i) and (ii). More problems occurred in parts (iii) and (iv), although these more challenging exercises were pleasingly well answered in many cases.

(iii) The most frequently seen wrong answers included a graph for Q that was identical to that for P, and a graph for which the speed was constant (with v = +8) for $0 \leq t \leq 16$, and decreasing uniformly to zero for $16 < t \leq 20$.

(iv) There were many correct graphs, but those of some candidates took the form of a continuous curve instead of having a straight section and a curved section. A common wrong answer was a graph that was the reflection of Q's graph in the line t = 10 rather than in the line x = 4.

(v) Many candidates were under the impression that the cyclists would meet at 'half-time', after 10s. There were many good attempts at finding the correct time of 11s, either by writing down equations for *t* or by realising that the cyclists would each have travelled 72 m, subtracting the 16m they travelled while accelerating in the first 4s and then calculating that the remaining distance of 56m would take a further 7s at a constant speed of 8ms^{-1}. An incorrect answer of 7s was quite frequently seen, resulting from the failure to add on the time for the first stage of the journey.

7) This more difficult question was very pleasingly answered by a large number of candidates.

(i) There were many completely correct solutions to this part. Errors included not finding the component of *B*'s weight down the plane, and occasionally, the omission of the frictional force on *A*. In the majority of cases when the frictional force was considered, it was nearly always correctly calculated. Some candidates preferred to use the 'complete system' method but frequently failed to use the total mass of 0.5 kg in their equation. Calculation of the value of *T* was omitted by quite a large number of candidates.

(ii) Many candidates did not take account of the fact that the acceleration would change after the string had broken and continued to use the same value as that found in part (i).

(iii) Most candidates who made an attempt to find the distance travelled by B used the equation $s = ut + ½ at^2$ with the relevant time, but commonly seen errors included using their value of acceleration from part (i), or simply *g*. Some candidates wrongly assumed that *B* would be stationary after travelling this distance and attempted to use the equation $v^2 = u^2 + 2as$ with v=0.

2637: Mechanics 1

General Comments

The level of achievement of candidates covered a wide range. There were some extremely good quality scripts seen by examiners. However, some candidates appeared to be entered before an appropriate standard had been reached.

Examiners commented that some candidates had problems expressing values less than 1 correct to 3 significant figures.

Candidates should be reminded of the need to show sufficient working when asked to obtain a required answer as in Q1(i) and Q6(iii).

Comments on Individual Questions

1) (i) This question was generally well answered, most candidates showing a good understanding of the concept of conservation of momentum. However, there were a few who did not take into account the vector nature of velocity, or were inconsistent with the use of signs, but still managed to get the given answer.

   (ii) This part was less successful for many candidates. Problems arose from an attempt to use velocity before and after the collision in constant acceleration formulae. Others did not use the relationship given in the first part to find *b*, but went back to a momentum equation, making some errors.

2) (i) Most candidates gained full marks.

   (ii) This was found to be more difficult. Most errors occurred as a result of the friction being assumed to be the same as in the previous part when in fact it should have been the coefficient of friction that remains the same. Of those who made correct mechanical statements, many found the algebraic manipulation difficult in attempting to find *P*.

3) (i) This question was well answered by the majority of candidates. The major error seen was incorrect use of sine or cosine in resolving forces.

   (ii) A variety of methods were used to answer this question. Use of vectors and components were the most successful. Attempts at combining two of the forces first were less successful. However, some candidates also worked out a direction when only the magnitude was required, which must have taken up valuable time. Others left the resultant force in component form, making no attempt to combine the components.

4) This question was not particularly well answered, with only a few totally correct answers seen by examiners. The use of constant acceleration formulae was disappointingly common. Some candidates who recognised integration was required in the first part used constant acceleration in the second part. Some attempted to differentiate, while others had problems dealing with the fractional indices.

(i) Many of those who used integration to solve the question either omitted a constant of integration, or did not use the fact that velocity was 60 m s^{-1} at time 9s, but merely used $v = 0$ at $t = 0$.

(ii) Some candidates did not use a value that they had found in part (i), but assumed that this constant was now 0. Hence it was common to see an incorrect answer of 194.

5) (i) Most candidates had the graph for the outward journey correct. However very few correct sketches were seen for the return. Most ignored the fact that the velocity on the return was negative. Although many such candidates had a correct shape, the graph was nevertheless above the *t*-axis instead of below it. Others had a sketch for the return below the *t*-axis but with the graph returning towards the *v*-axis.

(ii) The majority of candidates realised the need to find the area between the graph and the *t*-axis. Some however attempted to find the whole distance including the return. There were also a few candidates who attempted to use constant acceleration formulae with the same acceleration throughout.

(iii) Many candidates were aware of the method needed to calculate the required time but many careless errors were seen. These included solving $0.5t = 8$ to get $t = 4$ for the time accelerating on the return journey, and using distance divided by speed for the whole of the return journey. Some thought that the return journey took the same time as the outward journey. It was disappointing to see some wrong answers without any working seen.

6) Although some good solutions were seen, some confusion was evident in attempting to assign consistent signs to velocities and to the acceleration due to gravity.

(i) A common error was to assume the particle was at rest after 0.8s.

(ii) Many assumed that *A* and *B* had both travelled 8.864 at $t = 0.8$ or that they had the same speed.

(iii) Some good attempts were seen to reach the given answer. The time for *A* to travel to its highest point was found by most but some did not realise the time to come down again was the same, so made further calculations which in some cases led to errors. Inevitably candidates contrived to get the difference in times to be 1.5s even from incorrect work.

7) It was interesting to observe how many candidates felt it necessary to work out α.

(i) Two distinct methods were seen. Those who employed a whole system approach were usually less successful, omitting either the weight component of *A* or friction, or using an incorrect mass. Attempts at applying Newton's second law to the particles separately were better, but some thought *B* was not accelerating and had the tension in the string to be $0.32g$. A common arithmetic errors was $0.1a + 0.32a = 0.33a$.

(ii) Of the attempts seen, most were able to use their acceleration correctly to find a distance.

(iii) Very few correct answers were seen. Generally most either failed to calculate a new acceleration, using instead the acceleration found in part (i), or *g*. Many of those who made an attempt to find the acceleration failed to take account of the friction.

2638: Mechanics 2

General Comments

The majority of candidates showed a reasonable understanding of all topics, except for resolution of forces and taking moments, which were essential to the answering of Qs 5 and 7. Candidates very rarely showed that they were short of time. There were many excellent papers and only a small percentage of the candidature appeared to be totally unprepared. As general hints, candidates must attend to the precise requirements of questions, make clear diagrams and use sensible symbols for forces. In problems which involve friction, it is useful to distinguish between the actual force of friction (F) and the maximum possible frictional force (perhaps F_{max} is a good choice for this).

