OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 2646

Discrete Mathematics 2

Friday **14 JANUARY 2005** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 5 printed pages, 3 blank pages and an insert.

1 Four teachers are available to cover classes for absent colleagues. Four classes need to be covered, Karate (K), Latin (L), Maths (M) and Needlework (N). Mr Atkinson (A) is willing to cover Latin or Maths; Miss Bloggs (B) is willing to cover Latin or Needlework; Mrs Cook (C) is willing to cover Karate or Latin; Mr Davies (D) is willing to cover Needlework only.

(i) Draw a bipartite graph to show this information. Put the teachers (A, B, C and D) on the left hand side and the classes (K, L, M and N) on the right hand side. [1]

Mr Atkinson is sent to cover Latin, Miss Bloggs to cover Needlework and Mrs Cook to cover Karate.

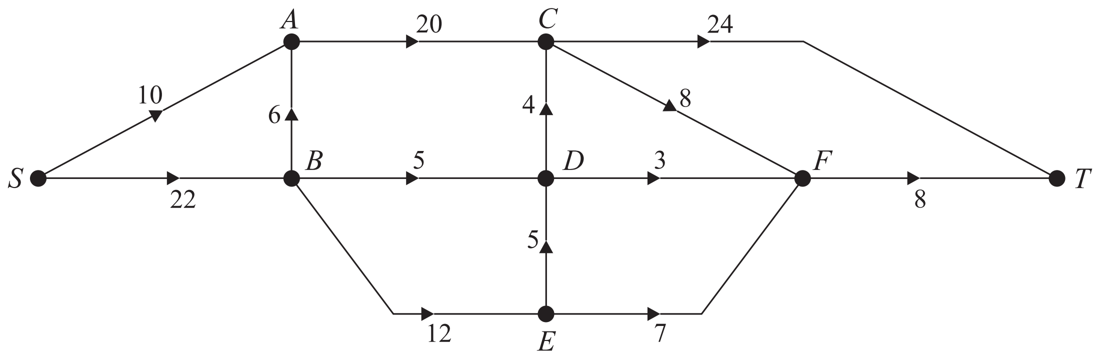
(ii) Draw a second bipartite graph to show this incomplete matching. [1]

(iii) Construct an alternating path, starting from M, to improve the matching. Show your alternating path on a third bipartite graph. [1]

(iv) Use your alternating path to find an improved matching. Show this final matching on a fourth bipartite graph. [1]

2 [Answer this question on the insert provided.]

A river flows from a source, S, to a town, T. At several points between the source and the town, the river divides and joins back together again. The network represents the river and shows the capacity of each section.



(i) On the first diagram on the insert show a flow of 18 from S to T in which the arcs BA, BE and DC are saturated. [2]

(ii) Mark the cut $\{S, B, D, E, F\}$, $\{A, C, T\}$ on the second diagram on the insert. Calculate the capacity of this cut. [2]

(iii) Add further flow to the solution to part **(i)** to give a maximum flow from S to T. Show your maximum flow on the third diagram on the insert. [1]

(iv) On the fourth diagram on the insert show a maximum flow in which the arcs BD and DF are saturated. [2]

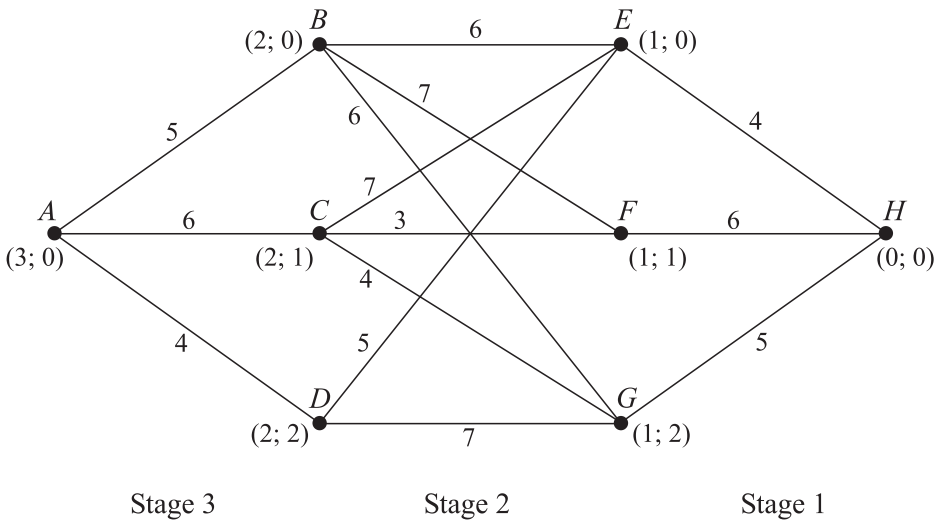
3 The table below shows a cost matrix for four workers, R, S, T, U and five jobs V, W, X, Y, Z. Row Q is a dummy row. Each worker will do one job, and the jobs will be done simultaneously. A minimum cost allocation between the four workers and four of the jobs is required.

