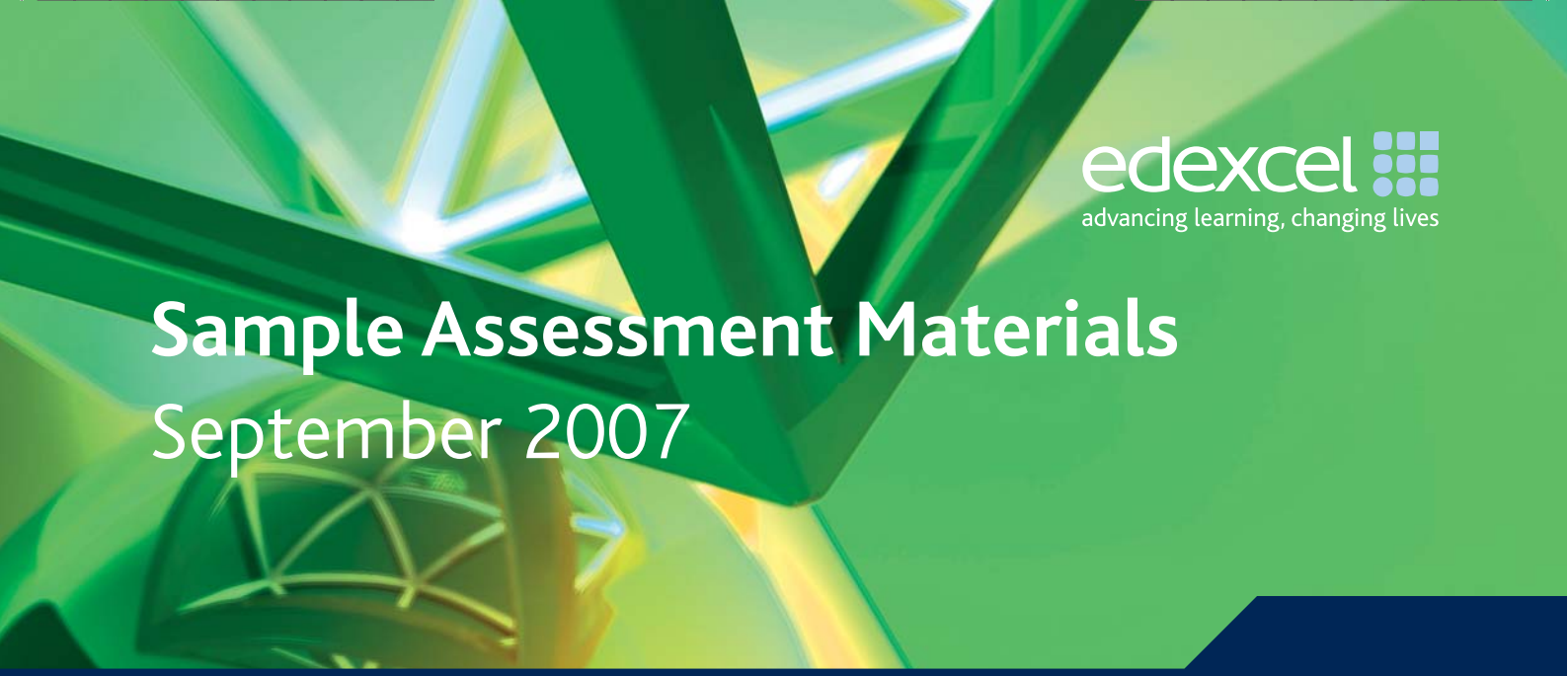

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# Sample Assessment Materials
## September 2007

GCE Mathematics

## Edexcel Advanced Subsidiary GCE in Mathematics (8371)
## Edexcel Advanced Subsidiary GCE in Further Mathematics (8372)
## Edexcel Advanced Subsidiary GCE in Pure Mathematics (8373)
## Edexcel Advanced Subsidiary GCE in Further Mathematics (Additional) (8374)
First examination 2009

## Edexcel Advanced GCE in Mathematics (9371)
## Edexcel Advanced GCE in Further Mathematics (9372)
## Edexcel Advanced GCE in Pure Mathematics (9373)
## Edexcel Advanced GCE in Further Mathematics (Additional) (9374)
First examination 2009

This document contains the new specimen assessment materials for the amended units FP1, FP2, FP3, D1 and D2.

The specimen assessment materials for Core, Statistics and Mechanics units are contained in the previous issue of the specimen papers UA014392 (2004).


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# Edexcel GCE e-Spec

## Your free e-Spec

This specification comes with a free e-Spec, Edexcel's electronic version of the specification. You will find the e-Spec disc inside the Specification book for this qualification.

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- A customisable student guide to help recruit students
- A course planner to make it easy to plan delivery
- Links to sample assessment materials so you can see what is expected
- Information on the products and services provided by Edexcel to support the specification.

## Easy-to-use

Just click on the walkthrough to see how easy and useful the e-Spec is and get more out of this specification today.

# Contents

# A Introduction

These sample assessment materials have been prepared to support the specification.

Their aim is to provide the candidates and centres with a general impression and flavour of the actual question papers and mark schemes in advance of the first operational examinations.

# B Sample question papers

Paper Reference(s)

**6667/01**

# Edexcel GCE

## Further Pure Mathematics FP1

## Advanced Subsidiary/Advanced

Sample Assessment Material

Time: 1 hour 30 minutes

| **Materials required for examination** | **Items included with question papers** |
|---|---|
| Mathematical Formulae | Nil |

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.**

### Instructions to Candidates

In the boxes on the answer book, write your centre number, candidate number, your surname, initial(s) and signature.
Check that you have the correct question paper.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

### Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 4 pages in this question paper. Any blank pages are indicated.

### Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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**1.** $f(x) = x^3 - 3x^2 + 5x - 4$

(a) Use differentiation to find $f'(x)$.

**(2)**

The equation $f(x) = 0$ has a root $\alpha$ in the interval $1.4 < x < 1.5$

(b) Taking 1.4 as a first approximation to $\alpha$, use the Newton-Raphson procedure once to obtain a second approximation to $\alpha$. Give your answer to 3 decimal places.

**(4)**

**(Total 6 marks)**

---

**2.** The rectangle $R$ has vertices at the points (0, 0), (1, 0), (1, 2) and (0, 2).

(a) Find the coordinates of the vertices of the image of $R$ under the transformation given by the matrix $\mathbf{A} = \begin{pmatrix} a & 4 \\ -1 & 1 \end{pmatrix}$, where $a$ is a constant.