Comments on individual questions

1) A straightforward first question. However, fewer than half of all candidates gained full marks. The first mark was often lost through not recognising that speed is a scalar quantity. In part (ii), many failed to take account of the change of direction, and more than a third of candidates did not give a direction for the impulse on the sphere.

2) Most candidates scored full marks in part (i). A small number of candidates confused the situation by unnecessarily calculating volumes. Part (ii) was less well answered.

3) Part (i) was well answered. Errors in part (ii) included using values of the driving force or resistance from part (i), and ignoring resistance.

4) There were many correct answers to part (i). However, a significant number of candidates did not multiply by cos30°, or included potential energy. In part (ii) it is pleasing to note that very few candidates tackled the problem by using Newton's second law thus ignoring the request to tackle the problem by considering energy. In considering energy, common errors were to fail to multiply R by the distance 25m and/or to have sign errors in the energy equation.

5) This was the least well attempted question and many candidates failed to score any marks. However, surprisingly, some candidates were more successful in the more complicated part (ii). A very common error in part (i)(a) was $R = 500g\cos 15°$ and in part (b) $R = 500g$. For the majority the combination, in part (ii), of circular motion, friction and resolution of forces was too complicated.

6) This question was generally well answered, particularly parts (i) and (ii). The impact of two spheres in part (iii) was unusual and many candidates over-complicated finding the speed of the spheres immediately after the collision. The methods used for calculating the speed and direction of motion of a sphere on hitting the ground were well understood. In only a minority of cases did candidates attempt to use displacements for finding the direction of motion on hitting the ground.

7) This question often caused difficulties. A common misconception was for the normal reaction forces at the ground and wall to be perpendicular to the ladder. There were many general problems with taking moments and with the use of sine and cosine. In part (i), there was greater success in finding the reaction at the ground, as opposed to the reaction at the wall. In part (ii), the vast majority failed to respond to the first request, 'find the frictional force at the ground'. Showing that the ladder was not in

danger of slipping was often unconvincing as candidates frequently continued to use their symbol for the reaction force at the wall. Many candidates continued to use the same reaction forces from part (i) in part (iii). Fewer than ten per cent of candidates were successful in this final part. Those candidates who were successful were usually those who scored more than 50 marks for the whole paper.

2639: Mechanics 3

General Comments

The many good scripts seen demonstrated the high level of skill and mathematical understanding of the majority of candidates. Nearly all scripts contained answers to every question, and many answers were entirely correct. Reference is made below to the most common errors that were seen. As will be seen, far more marks were lost through inaccurate work than through ignorance of the subject matter.

Where the question paper contained given answers, candidates were expected to refer clearly to the underlying physical principles involved, and carry out algebraic and arithmetic manipulations without ambiguity. In questions where there was no printed result, examiners could accept a less rigorous presentation of a candidate's work.

Comments on Individual Questions

1) Candidates who used a momentum or velocity triangle together with the cosine rule had the greatest success with this question. This was the most succinct approach, with fewest opportunities for error.

   Solutions that analysed the problem in terms of components tended to include some mistake such as the omission of mass from one or more terms, or have sign errors when calculating momentum change. Taking components parallel and perpendicular to the impulse (rather than the initial velocity) appeared to be the more successful choice with this method.

   The most effective method to find speed from components of velocity is the use of Pythagoras' theorem. However, finding the direction of motion and then working with a velocity component was commonly seen, and gave more scope for error. (This comment applies equally to Q3.)

2) (i) In a number of cases, candidates simply asserted that the acceleration was equal to g, from which the result swiftly followed, but full credit was not given. In other cases, the formula $mv^2/a = R - mg\cos\theta$ was quoted without justification, accompanied by the statements $\theta = 180^{\circ}$ and $R = 0$.

   (ii) In this more substantial part of the question, the candidates produced more convincing explanations. The energy change was clearly presented, and Newton's second law was generally used explicitly.

3) Most candidates understood this topic very well, and many excellent solutions were seen. Sign errors in the momentum and restitution equations were rare, and where marks were lost this was usually the result of errors in algebra or arithmetic. As in Q1, a significant number of candidates calculated the direction of motion of *B* and then deduced the speed of the sphere. Simple use of Pythagoras' theorem would have been adequate, and more convincing in those cases where a component of velocity was negative.

4) (i) This question was unusual in requiring the summation of the magnitudes of two forces, one tension, the other compression. The more common scenario, involving the calculation of the difference of two tensions, was clearly present in the minds of many candidates, for whom this was their sole error.

(ii) Virtually all candidates were able to continue successfully with their solution to part (i).

5) (i) Though many correct answers were seen, solutions often contained errors such as using the same trigonometric ratios in both terms of the moments equation, or omitting a distance from one of the terms.

(ii) A common problem for both candidate and examiner was the use of a diagram that showed four arrows attached to a single point *B*, like four compass points. The ambiguity could lose marks in part (ii), but also created confusion in many candidates' minds when working on part (iii).

(iii) Diagrams which showed clearly the magnitude and direction of forces at *B* on the rod *AB* alone were usually accompanied by the correct working. However, a significant number of candidates chose to take moments about *A* for the entire system, and these more complex expressions were easily marred by minor errors, or the consequences of premature approximation.

6) (i) The basic mechanical principles and calculus techniques needed to answer this question were well understood. The candidates who simplified their initial equations by eliminating decimal numbers and working with integers produced fluent solutions culminating in the printed result. Solutions worked with decimals contained much evidence of amending earlier steps after the printed answer had failed to emerge.

(ii) Though the majority of candidates knew how to proceed by integrating the printed result, answers were often wrong. Sometimes this was because the value of the integral was assumed to be zero when $t = 0$. However, many scripts showed candidates unable to evaluate $e^{-0.2x2.5}$.

Answers based on the use of constant acceleration formulae were frequently seen, and some candidates re-worked the problem using the $v\mathrm{d}v/\mathrm{d}x$ formula for acceleration.

7) (i) Though candidates understood well how to find the mass required, at times their work was imprecise. The ‘3.6’ in the question paper indicates an exact value, and numerically accurate working is needed. If an approximate value of 3.60 was intended it would have been clearly stated.

(ii) Nearly all candidates knew how to calculate the energy stored in an elastic string, and how to set up the energy balance equation. There were many instances of incorrect answers arising from the calculation of the energy for a single string. There were very few attempts, none successful, at a solution based on variable acceleration.

(iii) The most commonly overlooked assumption was that the strings were light. Some candidates scored only one mark by claiming both that there was ‘no air resistance’ and that ‘energy is conserved’. Examiners regarded these statements as essentially equivalent.