	V	W	X	Y	Z
Q	25	25	25	25	25
R	21	17	11	10	20
S	10	12	7	12	11
T	18	10	6	6	12
U	8	12	5	10	11

(i) Explain why the dummy row is needed. [1]

(ii) Apply the Hungarian algorithm, **reducing rows first**. Give your minimum cost allocation and state the associated cost. [8]

4 A wide load needs to be driven from A to H. The roads that can be used and the width limit (in metres) for each road are shown on diagram below. The towns A, B, C, D, E, F, G and H have been labelled with (stage; state) labels.



(i) The driver wants to find the route that allows the widest possible load to be driven from A to H. Explain why this involves solving a maximin problem. [2]

(ii) Set up a dynamic programming tabulation, working backwards from H, to find the route from A to H that allows the widest possible load. Give the route and the width of the widest load that can be driven from A to H using this route. [10]

5 The table lists the activities involved in a project to redesign a garden, their durations (in hours) and immediate predecessors.

Activity	Duration (hours)	Preceded by
A: Plan new garden	3	–
B: Visit builders merchants	1	A
C: Visit garden centre	2	A
D: Order turf	1	A
E: Prepare area for patio	5	B
F: Lay patio slabs	4	E
G: Build summerhouse	9	B, C
H: Lay turf	3	D
I: Planting	6	C, H
J: Tidy up	3	F, G, I

(i) Draw an activity network to represent these activities and their predecessors. Carry out a forward pass and a backward pass to determine the minimum completion time and the critical activities. [7]

The project is to be done in sessions of no more than 6 hours each. Activity G can be split between two sessions, but no other activity may be split.

(ii) Show how the project can be completed in four sessions. You may use a list or a diagram, but whichever you choose it must be clear when each activity starts and ends. [2]

(iii) By also splitting activity I and one other activity, the project can be completed in just three sessions. Which is the other activity that needs to be split? Describe the changes that this will make to your answer to part **(ii)**. [2]

Activities E, F, H and I need two people each. Activities G and J need four people each. Every other activity needs just one person each.

(iv) On graph paper, draw a resource histogram to show the number of people required when the project is completed in three sessions, as in part **(iii)**. How many people are needed for each session? [3]

6 A model of crop farming assumes that a field is in one of three states: growing corn (state C), growing flax (state F) or resting (state R).

The profit generated from the crop depends on the weather conditions while the crop is growing. The weather may be mostly wet (state W), mostly dry (state D) or a mixture of wet and dry (state M).

Table 1 shows the expected profit for one field in £1000's. This may be regarded as a zero-sum game in which the farmer plays against the weather.

		Weather		
		W	D	M
	C	−3	4	5
Farmer	F	−8	2	−1
	R	0	2	1

Table 1

(i) Show that the game has a stable solution and describe what playing safe would mean for the farmer. [3]

Under new rules, the farmer can apply for a grant if the weather is wet. This changes the profits to the values in Table 2.

		Weather		
		W	D	M
	C	1	4	5
Farmer	F	−1	2	−1
	R	2	2	1

Table 2

(ii) Show that state C dominates state F and explain what this means for the farmer. [2]

(iii) The farmer decides to grow corn with probability p and let the field rest with probability $1 - p$. Write down and simplify expressions for the expected profit for the farmer with each type of weather. [3]

(iv) Represent the expected profits on a graph. Use graph paper with the values of p, from 0 to 1, on the horizontal axis and the values of the expected profit, from −5 to 5, on the vertical axis. From your graph, find the value of p that the farmer should choose and calculate the expected profit with each type of weather for this value of p. [4]

(v) You are given that the probability of wet weather is 0.1, the probability of dry weather is 0.3 and the probability of mixed weather is 0.6. Calculate the expected profit for the farmer as a function of p. For what value of p is this expected profit a maximum? [2]

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Candidate Name	Centre Number	Candidate Number

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INSERT for Question 2

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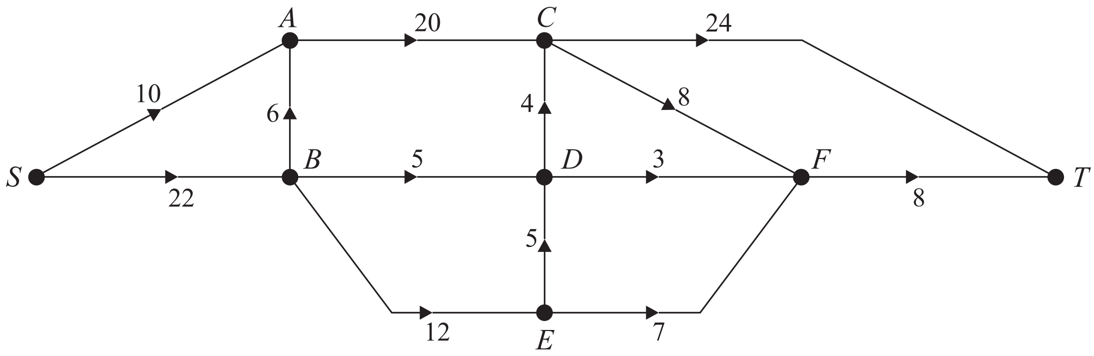
INSTRUCTIONS TO CANDIDATES