**(3)**

(b) Find det **A**, giving your answer in terms of $a$.

**(1)**

Given that the area of the image of $R$ is 18,

(c) find the value of $a$.

**(3)**

**(Total 7 marks)**

---

**3.** The matrix **R** is given by $\mathbf{R} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

(a) Find $\mathbf{R}^2$.

**(2)**

(b) Describe the geometrical transformation represented by $\mathbf{R}^2$.

**(2)**

(c) Describe the geometrical transformation represented by **R**.

**(1)**

**(Total 5 marks)**

---

**4.** $f(x) = 2^x - 6x$

The equation $f(x) = 0$ has a root $\alpha$ in the interval $[4, 5]$.

Using the end points of this interval find, by linear interpolation, an approximation to $\alpha$.

**(Total 3 marks)**

---

**5.** (a) Show that $\sum_{r=1}^{n}(r^2 - r - 1) = \frac{1}{3}(n-2)n(n+2)$.

**(6)**

(b) Hence calculate the value of $\sum_{r=10}^{40}(r^2 - r - 1)$.

**(3)**

**(Total 9 marks)**

---

**6.** Given that $z = -3 + 4i$,

(a) find the modulus of $z$,

**(2)**

(b) the argument of $z$ in radians to 2 decimal places.

**(2)**

Given also that $w = \frac{-14 + 2i}{z}$,

(c) use algebra to find $w$, giving your answers in the form $a + ib$, where $a$ and $b$ are real.

**(4)**

The complex numbers $z$ and $w$ are represented by points $A$ and $B$ on an Argand diagram.

(d) Show the points $A$ and $B$ on an Argand diagram.

**(2)**

**(Total 10 marks)**

---

**7.** The parabola $C$ has equation $y^2 = 4ax$, where $a$ is a constant.

The point $(4t^2, 8t)$ is a general point on $C$.

(a) Find the value of $a$.

**(1)**

(b) Show that the equation for the tangent to $C$ at the point $(4t^2, 8t)$ is

$$yt = x + 4t^2.$$

**(4)**

The tangent to $C$ at the point $A$ meets the tangent to $C$ at the point $B$ on the directrix of $C$ when $y = 15$.

(c) Find the coordinates of $A$ and the coordinates of $B$.

**(7)**

**(Total 12 marks)**

---

**8.** $$\mathrm{f}(x) \equiv 2x^3 - 5x^2 + px - 5, \quad p \in \mathbb{R}$$

Given that $1 - 2\mathrm{i}$ is a complex solution of $\mathrm{f}(x) = 0$,

(a) write down the other complex solution of $\mathrm{f}(x) = 0$,

**(1)**

(b) solve the equation $\mathrm{f}(x) = 0$,

**(6)**

(c) find the value of $p$.

**(2)**

**(Total 9 marks)**

---

**9.** Use the method of mathematical induction to prove that, for $n \in \mathbb{Z}^+$,

(a) $$\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}^n = \begin{pmatrix} n+1 & n \\ -n & 1-n \end{pmatrix}$$

**(7)**

(b) $\mathrm{f}(n) = 4^n + 6n - 1$ is divisible by 3.

**(7)**

**(Total 14 marks)**

---

**TOTAL FOR PAPER: 75 MARKS**

**END**

Paper Reference(s)

**6668/01**

# Edexcel GCE

## Further Pure Mathematics FP2

## Advanced

Sample Assessment Material

Time: 1 hour 30 minutes

| Materials required for examination | Items included with question papers |
|---|---|
| Mathematical Formulae | Nil |

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.**

### Instructions to Candidates

In the boxes on the answer book, write your centre number, candidate number, your surname, initial(s) and signature.
Check that you have the correct question paper.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

### Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 4 pages in this question paper. Any blank pages are indicated.

### Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
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**1.** Find the set of values of $x$ for which

$$\frac{x}{x-3} > \frac{1}{x-2}$$

**(Total 7 marks)**

---

**2.** (a) Express as a simplified single fraction $\frac{1}{r^2} - \frac{1}{(r+1)^2}$

**(2)**

(b) Hence prove, by the method of differences, that

$$\sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2} = 1 - \frac{1}{(n+1)^2}$$

**(3)**

**(Total 5 marks)**

---

**3.** (a) Show that the transformation $T$

$$w = \frac{z-1}{z+1}$$

maps the circle $|z| = 1$ in the $z$-plane to the line $|w-1| = |w+\mathrm{i}|$ in the $w$-plane.

**(4)**

The transformation $T$ maps the region $|z| \leq 1$ in the $z$-plane to the region $R$ in the $w$-plane.

(b) Shade the region $R$ on an Argand diagram.

**(2)**

**(Total 6 marks)**

---

**4.** $$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + y\frac{\mathrm{d}y}{\mathrm{d}x} = x, \quad y = 0, \ \frac{\mathrm{d}y}{\mathrm{d}x} = 2 \text{ at } x = 1$$

Find a series solution of the differential equation in ascending powers of $(x - 1)$ up to and including the term in $(x - 1)^3$.

**(Total 7 marks)**

---

**5.** (a) Obtain the general solution of the differential equation

$$\frac{\mathrm{d}S}{\mathrm{d}t} - 0.1S = t$$

**(6)**

(b) The differential equation in part (a) is used to model the assets, £$S$ million, of a bank $t$ years after it was set up. Given that the initial assets of the bank were £200 million, use your answer to part (a) to estimate, to the nearest £ million, the assets of the bank 10 years after it was set up.

**(4)**

**(Total 10 marks)**

---

**6.** The curve $C$ has polar equation

$$r^2 = a^2 \cos 2\theta, \quad \frac{-\pi}{4} \leq \theta \leq \frac{\pi}{4}$$

(a) Sketch the curve $C$.

**(2)**

(b) Find the polar coordinates of the points where tangents to $C$ are parallel to the initial line.

**(6)**

(c) Find the area of the region bounded by $C$.

**(4)**

**(Total 12 marks)**

---

**7.** (a) Given that $x = e^t$, show that

(i) $$\frac{dy}{dx} = e^{-t}\frac{dy}{dt}$$

(ii) $$\frac{d^2y}{dx^2} = e^{-2t}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)$$

**(5)**

(b) Use you answers to part (a) to show that the substitution $x = e^t$ transforms the differential equation

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3$$

into

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = e^{3t}$$

**(3)**

(c) Hence find the general solution of

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3$$

**(6)**

**(Total 14 marks)**

---

**8.** (a) Given that $z = e^{i\theta}$, show that

$$z^p + \frac{1}{z^p} = 2\cos p\theta,$$

where $p$ is a positive integer.