Probability and Statistics

Chief Examiner's Report

On all statistics units much excellent work was seen. As usual questions requiring verbal answers and explanation tend to be the least well answered. Although these units are part of a Mathematics specification Statistics is meaningless without application to the real world. The number of candidates who state modelling conditions and the conclusions to hypothesis tests in the context of the question (as opposed to repeating standard slogans such as 'they are independent') is also increasing. On the other hand there is some evidence that there is always a substantial minority of candidates who lose marks by forgetting aspects of the subject that are not explicitly in the specification but are part of assumed knowledge – for instance many still think that '0.77' is an answer correct to 3 significant figures.

4732: Probability and Statistics 1

General Comments

Almost all candidates found opportunities to display their knowledge and understanding and there were a large number of very good scripts.

In some cases, the written answers were less satisfactory than the calculations. Some candidates quoted, parrot-fashion, general statements making no reference to the context. These scored no marks. Other candidates wrote unnecessarily long explanations. Adequate written answers, in context, can always be given in a very few words.

A few candidates gave numerical answers without working. Perhaps they used the statistical functions on their calculator. In these cases, wrong answers automatically scored 0. Candidates should be encouraged to check twice every calculation carried out using these functions. In cases where the answer is given in the question, then the use of statistical functions, without written working, scores no marks.

Some candidates used the less convenient versions of formulae from the formulae booklet, e.g. $\sum(x-\mu)^2 p$ and $\sum(x-\bar{x})(y-\bar{y})$. Many candidates appeared to make no use of the formulae booklet, but preferred to use formulae of their own devising, usually incorrect.

There was evidence of some confusion between $\sum x/n$ and $\sum xp$.

A small number of candidates appeared to have rushed the last question or omitted parts of it, suggesting that they had run out of time.

A few candidates ignored the instruction on page 1 and rounded their answer to fewer than three significant figures.

Comments on Individual Questions

1) (i) The correct answer, *A*, was often supported by inadequate explanations such as ‘strong correlation’ or incorrect explanations such as ‘because the points are close together’. Reference to near-linearity was required. A few candidates thought that a poor fit to a straight line implies a high value of *r* and so chose *C*. Others thought that the correlation is highest where the line of best fit is the steepest, and chose *B*.

(ii) A few candidates chose *B*, stating that the points follow no particular pattern. Many correctly chose *C*, but failed to note the clear correlation that exists but is not linear.

2) (i) Some candidates used 15/2 etc. rather than 16/2 etc., and gave answers such as 7.5 instead of 8 for the median. Others found the median at the 8th data point, but then went on to find Q_1 at the (8+1)/4th data point. This happens to give the correct answer for Q_1, but a similar method for Q_2 at the 11.5th data point, gives $Q_2 = (12+24)/2 = 18$. Another common error was $Q_2 = 26$, presumably from counting along the penultimate row in the wrong order.

(ii) Mention of ‘outliers’ or ‘extreme values’ (not ‘anomalies’) was required. Some unacceptable explanations as to why the median is preferable were: because of the wide range of the data; because it is easier to calculate; because there are equal numbers of boys and girls; because the mean is not an integer and because the mean is not one of the original data.

(iii) Most candidates understood the main point here. However in some cases, when a particular advantage was described, the answer did not make clear which type of diagram had this advantage. A few candidates wrongly stated that a box and whisker plot shows the ‘trend’ of the data. Answers such as ‘it is easier to draw’ were not accepted. A few candidates gave unacceptably imprecise answers such as ‘a box and whisker plot shows the spread of the data better’. Another answer which was not accepted was that a box and whisker plot shows the skewness of the data better.

It was clear that to a few candidates the idea of statistical diagrams having advantages and disadvantages was new. They had to use their initiative and generally did not score marks.

3) (i) Most candidates succeeded in finding r_s correctly although a few found differences using the original data rather than ranks.

(ii) Answers such as ‘good correlation’ or ‘good rank correlation’ did not score. Mention of ‘agreement’ or ‘similar’ or equivalent was required. A common error was to state that the commentators agree on 75% of the marks.

4) (i) Most candidates knew how to find *k*, although a few made arithmetical errors.

(ii) Arithmetical errors were not uncommon in this part also. A few candidates divided by 5. Some candidates did not understand the notation $\sum x^2 p$ and found $\sum (xp)^2$ or $\sum xp^2$. Others failed to subtract μ^2. Some thought that $-(-0.1)^2 = +0.01$. A few candidates used their own incorrect version of the formula for variance, apparently unaware of those given in the formulae booklet.

5) (i) Some candidates used a binomial method in part (a) which is incorrect. In part (b), common incorrect solutions were: $\left(\frac{19}{20}\right)^9$, $\left(\frac{19}{20}\right)^{11}$, $1-\left(\frac{19}{20}\right)^{10}$, $\left(\frac{19}{20}\right)^{10}\times\frac{1}{20}$. Some candidates used the very long method of $1 - P(1 \le X \le 10)$. A few candidates used B(10, $\frac{1}{20}$) which in this part is a correct alternative to G($\frac{1}{20}$).

(ii) Almost all candidates gave the correct answer. A few understood 'expected' to mean that they should try to find, by trial and error, which number of people was the most likely.

6) (i) A few candidates omitted the binomial coefficient. Others just found $\frac{3}{8}\times\frac{5}{8}$.

(ii) Some candidates stated $\frac{1}{2}\times p = \frac{3}{8} \Rightarrow p = \frac{3}{4}$ without an intermediate step. This was inadequate because the answer of $\frac{3}{4}$ was given in the question. For full marks a step such as $p = \frac{3}{8}\div\frac{1}{2}$ was required.

(iii) This part was answered well on the whole.

7) (i) Many candidates gave a description of some or all of the conditions which give rise to a binomial distribution (fixed number of trials, only two outcomes, constant probability of success). Others stated that the values of n and p need to be known. Others gave correct conditions, but not in context (trials are independent, probability of success is constant). No marks were awarded unless the conditions were given in context. A common incorrect answer was 'boxes are selected at random'.

(ii) The majority of candidates used the formula rather than tables in both parts (a) and (b). Most did so correctly, but a few used $p = \frac{10}{42}$ or $\frac{1}{42}$. In part (b) common incorrect methods were P(Y‹2), P(≤2) and P(Y›2). Sometimes the coefficient was omitted.

(iii) The most common error was to omit '× 2'. Some candidates added their answers from part (i). Many failed to use their answers from part (i) but started again. Some candidates used their answer to parts (a) or (b) as the value of p in a new binomial distribution. A few used n = 16.

