OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 2636

Pure Mathematics 6

Wednesday **12 JANUARY 2005** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

1 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & -\frac{1}{2} \end{pmatrix}$.

(i) Give a complete geometrical description of the transformation represented by $\mathbf{M}$. [3]

(ii) Hence write down the smallest positive integer n for which $\mathbf{M}^n = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. [1]

2 A plane P has equation $(\mathbf{r} - \mathbf{a}).\mathbf{n} = 0$. With reference to this plane, give a geometrical interpretation of

(i) the vector $\mathbf{a}$, [1]

(ii) the vector $\mathbf{n}$, [1]

(iii) the set of points whose position vectors $\mathbf{r}$ satisfy the equation $(\mathbf{r} - \mathbf{a}) \times \mathbf{n} = \mathbf{0}$. [3]

3 The elements x, a, b, c, r, s belong to a non-commutative group G.

(i) Solve for x the equation $axb = c$. [2]

(ii) Given that $r^2s^2 = (rs)^2$, prove that $rs = sr$. [2]

4 **(i)** Given that $z = \mathrm{e}^{\mathrm{i}\theta}$, show that $z^n - \dfrac{1}{z^n} = 2\mathrm{i}\sin n\theta$. [2]

(ii) Express $\sin^5\theta$ in terms of sines of multiples of θ. [5]

5 **(i)** Find the values of the constant k for which the matrix

$$\begin{pmatrix} 7 & k & 1 \\ k & 3 & 1 \\ 1 & 7 & 3 \end{pmatrix}$$

is singular. [4]

(ii) Solve the simultaneous equations

$$\begin{aligned} 7x - y + z &= 9, \\ -x + 3y + z &= -3, \\ x + 7y + 3z &= -3, \end{aligned}$$

expressing your answers in terms of a parameter. [4]

6 In this question, z denotes the complex number $\frac{1}{2}\left(\cos\frac{1}{3}\pi + \mathrm{i}\sin\frac{1}{3}\pi\right)$.

(i) Write down z^2 and z^3 in polar form. [2]

(ii) The points in an Argand diagram which represent the numbers 1, $1 + z$, $1 + z + z^2$ and $1 + z + z^2 + z^3$ are denoted by A, B, C and D respectively. Sketch a diagram to show these points, and join AB, BC and CD. [2]

(iii) S_n denotes the sum of the series $1 + z + z^2 + \ldots + z^{n-1}$. Show that

$$S_{6n} = \frac{1}{3}\left(1 - \frac{1}{2^{6n}}\right)\left(3 + \mathrm{i}\sqrt{3}\right).$$ [5]

(iv) You are given that S_{6n} converges to S as $n \to \infty$. Write down the value of S. [1]

7 The point $P\,(2, 4, 1)$ lies on the line l_1 which has direction $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and the point $Q\,(1, 2, 3)$ lies on the line l_2 which has direction $-2\mathbf{i} + \mathbf{j}$.

(i) Giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$, write down an equation of the plane which passes through the mid-point of PQ and which is parallel to both l_1 and l_2. [2]

(ii) Find a vector normal to the plane in part **(i)** and hence, or otherwise, express the equation of the plane in the form $fx + gy + hz = d$. [4]

(iii) Find the unit vector in the direction $\overrightarrow{PQ}$. [2]

(iv) **Hence** calculate the shortest distance between the lines l_1 and l_2. Explain briefly why your calculation gives the shortest distance. [3]

8 The function f is defined by $\mathrm{f} : x \mapsto \dfrac{1}{1 - x}$ for $x \in \mathbb{R},\ x \neq 0,\ x \neq 1$.

(i) Show that $\mathrm{ff}(x) = 1 - \dfrac{1}{x}$. [2]

It is given that f is an element of a group F under the operation of composition of functions.

(ii) Show that the order of f is 3. [3]

The group F is a proper subgroup of a group H of order 6. Four of the elements of H are e, f, ff and h, where $\mathrm{e} : x \mapsto x$ and $\mathrm{h} : x \mapsto \dfrac{1}{x}$.

(iii) List the elements of another proper subgroup of H. [2]

(iv) Express the other two elements of H in terms of x. [4]

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