**(2)**

(b) Given that

$$\cos^4\theta = A\cos 4\theta + B\cos 2\theta + C,$$

find the values of the constants $A$, $B$ and $C$.

**(6)**

The region $R$ bounded by the curve with equation $y = \cos^2 x$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, and the $x$-axis is rotated through $2\pi$ about the $x$-axis.

(c) Find the volume of the solid generated.

**(6)**

**(Total 14 marks)**

---

**TOTAL FOR PAPER: 75 MARKS**

**END**

Paper Reference(s)

**6669/01**

# Edexcel GCE

## Further Pure Mathematics FP3

## Advanced

Sample Assessment Material

Time: 1 hour 30 minutes

| Materials required for examination | Items included with question papers |
|---|---|
| Mathematical Formulae | Nil |

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.**

### Instructions to Candidates

In the boxes on the answer book, write your centre number, candidate number, your surname, initial(s) and signature.
Check that you have the correct question paper.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

### Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 4 pages in this question paper. Any blank pages are indicated.

### Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
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**1.** Find the eigenvalues of the matrix $\begin{pmatrix} 7 & 6 \\ 6 & 2 \end{pmatrix}$

**(Total 4 marks)**

---

**2.** Find the values of $x$ for which

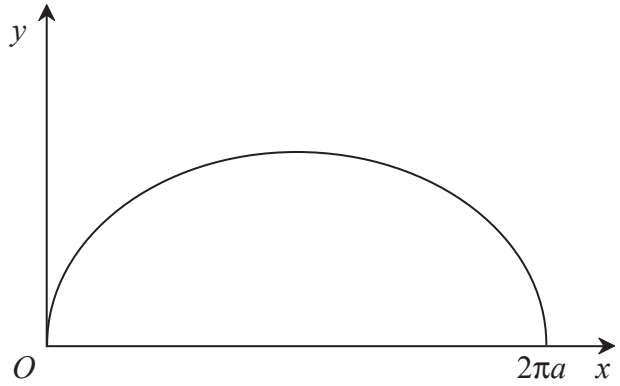
$$9\cosh x - 6\sinh x = 7$$

giving your answers as natural logarithms.

**(Total 6 marks)**

---

**3.** **Figure 1**




The parametric equations of the curve $C$ shown in Figure 1 are

$$x = a(t - \sin t), \quad y = a(1 - \cos t), \quad 0 \leq t \leq 2\pi$$

Find, by using integration, the length of $C$.

**(Total 6 marks)**

---

**4.** Find $\int \sqrt{(x^2+4)}\,\mathrm{d}x$.

**(Total 7 marks)**

---

**5.** Given that $y = \arcsin x$ prove that

(a) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\sqrt{(1-x^2)}}$ **(3)**

(b) $(1-x^2)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} - x\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$ **(4)**

**(Total 7 marks)**

---

**6.** $$I_n = \int_0^{\frac{\pi}{2}} x^n \sin x \, \mathrm{d}x$$

(a) Show that for $n \geq 2$

$$I_n = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2}$$

**(4)**

(b) Hence obtain $I_3$, giving your answers in terms of $\pi$.

**(4)**

**(Total 8 marks)**

---

**7.** $$\mathbf{A}(x) = \begin{pmatrix} 1 & x & -1 \\ 3 & 0 & 2 \\ 1 & 1 & 0 \end{pmatrix}, x \neq \frac{5}{2}$$

(a) Calculate the inverse of $\mathbf{A}(x)$.

$$\mathbf{B} = \begin{pmatrix} 1 & 3 & -1 \\ 3 & 0 & 2 \\ 1 & 1 & 0 \end{pmatrix}$$

**(8)**

The image of the vector $\begin{pmatrix} p \\ q \\ r \end{pmatrix}$ when transformed by $\mathbf{B}$ is $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$

(b) Find the values of $p$, $q$ and $r$.

**(4)**

**(Total 14 marks)**

---

**8.** The points $A$, $B$, $C$, and $D$ have position vectors

$$\mathbf{a} = 2\mathbf{i} + \mathbf{k}, \mathrm{b} = \mathbf{i} + 3\mathbf{j}, \mathbf{c} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k}, \mathbf{d} = 4\mathbf{j} + \mathbf{k}$$

respectively.

(a) Find $\overrightarrow{AB} \times \overrightarrow{AC}$ and hence find the area of triangle $ABC$.

**(7)**

(b) Find the volume of the tetrahedron $ABCD$.

**(2)**

(c) Find the perpendicular distance of $D$ from the plane containing $A$, $B$ and $C$.

**(3)**

**(Total 12 marks)**

**9.** The hyperbola $C$ has equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

(a) Show that an equation of the normal to $C$ at $P(a \sec \theta, b \tan \theta)$ is

$$by + ax \sin \theta = (a^2 + b^2)\tan \theta$$

**(6)**

The normal at $P$ cuts the coordinate axes at $A$ and $B$. The mid-point of $AB$ is $M$.

(b) Find, in cartesian form, an equation of the locus of $M$ as $\theta$ varies.

**(7)**

**(Total 13 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**

Paper Reference(s)

**6689/01**

# Edexcel GCE

## Decision Mathematics D1

## Advanced/Advanced Subsidiary

Sample Assessment Material

Time: 1 hour 30 minutes

**Materials required for examination**
Nil

**Items included with question papers**
D1 Answer book

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions to Candidates

Write your answers for this paper in the D1 answer book provided.
In the boxes on the answer book, write your centre number, candidate number, your surname, initial(s) and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.
Complete your answers in blue or black ink or pencil.
Do not return the question paper with the answer book.

### Information for Candidates

Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 12 pages in this question paper. The answer book has 16 pages. Any blank pages are indicated.

### Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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**Write your answers in the D1 answer book for this paper.**

**1.** Use the binary search algorithm to try to locate the name NIGEL in the following alphabetical list. Clearly indicate how you chose your pivots and which part of the list is being rejected at each stage.