8) (i) Much calculation with 8! and either 6! or 7! was seen. Some candidates failed to multiply by 2. Some had a numerator of 2 x 7 instead of 2 x 7!. A few candidates calculated only the number of arrangements, without proceeding to the probability.

In parts (ii) and (iii) candidates who used direct probability methods rather than arrangements produced much more efficient solutions, which were often correct.

(ii) The denominator of 4! x 4! was frequently correct although 8! was common. Incorrect numerators such as 2 x 3! or 1 x 1 or just 2 were common. Others added 4! + 4! and/or 3! + 3!. Only a few candidates avoided all the above confusion about arrangements and used a direct probability method, $(\frac{1}{4})^2$.

(iii) Some candidates identified the three cases which were not required, sometimes correctly finding (ii) + 2 x (ii) even if (ii) was wrong. Some candidates listed the 13 required cases correctly, but then calculated $^{13}/_{4!\times 4!}$. Others listed the categories as 'separated by 2 or by 3 or by 4 etc.', but then just counted these without reference to the different numbers of arrangements which give rise to them. Only very few candidates realised that by considering probabilities directly, only the positions of the two relevant questions need to be considered and the other six questions can be ignored.

9) (i) Some candidates calculated S_{yy} or even r. The summary data given in the question was intended to make the calculation of b simple, using $S_{xx} = \sum x^2 - \left(\sum x\right)^2 / n$ etc. Many candidates ignored this help and used $\sum\left(x-\bar{x}\right)^2$ etc. Some candidates thought that $\sum\left(x-\bar{x}\right)^2$ meant $\left(\sum x-\bar{x}\right)^2$, i.e. $(90 - 18)^2$. Others used $\left(\sum x\right)^2 - \left(\sum x\right)^2 / n$. Some candidates who made an error in finding b persisted in using their value rather than the given value in the rest of the question. A common error was losing the sign of b when substituting to find a.

A few candidates showed no working. Since the value 0.06 was given in the question, they scored no marks for finding this value, but they could still gain partial credit for the final equation.

(ii) Common errors were to substitute 20 or 20.4 or 20.49 or 13 and 23.

In parts (iii), (iv) and (v) it appeared that most candidates just did what they were asked mechanically, without realising that the e values are the residuals.

(iii) Candidates who arrived at huge values for e_4 and e_5 appeared unmoved by the discrepancy between these values and the given values of e_1 etc.

(iv) Some candidates calculated $\sum e$. Hardly any candidates stated that the line of regression minimises the value of $\sum e^{2}$. Some candidates stated that if $\sum e^2$ is small then correlation is good – which does not answer the question.

(v) Few candidates realised that ē must be 0 because of the definition of the line of regression. In finding the variance, some candidates squared their value of $\sum e^2$ found in part (iv). Others used incorrect formulae such as $\sum e^2 - (\sum e)^2$.

A few candidates (even after finding ē correctly), tried to use $\sum xp$ and $\sum x^2p$ for variance and calculated 1 x e_1 + 2 x e_2 + 3 x e_3 etc. and 1 x e_1 + 2^2 x e_2 + 3^2 x e_3 etc.

2641: Probability and Statistics 1

General Comments

The overall standard of the candidates on this paper seemed to be quite good. However there were fewer outstanding candidates than usual. This may be due to there being an unusually large number of parts of questions requiring written answers. These were generally answered less well than numerical parts. Many candidates are still not giving written answers in context. Few candidates seemed to run out of time.

Comments on Individual Questions

1) (i) This question was answered correctly by most candidates from across the ability range. Some careless arithmetical errors were seen, e.g. 1-0.63=0.27.

(ii) Again, this part was usually answered correctly. Those who lost marks made mistakes in one of the probabilities to be added. A few multiplied the two correct probabilities. Although time-consuming, tree diagrams helped many candidates to answer both parts correctly.

2) (i) This question was answered correctly by most candidates. $^{22}P_{11}$ was the most common incorrect answer.

(ii) This question was answered correctly by many candidates. However, many candidates considered only $^{11}C_6$ or $^{11}C_5$. If they did consider both, many added them. Some produced $^{11}C_6$x$^{11}C_5$, but did not divide by their answer to part (i). Those using a probability method usually scored two marks for (11/22x…x6/17)x(11/16x…x7/12). Very few realised that this needed to be multiplied by 11!/(6!5!).

3) (i) Many did not use probability tables. Of these, many considered the five correct probabilities, but premature approximation cost them the final accuracy mark. Of those who did use tables, F(10)-F(6) was a very common error. Others used F(10)-(1-F(5 or 6)).

(ii) Most candidates answered this part correctly. Only a few left out $^{23}C_{10}$.

(iii) Few gave a valid reason in context.

4) (i) Most answered this part correctly. A few ranked the two sets of data in opposite directions. Some ranked only one set of data and compared with the student numbers.

(ii) Most answered this question correctly. This part produced the best written answers on the paper.

(iii) Few gave a valid reason. The obvious ‘six is a very small sample’ was rarely seen.

5) (i) Most, but by no means all, produced the correct frequencies. This is surprising, as the 3rd frequency, 50, was given. Far fewer produced and used the correct mid-points. Some used upper class boundaries, far more the class widths.

(ii) There was much confusion about the sum of ft^2. $(ft)^2$ was a common error. Most knew that the square of the mean should be subtracted. A few forgot to take the square root. Premature approximation cost many the final accuracy mark. A significant number split the final class into two, viz 6-8 and 8-10, each with a frequency of 9. This produced the correct answer in (i), but produced an inaccurate s.d.

6) (i) The given answer produced the usual crop of spurious answers. The correct method had to be based on (1+1+4)/16. Some who were on the right lines did not consider all four cases of HT matchings.

(ii) Most realised that the distribution was Geometric. Better candidates used q^2. Those trying 1-P(2 or less) often included P(3) as well.

(iii) Most produced 8/3. Less than half mentioned mean or average to score the final mark.

7) (i) Most obtained the median correctly. A variety of methods for finding the quartiles were used, and allowed, even though they produced answers as diverse as 1.4 and 2.4. A minority produced the quartiles but did not subtract them.

(ii) Almost all candidates used graph paper, and usually scored full marks.

(iii) Most scored the marks available for a comment on variability, but fewer considered skewness. Of these, most scored the mark, but some confused positive and negative skew.

8) (i) This question was answered correctly by many candidates. Most candidates realised that they had to try to solve the simultaneous equations, but some of the algebra seen was confused. Some attempted to use the answer given in part (ii).

(ii) All candidates who answered part (i) correctly scored this mark as well.

(iii) This question was answered correctly by most candidates.

(iv) Most used y on x and produced the correct answer. Many did not score the mark for justification of choice of line by stating that x was a controlled variable.

