RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 2632

Pure Mathematics 2

Monday **10 JANUARY 2005** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

1 Find

(i) $\int \frac{3}{x}\,dx$, [1]

(ii) $\int 4e^{\frac{1}{2}x}\,dx$. [2]

2 **(i)** Find the first three terms in the expansion of $(2+x)^8$ in ascending powers of x, simplifying the coefficients. [4]

(ii) Hence, or otherwise, determine the coefficient of y^4 in the expansion of $\left(2+\frac{1}{2}y^2\right)^8$. [2]

3 The polynomial $f(x)$ is defined by

$$f(x) = x^3 + px + q,$$

where p and q are constants. It is given that $x+1$ and $x-3$ are factors of $f(x)$.

(i) Find the values of p and q. [4]

(ii) Solve the equation $f(x) = 0$. [2]

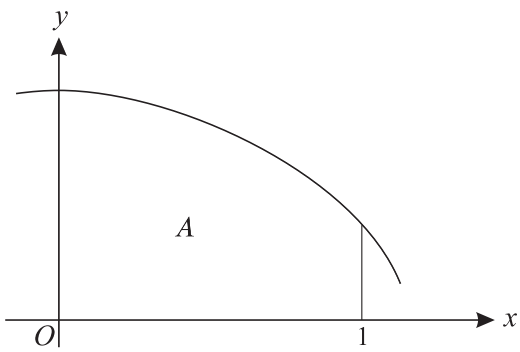
4 At time t minutes after a pollution incident, the area of sea covered by oil is $X\text{ m}^2$. Two models giving X in terms of t are as follows.

Model 1: $X = 3e^{0.4t}$

Model 2: $X = \sqrt{(2t^4 + 9)}$

Show by differentiation that the two models give approximately the same value for the rate of increase of X when $t = 7$. [6]

5



The diagram shows part of the curve $y = \ln(16 - 12x^2)$. The region A is bounded by the curve and the lines $x = 0$, $x = 1$ and $y = 0$.

(i) Show that the trapezium rule, with two strips each of width $\frac{1}{2}$, gives a value of $\frac{1}{2}\ln 104$ for the area of A. [5]

(ii) Explain how the diagram indicates that $\frac{1}{2}\ln 104$ is an underestimate of the area of A. [1]

6

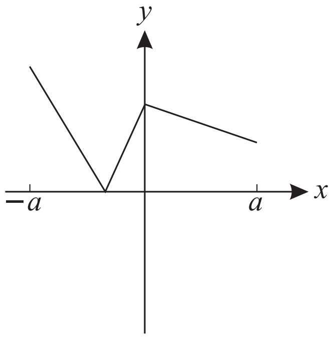


Fig. 1

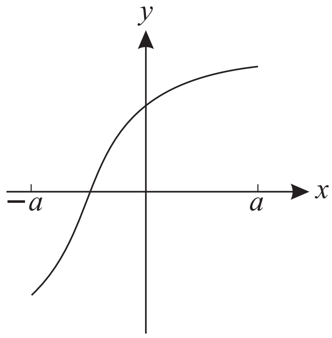


Fig. 2

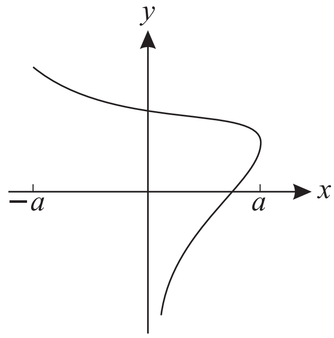


Fig. 3

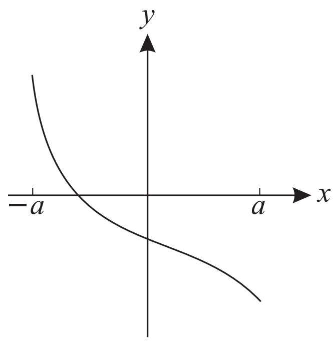


Fig. 4

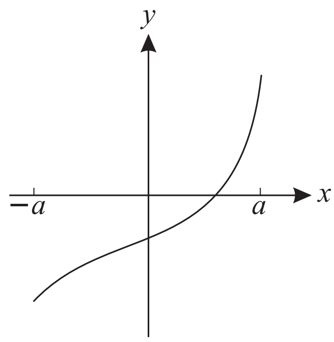


Fig. 5

The diagrams show five different graphs, each for values of x such that $-a \leqslant x \leqslant a$ where a is a constant.