1. Bhavika
2. Clive
3. Elizabeth
4. John
5. Mark
6. Nicky
7. Preety
8. Steve
9. Trevor
10. Verity

**(Total 4 marks)**

**2.**

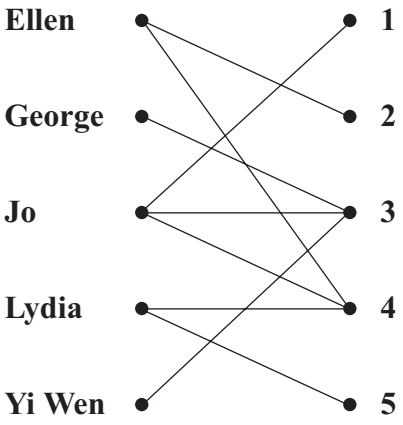


**Figure 1**

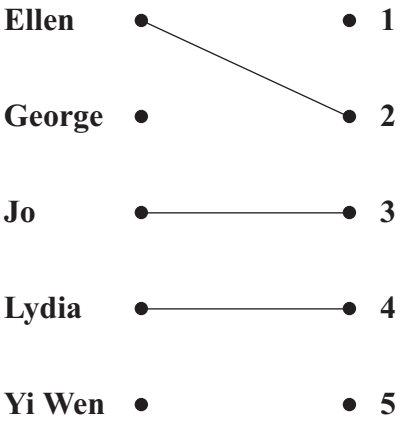


**Figure 2**

Figure 1 shows the possible allocations of five people, Ellen, George, Jo, Lydia and Yi Wen to five tasks, 1, 2, 3, 4 and 5.

Figure 2 shows an initial matching.

(a) Find an alternating path linking George with 5. List the resulting improved matching this gives.

**(3)**

(b) Explain why it is not possible to find a complete matching.

**(1)**

George now has task 2 added to his possible allocation.

(c) Using the improved matching found in part (a) as the new initial matching, find an alternating path linking Yi Wen with task 1 to find a complete matching. List the complete matching.

**(3)**

**(Total 7 marks)**

**3.**

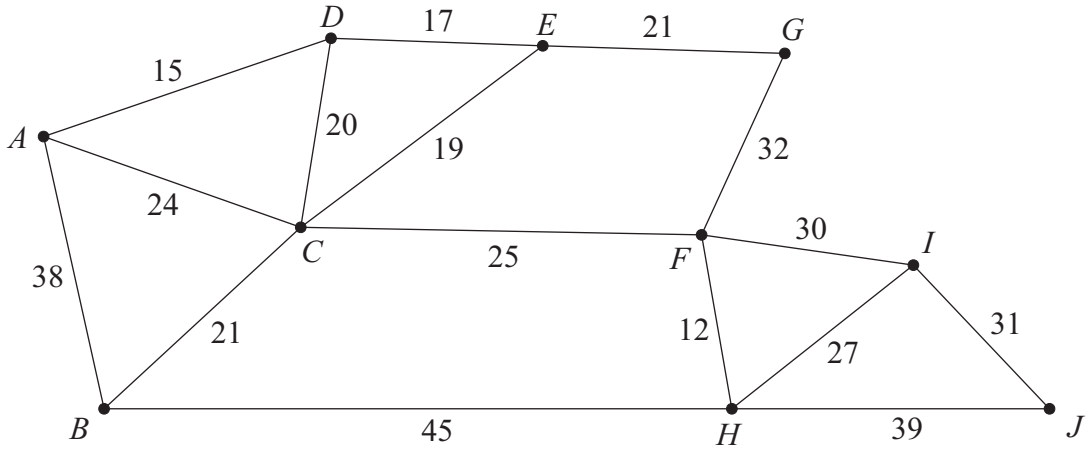


**Figure 3**

The network in Figure 3 shows the distances, in metres, between 10 wildlife observation points. The observation points are to be linked by footpaths, to form a network along the arcs indicated, using the least possible total length.

(a) Find a minimum spanning tree for the network in Figure 3, showing clearly the order in which you selected the arcs for your tree, using

(i) Kruskal's algorithm,

**(3)**

(ii) Prim's algorithm, starting from $A$.

**(3)**

Given that footpaths are already in place along $AB$ and $FI$ and so should be included in the spanning tree,

(b) explain which algorithm you would choose to complete the tree, and how it should be adapted. (You do **not** need to find the tree.)

**(2)**

**(Total 8 marks)**

  

**4.** 650 431 245 643 455 134 710 234 162 452

(a) The list of numbers above is to be sorted into **descending** order. Perform a Quick Sort to obtain the sorted list, giving the state of the list after each pass, indicating the pivot elements.

**(5)**

The numbers in the list represent the lengths, in mm, of some pieces of wood. The wood is sold in one metre lengths.

(b) Use the first-fit decreasing bin packing algorithm to determine how these pieces could be cut from the minimum number of one metre lengths. (You should ignore wastage due to cutting.)

**(4)**

(c) Determine whether your solution to part (b) is optimal. Give a reason for your answer.

**(2)**

**(Total 11 marks)**

**5.** (a) Explain why a network cannot have an odd number of vertices of odd degree.

**(2)**

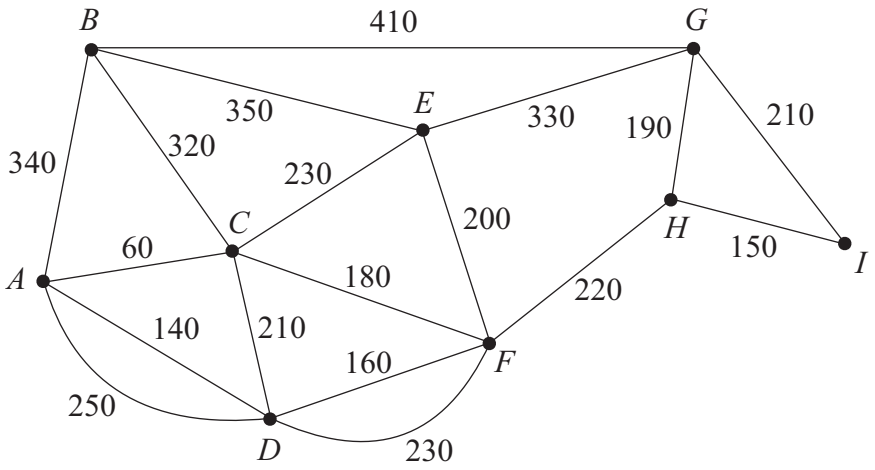


**Figure 4**

Figure 4 shows a network of paths in a public park. The number on each arc represents the length of that path in metres. Hamish needs to walk along each path at least once to check the paths for frost damage starting and finishing at *A*. He wishes to minimise the total distance he walks.

(b) Use the route inspection algorithm to find which paths, if any, need to be traversed twice.

**(4)**

(c) Find the length of Hamish's route.
[The total weight of the network in Figure 4 is 4180m.]