(v) Most realised that their estimate was unreliable due to the low value of r.

2642: Probability and Statistics 2

General Comments

This paper contained several hard questions, while the easier ones were found very accessible. There was no indication that candidates were short of time.

Two errors seemed more widespread than in the past: in significance tests it is important to use the theoretical value as the null hypothesis and in calculations, rather than the data value; and hypotheses were often stated poorly. These are not merely pedantic points; the whole theoretical basis of significance testing depends on the correct selection and use of hypotheses. It is not insisted that parameters used in statements of hypotheses are defined, but if they are not (as is usually the case), the standard notation is required.

Candidates tend to reveal limited thought processes in their answers to verbal questions. Such questions generally require understanding and overview, rather than manipulation of numbers or expressions; many candidates seem unable to think about such questions except at a numerical level.

Comments on Individual Questions

1) Most found this an easy start, apart from the many who wrote P(>3) = 1 – P(≤2). Quite a number calculated probabilities by hand instead of using the tables.

2) The most important reason why the results obtained from a self-selecting sample are biased is of course that only those who feel strongly about a subject are likely to fill in a questionnaire about it. The *Radio Times* did exactly this some years ago and duly reported that about 90% of their readers were seriously interested in environmental issues.

   Some centres have noted that the specification requires knowledge of exactly one type of sampling, namely simple random sampling from a sampling frame using random numbers. Many have not, and a large number of scripts showed no more than a rough idea of what is meant by random sampling. It is again emphasised that selecting from a hat, or equivalent, is not acceptable. In this context, as the most important reason for the original method being biased was that only those who were interested would be likely to respond, it was a pity that more candidates did not include an attempt to overcome this problem, such as contacting each member of the sample personally.

3) The first part of this question was generally very well answered, although the rider was not. As in the general comment above, far too many try to think in terms of whether calculations are possible or ‘accurate’, instead of considering the shape of the distribution – which is essentially what modelling questions are about.

4) This was probably the least well-answered question. First, there was the usual high proportion of candidates who do not seem to have learnt how to do hypothesis tests for discrete distributions and invariably reach for the normal distribution (which is not valid here as nq = 4). Some used the correct distribution, but found P(= 10) rather than P(≤ 10). The words ‘at least three-quarters’ caused difficulties for many candidates who were unsure whether to consider a left-hand tail, a right-hand tail or both. And, as noted in the General Comments, there were many who took H_0 to be p = 10/16 and found the probability of 12 or more.

5) Most answered part (i) correctly. However, in part (ii) there were many who calculated $P(Y < 22)$ or $P(Y \le 22.5)$ instead of $P(21.5 \le Y \le 22.5)$, and there was the usual large number of errors with continuity corrections.

Many had forgotten how to calculate a percentage error.

6) Most could explain what was meant by a Type I error (though it is not a 'probability'). However, the scenario for part (ii) confused many candidates, who usually attempted to bring in the value $\mu = 25$. Only the better candidates realised that the key to whether or not the outcomes of the tests had to be independent was to be found in the use of a binomial distribution.

7) Those who knew what to do had no difficulty in finding q in part (i), and many excellent solutions were seen. Others were way off target.

Part (ii) showed that candidates struggle to combine the demands of curve-sketching with the requirements of a probability density function. Some drew the curve going below the x-axis; few drew curves that plainly had the same area as that of $f_1(x)$. A large majority of those who recognised the curve as a sad parabola drew it with its vertex well to the right of the y-axis; they may have done too many questions where the pdf is of the form $kx(2 - x)$, or they may have thought that the k factor was equivalent to an x-factor in requiring the curve to go through the origin. Explanations in both parts (ii) and (iii) again showed widespread misunderstanding of the meaning of the pdf; weaker candidates reached for the familiar techniques of integration.

8) The first two parts of this question were often found straightforward by competent candidates, although there were the usual omissions of the $\sqrt{n}$ factor, and here again many attempted to work with null hypothesis $\mu = 36.6$. This can give an equivalent set of calculations but is not acceptable.

The last part was intentionally testing. Only the best candidates were able to produce a clear answer in terms of the scientists' intentions; most were limited to vague comments such as 'they want to be more accurate'.

2643: Probability and Statistics 3

General Comments

The entry for this paper was significantly larger than in January 2004 but most candidates were able to approach most of the questions with accuracy and confidence.

Presentation was generally good and in only a few cases was handwriting very difficult to decipher.

It is worth reiterating that, when candidates are asked to explain an answer which is given on a paper (Qs 6 and 7(iii)), sufficient detail must be provided. This was not always seen.

The questions on hypothesis testing were generally well answered but sometimes candidates are overassertive in stating their conclusions. Many do state 'Accept H1', 'there is evidence that $\mu < 3.5$', but often something like 'Reject H0, so $\mu < 3.5$' appears. The former is preferable.

Comments on Individual Questions

1) Many were able to earn full marks for the question but some reversed the hypotheses and others miscalculated the number of degrees of freedom. It is hoped that in hypotheses tests a statement will be given as to why the null hypothesis is rejected, in this case $X^2>5.991$. This was usually seen but some candidates merely give the critical value.

2) (i) This was generally well-answered. The binomial distributions were mostly recognised and candidates could proceed successfully to E(*F*).

   (ii) Most candidates then approximated the two binomial distributions to Poisson distributions leading to a Po(5).

3) The general performance was very high on this question. Most could negotiate the integrals in part (i) successfully and the relevance of the answer to part (ii) was usually seen. Some did, however, successfully carry out the long integration.

4) The procedure for the confidence interval was usually known but some did not know the correct formula for $\hat{\sigma}^2$. It does appear, as S^2, in the formulae booklet, but in two parts.

   The symbol μ was sometimes used instead of $\overline{x}$ which caused some difficulty.and some candidates used a *t* rather than *z* critical value. This was acceptable if the value used was between the two relevant tabular values.

   The reason for the use of the CLT in part (iii) was that the normality of the population was not given. Many stated that *n* was large or that the variance was unknown and there were often misconceptions about the CLT.

   The final part was mostly answered correctly.

5) As was expected, this proved to be a testing question, but most candidates could score some marks. For part (ii) it was hoped that F(1.5) would be compared with 0.5 but many obtained a cubic equation for the median. Some then either solved the equation using their graphical calculator, or obtained a suitable interval. This was a lot of work for 2 marks maximum.

Only the most able could obtain the result in part (iii) which required correct use of V and v.

Parts (iv) and (v) could be done by those who scored little in the previous parts.

6) (i) Many candidates could fit the Geo(½) and carry out the requisite χ^2 test. Some candidates had E-values which totalled 127.5 but this made little difference to the test statistic. It was gratifying to see that most candidates knew to combine classes which were too small.

