OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 2635

Pure Mathematics 5

Monday **10 JANUARY 2005** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

1 **(i)** Show that the substitution $x = y + 1$ transforms the equation $x^4 - 4x^3 + x^2 + 6x + 2 = 0$ to

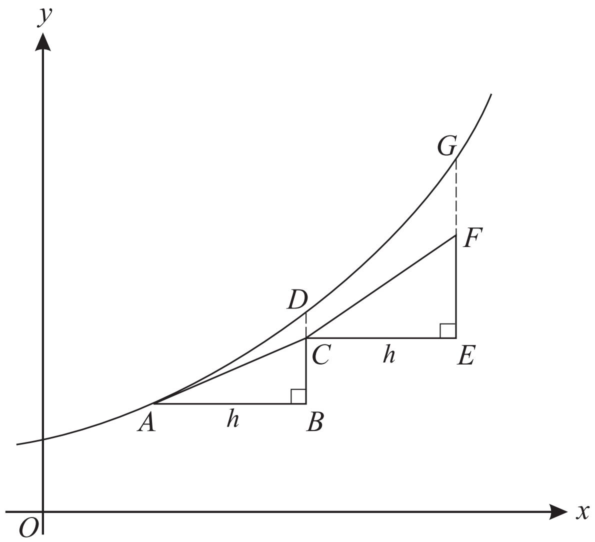
$$y^4 - 5y^2 + 6 = 0. \quad [2]$$

(ii) Hence find the exact roots of $x^4 - 4x^3 + x^2 + 6x + 2 = 0$. [3]

2 Use the Newton-Raphson method to find the x-coordinate of the point where the curves $y = \ln x$ and $y = \dfrac{3}{x}$ meet. Give your answer correct to 2 decimal places. [5]

3 Use the substitution $x = \frac{1}{2}\sinh u$ to find $\int \sqrt{(1 + 4x^2)}\,dx$. [6]

4



The diagram illustrates the working of Euler's method for the solution of a differential equation of the form $\dfrac{dy}{dx} = f(x, y)$. The curve represents the solution of the differential equation and $A\ (x_0, y_0)$ is the initial point. The first two steps of Euler's method with step-length h are shown.

(i) State the relation between the line AC and the solution curve. [1]

(ii) Write down an expression for

(a) the y-coordinate of C, [1]

(b) the gradient of the line CF. [1]

The differential equation

$$\frac{dy}{dx} = \sqrt{(x^3 + y^3)},$$

where $y = 1$ when $x = 1$, is to be solved using Euler's method.

(iii) Use a step-length of 0.1 to obtain an estimate of y when $x = 1.2$, giving your answer correct to 3 decimal places. Show your working clearly. [3]

5 It is given that α, β and γ are three numbers such that

$$\alpha + \beta + \gamma = 3, \quad \alpha^2 + \beta^2 + \gamma^2 = 19 \quad \text{and} \quad \alpha\beta\gamma = 1.$$

Find

(i) $\alpha\beta + \beta\gamma + \gamma\alpha$, [2]

(ii) a cubic equation with roots α, β and γ, [2]

(iii) exact values for α, β and γ. [4]

6 It is given that $I_n = \int_0^1 x^n (1-x)^{\frac{1}{2}}\,dx$, for $n = 0, 1, 2, \ldots$.

(i) Show that $I_n = \frac{2}{3}n \int_0^1 x^{n-1}(1-x)^{\frac{3}{2}}\,dx$. [3]

(ii) By writing $(1-x)^{\frac{3}{2}}$ as $(1-x)(1-x)^{\frac{1}{2}}$, or otherwise, show that $I_n = \dfrac{2n}{2n+3} I_{n-1}$. [3]

(iii) Evaluate I_2, giving your answer as a fraction. [3]

7 **(i)** Sketch the curve $y = \operatorname{sech} x$. [1]

(ii) Using the substitution $e^x = u$, show that the area of the region bounded by the curve $y = \operatorname{sech} x$, the line $x = 1$ and the positive x- and y-axes is

$$2\tan^{-1} e - \tfrac{1}{2}\pi. \qquad [6]$$

(iii) The region defined in part **(ii)** is rotated through 2π radians about the x-axis. Prove that the volume of the solid formed is

$$\pi\left(\frac{e^2 - 1}{e^2 + 1}\right). \qquad [3]$$

8 **(i)** Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 10y = 10. \qquad [5]$$

(ii) Find the particular solution representing a curve which has tangent $y = x$ at the point (0, 0). [6]

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