**(1)**

**(Total 7 marks)**

**6.**

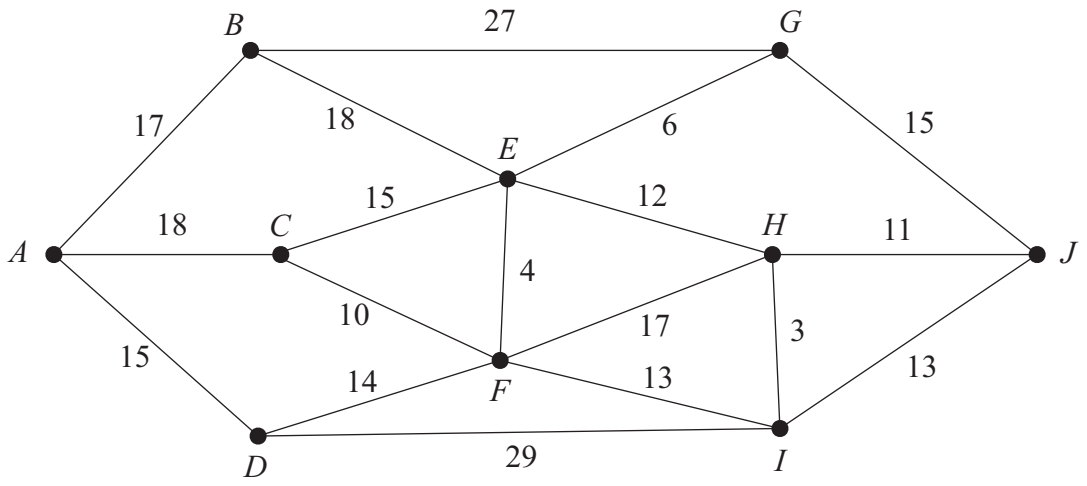


**Figure 5**

Figure 5 shows a network of roads. The number on each arc represents the length of that road in km.

(a) Use Dijkstra's algorithm to find the shortest route from *A* to *J*. State your shortest route and its length.

**(5)**

(b) Explain how you determined the shortest route from your labelled diagram.

**(2)**

The road from *C* to *F* will be closed next week for repairs.

(c) Find a shortest route from *A* to *J* that does not include *CF* and state its length.

**(2)**

**(Total 9 marks)**

**7.**

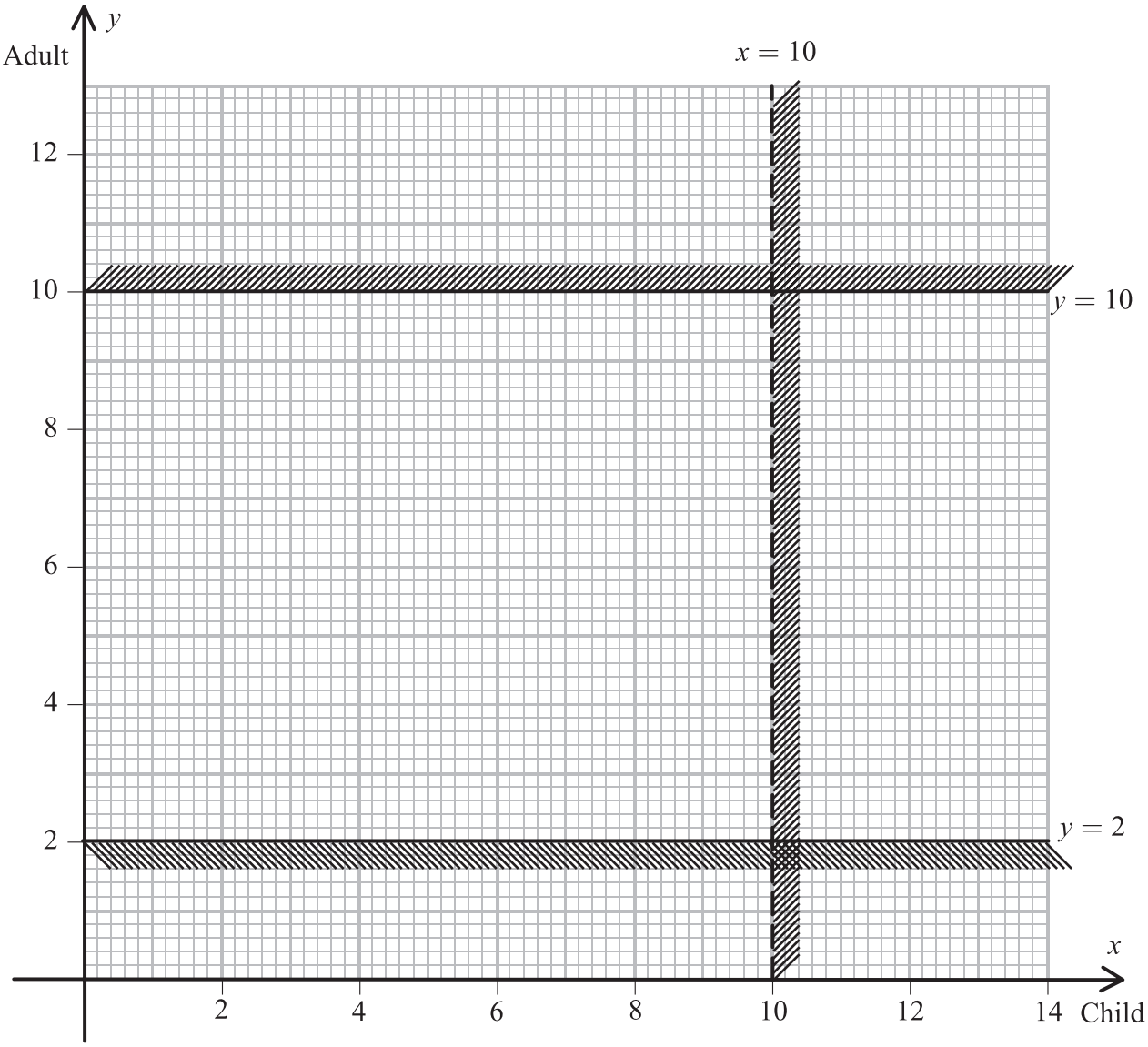


**Figure 6**

The captain of the *Malde Mare* takes passengers on trips across the lake in her boat.

The number of children is represented by $x$ and the number of adults by $y$.

Two of the constraints limiting the number of people she can take on each trip are

$$x < 10$$

and

$$2 \leqslant y \leqslant 10$$

These are shown on the graph in Figure 6, where the rejected regions are shaded out.

(a) Explain why the line $x = 10$ is shown as a dotted line.

**(1)**

(b) Use the constraints to write down statements that describe the number of children and the number of adults that can be taken on each trip.