Several candidates attempted a *z*-test of $\mu_x = 2$ against $\mu_x > 2$, but this requires a normal population and is not a goodness of fit test.

(ii) Some appealed to the results in part (iii) but a majority considered the O and E values to decide that the coin was biased towards a tail.

(iii) The answers being given, the two values 304 and 128 needed convincing reasons, which were not always forthcoming.

(iv) The method for finding the confidence interval was usually known but variance estimate of $\hat{p}$ was often incorrect.

7) (i) This was a straightforward question and often well carried out. Some candidates gave the critical region as $|t| > 2.015$, rather than $t < -2.015$. The former is incorrect and can lead to an incorrect test conclusion.

(ii) Many knew that the population variances should be equal, but some thought independent samples were required. It is not easy to conceive how the given samples could be otherwise, so this (without the common variance) was not accepted. The formula for pooled variance is given in the formulae booklet but it requires unbiased estimate of the population variances. These are most easily obtained from the SD mode of a calculator.

(iii) Most candidates realised that 0.5 was well inside the confidence interval and so the data is consistent with the given statement.

4736: Decision Mathematics 1

General Comments

A good spread of marks was achieved. There were only a few exceptionally good scripts, but equally there were few exceptionally poor scripts. Most candidates were able to complete the examination in the time allowed, although a few wasted time copying out the diagrams or tables from the insert.

In general, the quality of the presentation of the candidates' answers was good. A few candidates submitted messy work that was poorly organised; invariably these candidates were unable to read their own working and incurred a consequent loss of marks.

Comments on Individual Questions

1) This was a straightforward start for most candidates, although a few candidates used bubble sort instead of shuttle sort. Some candidates did not clearly identify the list resulting from each pass, as instructed in the question, and several of these did not show that the second pass caused no change.

2) (i) Some candidates were able to present an argument based on the sum of the orders at the vertices being twice the number of arcs. Rather more candidates chose to draw a specific example of a graph with the required properties, and a few tried to apply Euler's theorem to a specific example of a graph with the required properties.

(ii) Allowing for spelling errors, nearly all the candidates were able to state that the graph was semi-Eulerian because it had exactly two odd vertices.

(iii) Only the better candidates were able to explain, in the general case, why such a graph is connected. Many candidates seemed to assume that the specific example that they had drawn was the only possibility and essentially said that the diagram showed that the graph was connected. Others calculated the minimum number of arcs required for a connected graph and assumed that since the number of arcs in this graph was greater the graph must be connected. The best answers were those where the candidate argued that in a simple graph a vertex of order 4 must connect five of the vertices and that the sixth vertex did not have order 0 so it must also be connected. A few candidates were able to explain that if the graph were not connected then it could have at most 10 arcs and hence this graph must be connected, or that the complete graph on six vertices would have 15 arcs and that only removing four of these could not cause the graph to become disconnected.

3) (i) Answered well. A few candidates found the length of the minimum spanning tree for the reduced network but did not join *U* back in again, and some candidates insisted on finding a cycle in the reduced network.

(ii) Many good answers. Some candidates needed to try out each vertex as a potential starting point and a small, but significant, minority just wrote down cycles without using nearest neighbour.

4) (i) Answered well. For many of the weaker candidates this was where they gained most of their marks.

(ii) Answered well, with nearly all the candidates scoring both marks. A few candidates drew their minimum spanning tree from part (i) instead of showing the whole network.

(iii) Candidates' explanations often assumed that a specific route was being used. Essentially, if arc *AC* is used then it is not possible to visit both *B* and *D* without repeating a vertex.

(iv) Candidates often assumed that *E* had been reached before *F*, rather than arguing generally.

(v) Answered well. Only a few candidates gave routes that did not fit the requirements. Most candidates were able to identify the quicker route, even with minor arithmetic errors.

5) (i) Some candidates had difficulty in finding the equations of the lines drawn on the diagram in the question or, having found the equations, in getting the inequality signs the right way round.

(ii) Most candidates were able to identify (3, 0), (0, 1) and (0.9, 2.8), but several omitted the origin. Some candidates only found (0.9, 2.8) and some candidates swapped over the *x* and *y* coordinates. Candidates who used a sliding 'line of constant profit' often overlooked the instruction in the question to 'obtain the coordinates of the vertices of the feasible region'. Some candidates assumed that the maximum value of *P* occurred at (0.9, 2.8) but most were able to calculate that the maximum value was 15 and that this occurred at $x = 3$, $y = 0$.

(iii) Only the better candidates were able to make progress with this part. Some calculated the gradients of the boundary lines for the feasible region, but most of the candidates who attempted this part calculated algebraic expressions for the objective and deduced the value of *a* that would make $Q = 3$. Several of these candidates achieved the values $a = 1$ and $a = -6$, but only a very small number of candidates were able to state that $Q = 3$ for all values of *a* that are less than or equal to -6.

6) (i) Most candidates were able to write down the constraints as equations, often adding that *s* and *t* needed to be non-negative.

(ii) Many correct answers. Some candidates went on to give a second iteration, and subsequently repeated the work in part (iii). Candidates who gave the objective row as 2, -5, -1 instead of -2, 5, 1 often went on to choose the pivot from the wrong column. Most candidates applied appropriate pivoting operations accurately but a few candidates just combined arbitrary linear combinations of the rows rather than using 'pivot row ÷ pivot value' and 'current row $\underline{+}$ multiple of pivot row'.

(iii) Several candidates stopped after one iteration and just read the values resulting from the table at this stage. Others insisted on writing down the equations from the table and did not seem to understand that they could read the values directly from the table.

(iv) Despite being told that the pivot changes, some candidates insisted that the pivoting was as before. The best candidates showed the result of the first iteration, using algebraic expressions where necessary, and then deduced the effect on the value of *y*.

7) (a)(i) Many good answers. Only a few candidates wrote down 'extra' temporary labels; there is no need to record a temporary label if it is larger than the current temporary labels at a vertex. Some candidates appeared to have tried to use common sense and then 'fiddled' the values in the boxes to make it look like Dijkstra. Almost without exception, these candidates lost the value 9 at *E*.

(ii) Most candidates were able to either write down the correct route and its length or to combine their answers from part (i) appropriately.

(iii) Some candidates said that, since *G* and *H* were close to each other, the shortest route would just be the arc *GH.* Most candidates realised that the question was asking about the shortest route between *G* and *H* that passed through every vertex, although some of these candidates thought that the shortest route from *A* to *H* passed through *G*. By writing down the route from (ii) with the end vertices removed, it was clear that the route from *G* to *H* would leave out *E* and *J*.