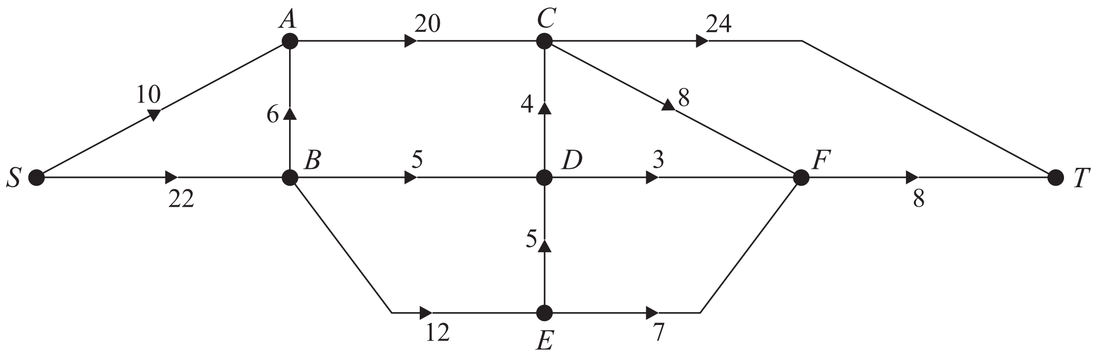
- This insert should be used to answer Question **2**.
- Write your Name, Centre Number and Candidate Number in the spaces provided at the top of this page.
- Write your answers to Question **2** in the spaces provided in this insert, and attach it to your answer booklet.

This insert consists of 2 printed pages.

2 (i)



(ii)



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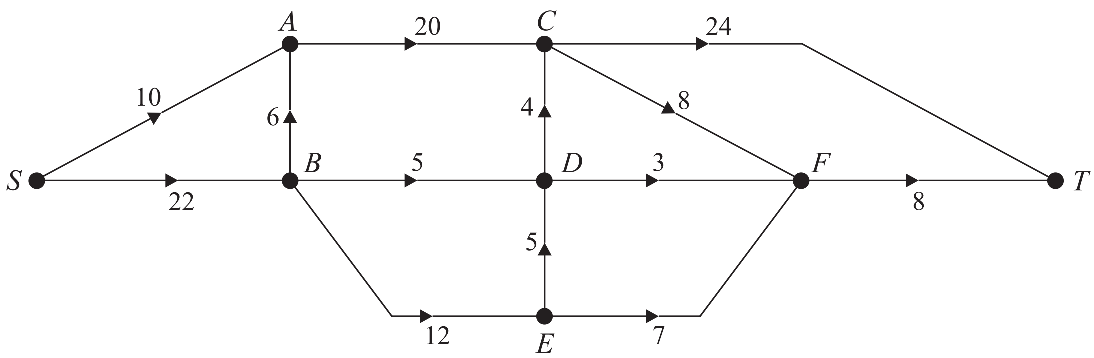
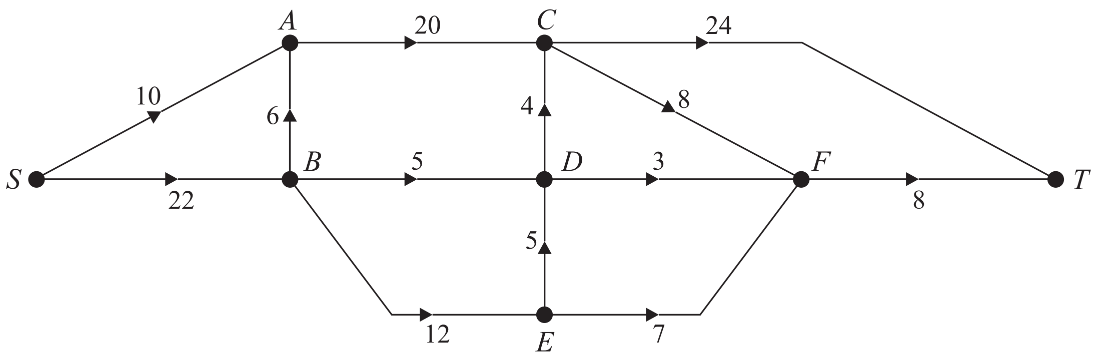
Capacity of cut = ...

(iii)



(iv)



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