**(3)**

For each trip she charges £2 per child and £3 per adult. She must take at least £24 per trip to cover costs.

The number of children must not exceed twice the number of adults.

(c) Use this information to write down two inequalities.

**(2)**

(d) Add two lines and shading to Diagram 1 in your answer book to represent these inequalities. Hence determine the feasible region and label it R.

**(4)**

(e) Use your graph to determine how many children and adults would be on the trip if the captain takes:

(i) the minimum number of passengers,

(ii) the maximum number of passengers.

**(4)**

**(Total 14 marks)**

**8.**

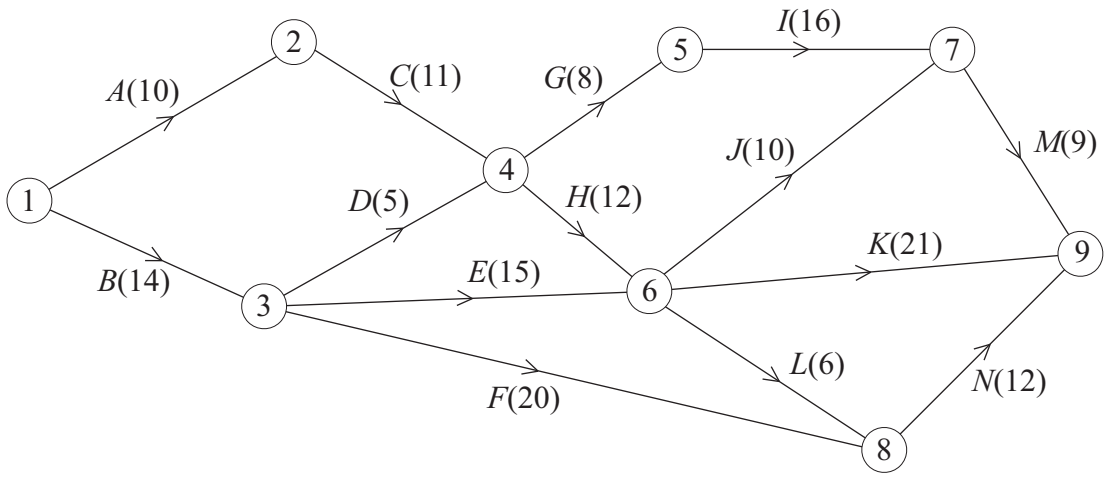


**Figure 7**

An engineering project is modelled by the activity network shown in Figure 7. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires one worker. The project is to be completed in the shortest time.

(a) Calculate the early time and late time for each event. Write these in the boxes in Diagram 1 in the answer book.

**(4)**

(b) State the critical activities.

**(1)**

(c) Find the total float on activities *D* and *F*. You must show your working.

**(3)**

(d) On the grid in the answer book, draw a cascade (Gantt) chart for this project.

**(4)**

The chief engineer visits the project on day 15 and day 25 to check the progress of the work. Given that the project is on schedule,

(e) which activities **must** be happening on each of these two days?

**(3)**

**(Total 15 marks)**

**TOTAL FOR PAPER: 75 MARKS**

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Candidate No. | | | | | | 6 | 6 | 8 | 9 | / | 0 | 1 | Signature | |

Paper Reference(s)

**6689/01**

# Edexcel GCE

## Decision Mathematics D1

## Advanced/Advanced Subsidiary

Sample Assessment Material

## Answer Book

## Do not return the question paper with the answer book

Examiner's use only

| | | |
|---|---|---|
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Team Leader's use only