(b) Several candidates found the six odd vertices but did not exclude *A* and *E* as being the end points (and therefore needing to be odd). Sometimes these candidates went on to fill half a page with calculations of shortest pairings, but often they just gave the value 12 (*AC* + *DF* + *EG*). Many candidates then added the total weight of the arcs, or their calculation of the total weight, but few gave a completely correct answer with the units correct. The shortest route was of length (10 + 147) × 100 metres = 15700 metres.

2645: Discrete Mathematics 1

General Comments

Candidates who had learnt the standard algorithms were usually able to score well on the paper. Most candidates were able to score some marks on every question and few candidates scored less than 20 marks out of the maximum possible of 60. The standard of work produced by the best candidates was impressive. All candidates appeared to have ample time to complete the paper.

Comments on Individual Questions

1) Most candidates were able to answer the last two parts to this question correctly.

   (i) This question was answered poorly by many candidates. A lot of candidates simply referred to nodes being of even or odd order, or attempted to use Euler's relationship, failing to make the connection between the sum of the orders equalling twice the number of arcs correctly.

   (ii) Most candidates identified arc *AB* correctly.

   (iii) Most candidates answered this question correctly. Where candidates did make errors the majority thought the answer was equal to *n*-1 and therefore 7.

2) This question was generally answered well, with many candidates scoring full marks. If errors did occur it was usually on part (ii).

   (i) Generally well answered with many fully correct answers seen. Most candidates were able to use the nearest neighbour algorithm to give the correct cycle and appropriate length. Some drew a graph to represent the route.

   (ii) This question was less well answered. Some candidates did not know how to find the lower bound correctly and merely used the nearest neighbour algorithm again to give a cycle without node *C*.

3) Most candidates were able to make a reasonable attempt at this question and gained some marks on each part of the question. Some candidates, however, did overlook the word decreasing from the first-fit decreasing algorithm.

   (i) Most candidates were able to use the first-fit algorithm correctly but some failed to put the numbers in decreasing order first.

   (ii) Most candidates answered this part correctly.

   (iii) Some candidates failed to use the lengths for plain paper and calculated 42÷3=14m as the length of roll required. Candidates who correctly worked out 38÷3=12.66… and identified 13m as the required length generally gained full marks.

4) Most candidates drew the correct graph and were therefore able to answer the rest of the question well. Where errors did occur it was sometimes due to candidates placing weights in ambiguous places close to two different arcs, or not realising that giving the shortest route means listing nodes.

(i) Most candidates were able to draw the required network diagram and write the correct weights on the correct arcs.

(ii) Most candidates appeared to know how to use Dijkstra's algorithm correctly. Some candidates failed to update their temporary labels correctly and had missing values at nodes E or F. Some candidates failed to list the shortest route the algorithm gave and merely stated the shortest length.

(iii) This question was generally answered well by candidates.

5) Most candidates were able to gain several marks on this question but few managed to get the whole question correct.

(i) Most candidates were able to write down the constraints $x \geq 0$ and $y \geq 0$, but some candidates found difficulty in formulating all the constraints correctly. Some had inequality signs the wrong way round or were unable to calculate the equations of the lines correctly.

(ii) The majority of candidates successfully identified (3,0), (0,1) and (0.9,2.8) as vertices of the region but some candidates failed to give the vertex (0,0). Candidates who failed to identify (0.9,2.8) correctly had often used simultaneous equations instead of the graph to find this vertex.

Having successfully found any vertex most candidates were able to calculate the corresponding value of the objective function at their point or points correctly. However, some candidates then incorrectly identified (0.9,2.8) and P=12.9 as the maximum point. Other candidates failed to clearly state the maximum value of P as 15 at (3,0) as required.

(iii) Many candidates found this question difficult and scored no marks. Those candidates who did well, successfully calculated Q at their vertices and often gained values for a of 1 and -6 but in most cases then failed to give $a \leq -6$ as the required set of values. There was little evidence of candidates considering the gradient of the profit line Q and the gradients of the boundary lines to solve this question.

6) Most candidates were able to work through the flow chart, although some failed to appreciate how the output related to the input.

(i) This question was generally answered well, with a lot of candidates gaining full marks. Errors that did occur were often caused by candidates not being able to work through the flow chart correctly, or by giving a larger factor than the smallest prime factor as the output.

(ii) Most candidates were able to give a valid reason as to why 6 or 9 would not be output by the flowchart. Clear and concise responses were a rarity however.

(ii) Some candidates failed to give a full definition as to the relationship between the output and input. A common omission made by candidates was stating that the output was merely the lowest factor of the input with no reference to excluding 1 or the number being prime. A number of candidates stated that the output was a multiple of the input instead of a factor.

(iii) Many candidates successfully referred to the condition that eventually D would increase until $D^2>N$. Many candidates, however, failed to refer to the stopping conditions prior to this when a prime factor is found and the algorithm jumps to step 4 or 5.

7) There were many good answers to parts (i) and (ii) of this question. Most candidates were able to set up and apply the Simplex algorithm, but a few candidates could not read off their solution from their final tableau.

(i) A few candidates left the inequality signs in the constraints even after adding slack variables.

(ii) Many good answers, but some candidates made arithmetic errors or failed to ensure that there were two basis columns in their tableau, other than P.

(iii) Many candidates failed to gain marks on this question because they failed to recognise that the pivot point changed as a result of $c<6$. The few candidates who did recognise this and used algebraic entries often gained full marks. The candidates who substituted a numerical value for $c<6$ into their tableau, in most cases gained the method mark but did not always give their results for x and P in terms of c.

2646: Discrete Mathematics 2

General Comments

A good spread of marks was achieved and most candidates were able to complete the examination in the time allowed.

In general, the quality of the presentation of the candidates' answers was good. A few candidates submitted messy diagrams that invariably led to a loss of accuracy and a consequent loss of marks.

Comments on Individual Questions

1) (i) Almost all the candidates were able to draw the correct bipartite graph.

(ii) Only a tiny number of candidates did not follow the instruction to draw a second diagram showing the incomplete matching.

(iii) Some candidates superimposed their alternating path on the answer to part (ii), and many used the Dijkstra method starting their path from *D* instead of *M*, as instructed in the question.

(iv) Most candidates found the complete matching, but a few gave it only in list form or drew it on the same diagram as they had used in part (iii).

2) (i) Most candidates did seem to be trying to find a flow, but some made arithmetic errors resulting in vertices for which the flow in did not equal the flow out. A few candidates did not make it clear what was happening on arcs that were either saturated or empty.

(ii) Almost all the candidates who attempted this question were able to mark the cut appropriately on the diagram. The majority were also able to calculate the capacity of the cut as being 28 but, inevitably, a few tried to include the capacity of the arc *CF* and gave an answer of 36.