(i) State which diagram does not show the graph of a function. Justify your answer. [2]

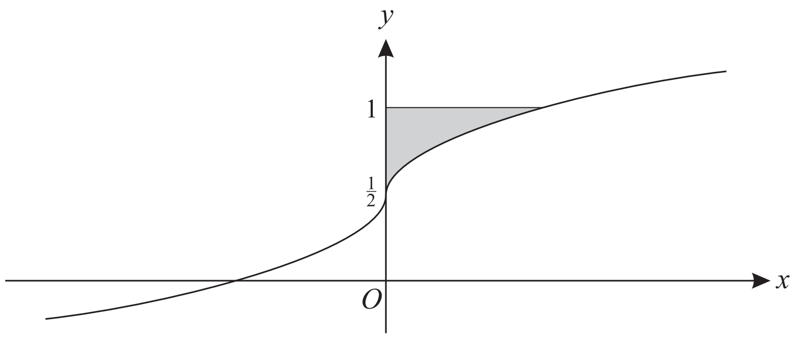
(ii) State which diagram shows the graph of a function which is not 1-1. Justify your answer. [2]

(iii) It is given that two of the diagrams illustrate functions which are inverses of each other. Identify these two diagrams. [1]

(iv) The graph in Fig. 5 has equation $y = \mathrm{f}(x)$. Sketch the graph of $y = |\mathrm{f}(x)|$. [2]

7 **(i)** Given that $y = \frac{1}{4}\left(2 + \sqrt[5]{x}\right)$, show that x may be expressed in the form $(ay + b)^5$, where the values of the constants a and b are to be stated. [2]

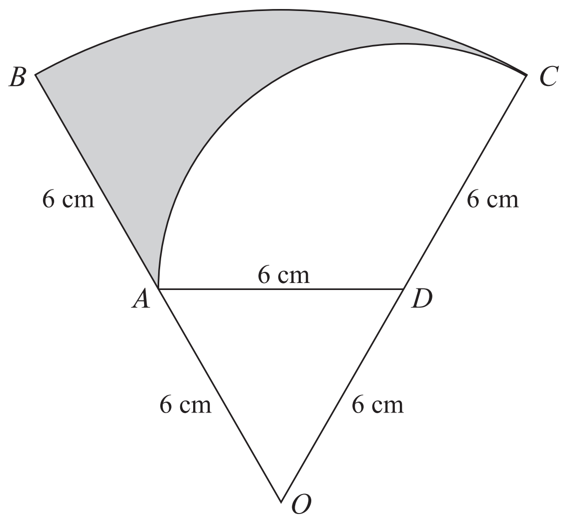
(ii)



The diagram shows a sketch of the curve $y = \frac{1}{4}\left(2 + \sqrt[5]{x}\right)$. The shaded region is bounded by part of the curve and the lines $x = 0$ and $y = 1$. The shaded region is rotated through four right angles about the ***y*-axis**. Find the exact volume of the solid produced. [4]

[Turn over

8



The diagram shows a sector OBC of a circle, centre O and radius 12 cm. The mid-points of OB and OC are A and D respectively. The length of AD is 6 cm. AC is an arc of the circle, centre D and radius 6 cm. The shaded region is bounded by the line AB and the arcs AC and BC.

(i) Show that the angle $ADC = \frac{2}{3}\pi$ radians. [1]

(ii) Show that the perimeter of the shaded region is $(8\pi + 6)$ cm. [3]

(iii) Find the exact area of the shaded region. [4]

9 A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 7, \quad u_{n+1} = u_n + 15.$$

The sum of the first n terms of this sequence is denoted by S_n. The terms of a second sequence $v_1, v_2, v_3, \ldots$ form a geometric progression with first term 1.2 and common ratio 1.2.

(i) Show that $u_3 + v_3 = 38.728$. [2]

(ii) Show that $S_{70} = 36\,715$. [3]

(iii) Find the largest value of p such that $v_p < S_{70}$. [3]

(iv) Find the largest value of q such that $S_q < v_{70}$. [4]

BLANK PAGE