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**1.**

**Q1**

**(Total 4 marks)**

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**2.**

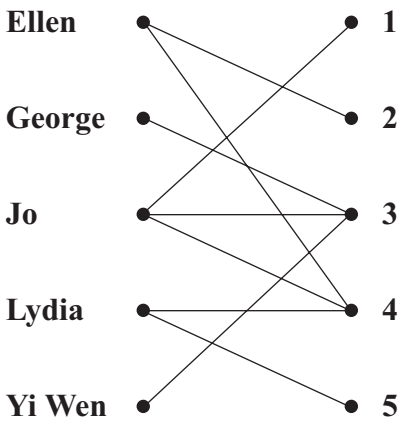


**Figure 1**

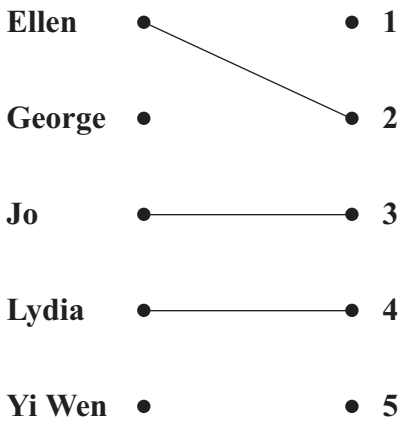


**Figure 2**

**Q2**

**(Total 7 marks)**

Leave blank

**3.**

**Q3**

**(Total 8 marks)**

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**4.**

**Q4**

**(Total 11 marks)**

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**5.**

**Q5**

**(Total 7 marks)**

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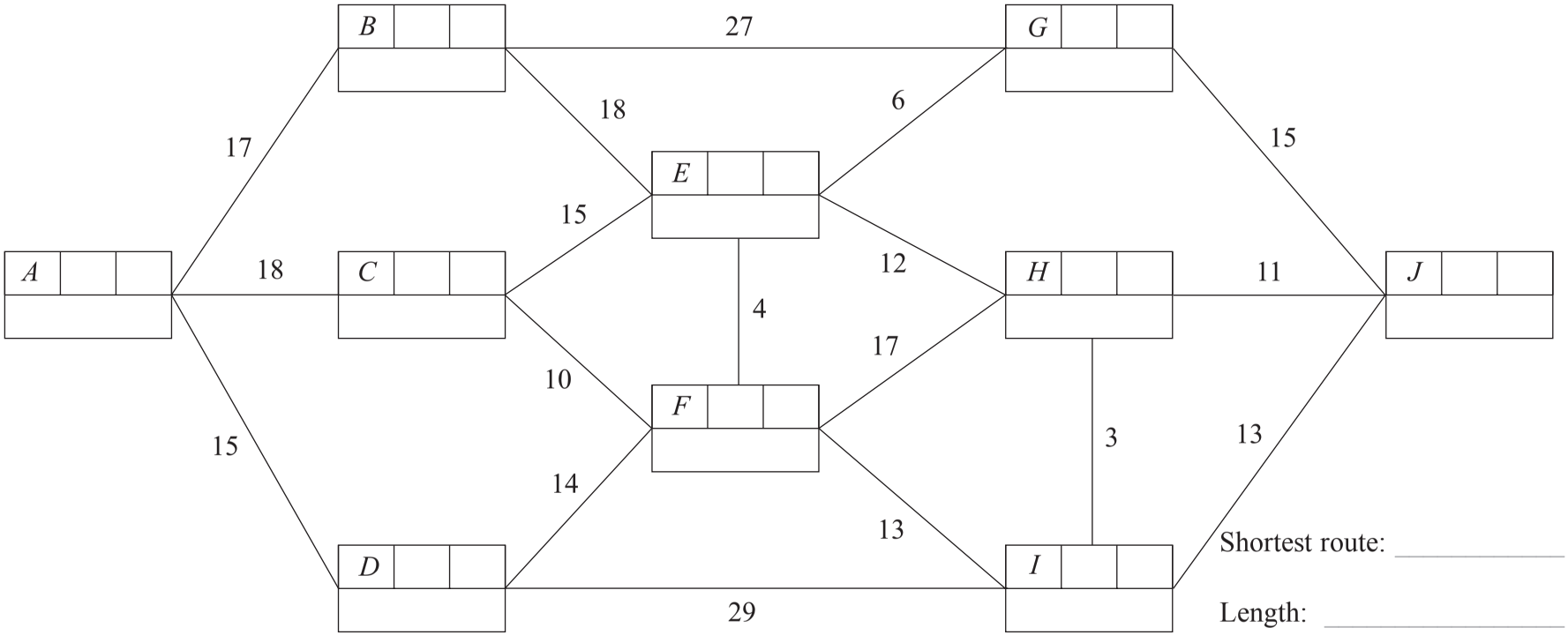
**6.**