(iii) Some candidates opted out of the latter parts of this question, but those who had answered part (i) correctly were usually able to add 10 to the flow to give the maximum flow.

(iv) Only a few of the candidates who attempted this part failed to find a feasible flow in which the given arcs were saturated. The majority of these gave a flow that was also maximal.

3) (i) Answered well, virtually all the candidates were able to say in some way or another that the matrix needs to be square.

(ii) Apart from a few candidates who augmented in steps of one unit at a time, and some who reduced columns first instead of rows, this was generally done quite well. Several candidates failed to gain full marks through arithmetic mistakes, through not stating the matching or its cost, or through including the dummy in the final costing.

4) (i) Candidates’ explanations were usually muddled, or just repeated the wording from the question. Candidates needed to explain why the minimum width on each route needed to be identified and then explain that the maximum of these route minima would give the width of the widest load that could pass through.

(ii) Only a few candidates were able to set up the stage, state and action columns correctly. The state column was often labelled with (stage; state) variables. The action values should be the state label of the vertex being moved into. The candidates who remembered that this was a maximin problem, and not a maximum problem, were generally able to complete the dynamic programming tabulation and to read off the route from their completed table. Most of the candidates realised that the widest load that could pass was 5, but only a few gave the answer as 5 metres.

5) (i) The majority of the activity networks were structurally incorrect, usually due to insufficient dummy activities resulting in activity *I* following not just *C* and *H* but also *B*. Other candidates gave far too many dummy activities. Many candidates lost marks for not stating either the minimum completion time or the critical activities, or both. Centres should note that the new specification requires the activity on arc formulation.

(ii) Many candidates gave up on the latter parts of this question. Those who persevered were usually able to complete a correct schedule, provided they realised that more than one activity could happen at a time.

(iii) The candidates who had given a valid schedule in part (ii) were usually able to realise that activity *I* needed to start earlier so that activity *J* could come forwards. Some candidates said that activity *E* needed to be split over two sessions, but this would not have left enough room for activity *F* to be completed in time whilst still preserving the precedences. The only way to complete the project with these restrictions was to split activity *F*.

(iv) Despite the question requesting that the graph was to be drawn on graph paper, some candidates gave freehand diagrams in their answer booklet. Often these were too inaccurate to be of any real use. Some candidates did not use a continuous time axis, but instead split the diagram into three separate histograms.

6) (i) Most candidates were able to correctly identify the play-safe strategies and explain how they knew that the game has a stable solution. Only a few candidates were able to describe what playing safe would mean for the farmer, in context (‘leave the field resting and, at worst, make no profit’).

(ii) Most candidates were able to show that state *C* dominates state *F*, although a few just described what dominance means without actually showing it. Again, several candidates did not interpret the consequence of this in context (‘never grow flax’).

(iii) Most candidates were able to find the three equations correctly.

(iv) Most candidates were able to graph their equations, although some of the graphs were far too inaccurate to be useful. Several candidates lost a mark for not calculating all three pay-offs, as asked in the question.

(v) Only a few candidates attempted this part, those who did were usually able to give a correct expression and to realise that it was maximised when *p* took the value 1.

7840, 7842, 7844, 3840, 3841, 3842, 3843, 3844 AS and A2 Mathematics
January 2005 Assessment Session

Unit Threshold Marks

Unit		Maximum Mark	A	B	C	D	E	U
All units	UMS	100	80	70	60	50	40	0
2631	Raw	60	47	41	35	29	23	0
2632	Raw	60	47	41	35	30	25	0
2633	Raw	60	45	39	33	28	23	0
2634	Raw	60	45	40	35	30	25	0
2635	Raw	60	48	41	35	29	23	0
2636	Raw	60	46	40	34	29	24	0
2637	Raw	60	45	39	33	28	23	0
2638	Raw	60	41	36	31	26	21	0
2639	Raw	60	50	43	36	30	24	0
2641	Raw	60	47	41	35	29	24	0
2642	Raw	60	46	40	34	29	24	0
2643	Raw	60	46	40	34	29	24	0
2645	Raw	60	47	41	36	31	26	0
2646	Raw	60	43	38	33	28	23	0
2647	Raw	60	48	42	36	30	24	0

Specification Aggregation Results

Overall threshold marks in UMS (i.e. after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
7840	600	480	420	360	300	240	0
7842	600	480	420	360	300	240	0
7844	600	480	420	360	300	240	0
3840	300	240	210	180	150	120	0
3841	300	240	210	180	150	120	0
3842	300	240	210	180	150	120	0
3843	300	240	210	180	150	120	0
3844	300	240	210	180	150	120	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
2631	23.2	39.4	53.7	67.5	79.6	100.0	1364
2632	31.7	50.4	66.8	76.1	83.6	100.0	4455
2633	31.8	49.5	65.0	74.5	81.5	100.0	1461
2634	35.3	54.3	68.3	79.4	86.3	100.0	983
2635	49.1	72.7	85.5	90.9	92.7	100.0	55
2636	22.2	39.7	46.0	60.3	76.2	100.0	60
2637	27.9	44.4	60.2	73.6	83.1	100.0	1789
2638	33.4	46.3	59.9	73.3	83.6	100.0	1173
2639	42.6	66.0	79.4	85.1	89.0	100.0	282
2641	25.9	42.2	57.9	73.3	83.9	100.0	1164
2642	33.4	50.7	62.9	74.1	84.1	100.0	1241
2643	38.8	56.2	66.1	76.5	89.7	100.0	242
2645	40.3	63.9	77.6	87.8	95.0	100.0	833
2646	35.7	55.3	72.7	83.9	90.0	100.0	311
2647	0	100.0	100.0	100.0	100.0	100.0	1

3890 AS Mathematics
January 2005 Assessment Session

Unit Threshold Marks

Unit		Maximum Mark	A	B	C	D	E	U
All units	UMS	100	80	70	60	50	40	0
4721	Raw	72	56	49	42	35	28	0
4722	Raw	72	54	47	40	33	27	0
4728	Raw	72	57	49	42	35	28	0
4732	Raw	72	54	47	40	34	28	0
4736	Raw	72	54	47	40	34	28	0

Specification Aggregation Results

Overall threshold marks in UMS (i.e. after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
3890	300	240	210	180	150	120	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
4721	34.2	49.8	64.0	75.5	84.9	100.0	8885
4722	38.4	55.3	69.7	81.7	89.0	100.0	2210
4728	51.6	68.5	79.1	87.0	91.5	100.0	708
4732	36.1	51.9	66.0	75.8	85.0	100.0	1718
4736	20.0	42.8	65.5	80.4	89.0	100.0	1285