Shortest route: _____

Length: _____

Leave blank

**(Total 9 marks)**

**Q6**

Leave blank

**7.**

Leave blank

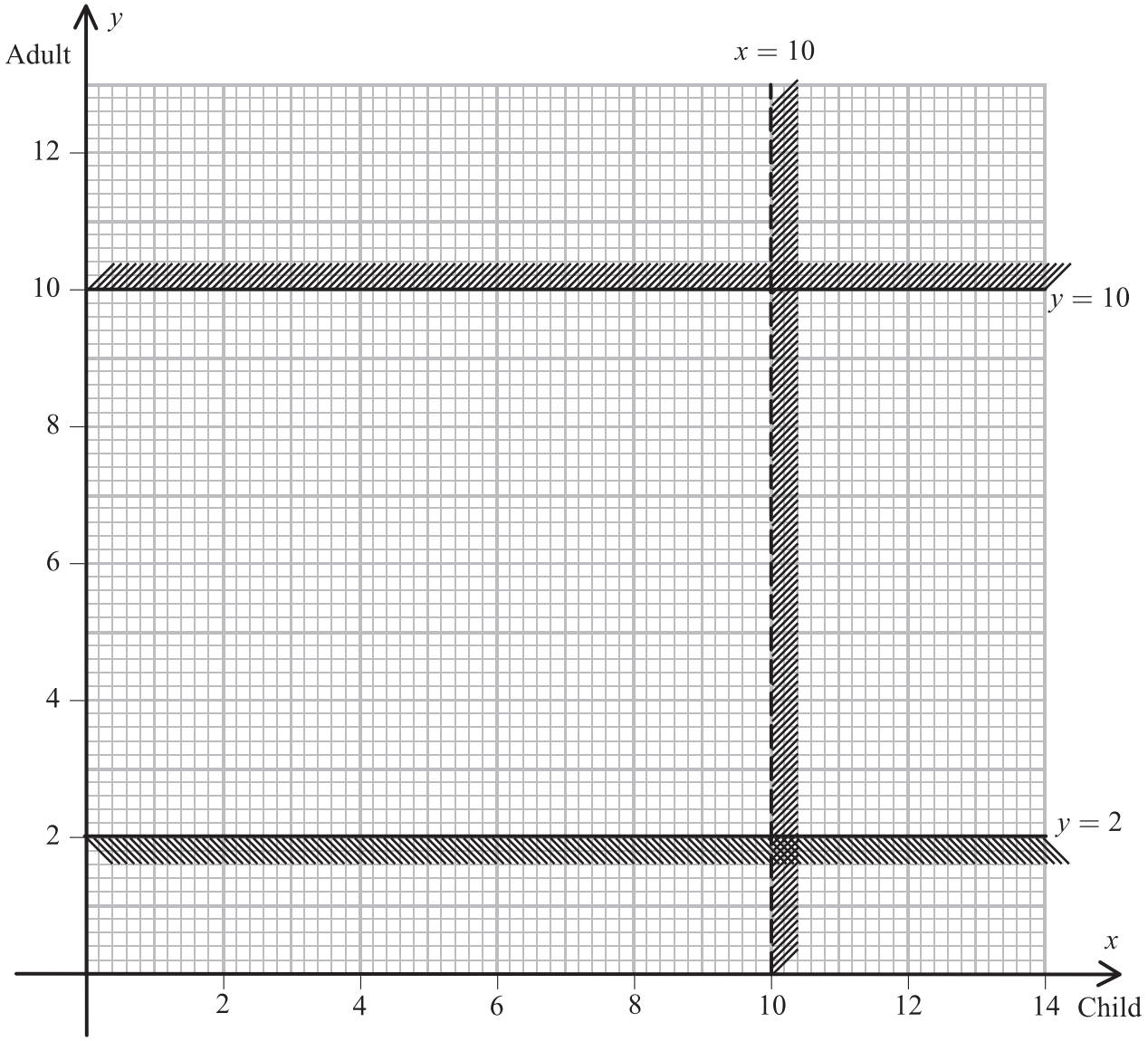



**Figure 6**

**Q7**

**(Total 14 marks)**

**8.**

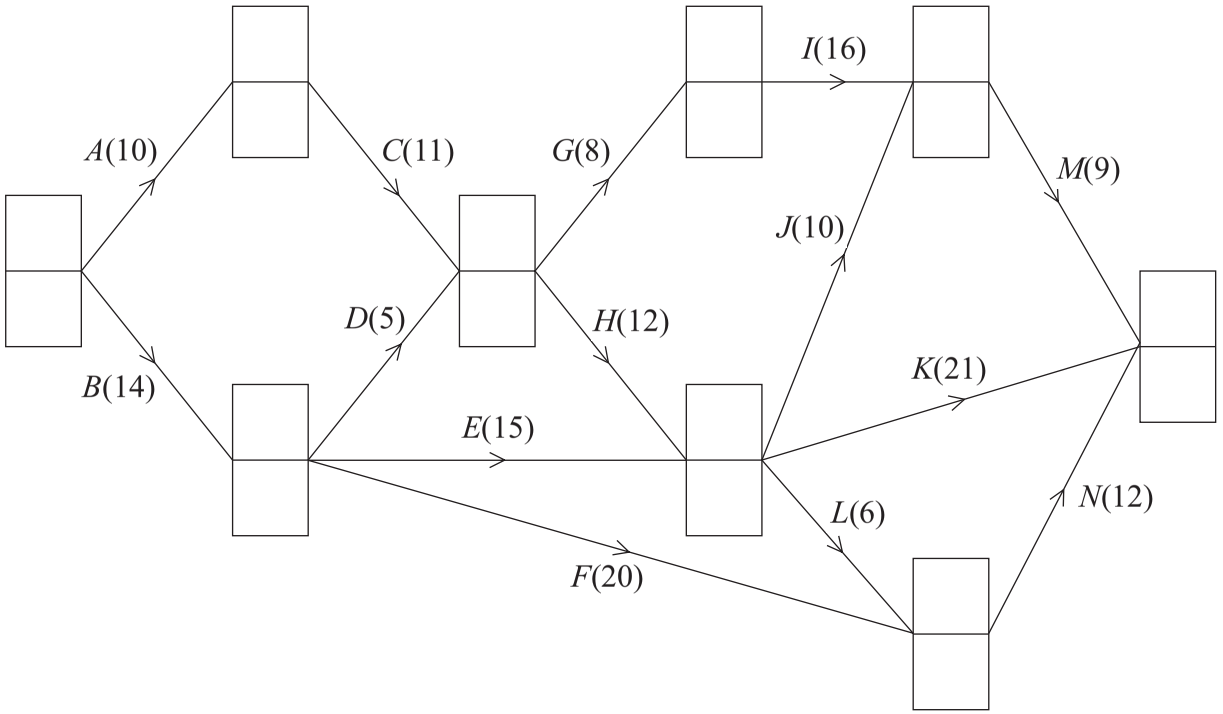



**Diagram 1**

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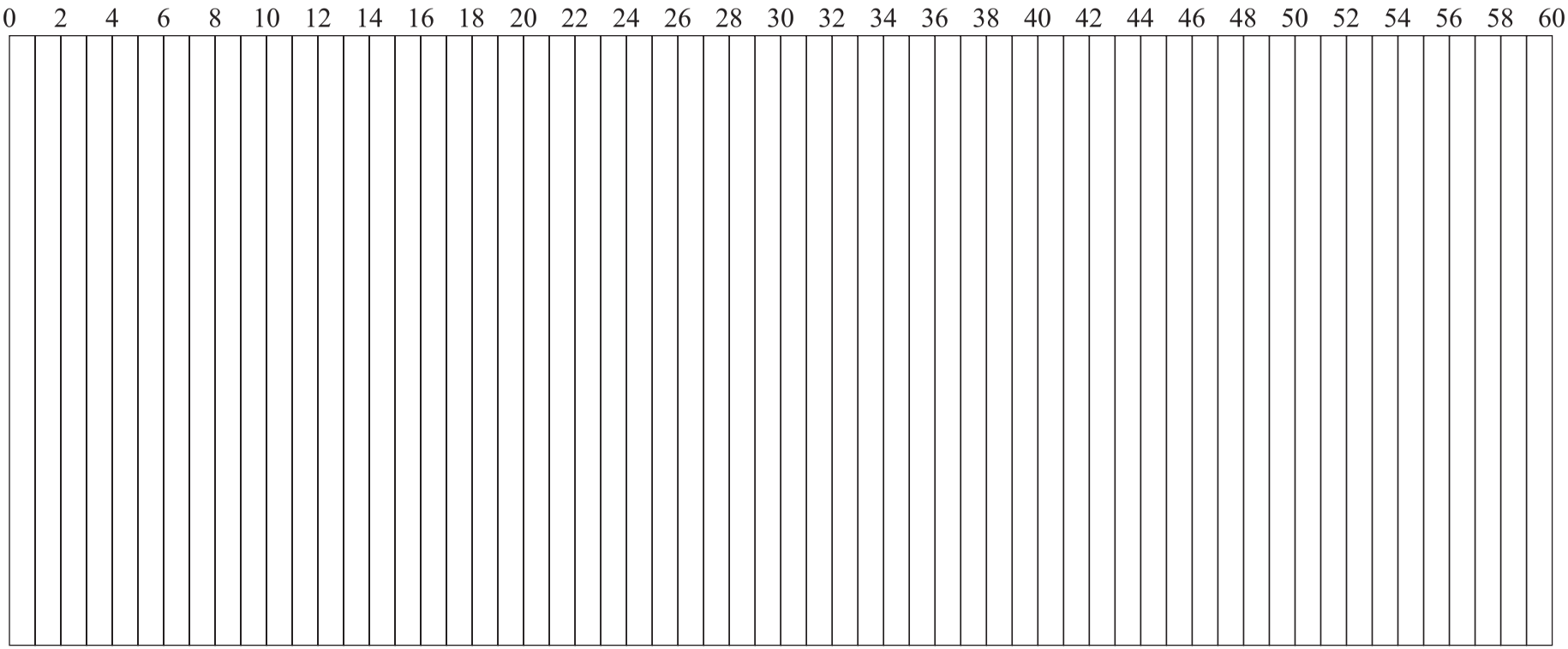


Activities that **must** be taking place on (i) day 15 _____

(ii) day 25 _____

**Q8**

**(Total 15 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**

Leave blank

Paper Reference(s)

**6690/01**

# Edexcel GCE

## Decision Mathematics D2

## Advanced/Advanced Subsidiary

Sample Assessment Material

Time: 1 hour 30 minutes

| **Materials required for examination** | **Items included with question papers** |
|---|---|
| Nil | D2 Answer Book |

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions to Candidates

Write your answers for this paper in the D2 answer book provided.
In the boxes on the answer book, write your centre number, candidate number, your surname, intials and signature.
Check that you have the correct question paper.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.
Do not return the question paper with the answer book.

### Information for Candidates

Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this question paper is 75.
There are 8 pages in this question paper. The answer book has 8 pages. Any blank pages are indicated.

### Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the examiner.
Answers without working may not gain full credit.

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**Write your answers for this paper in the D2 answer book.**

**1.**

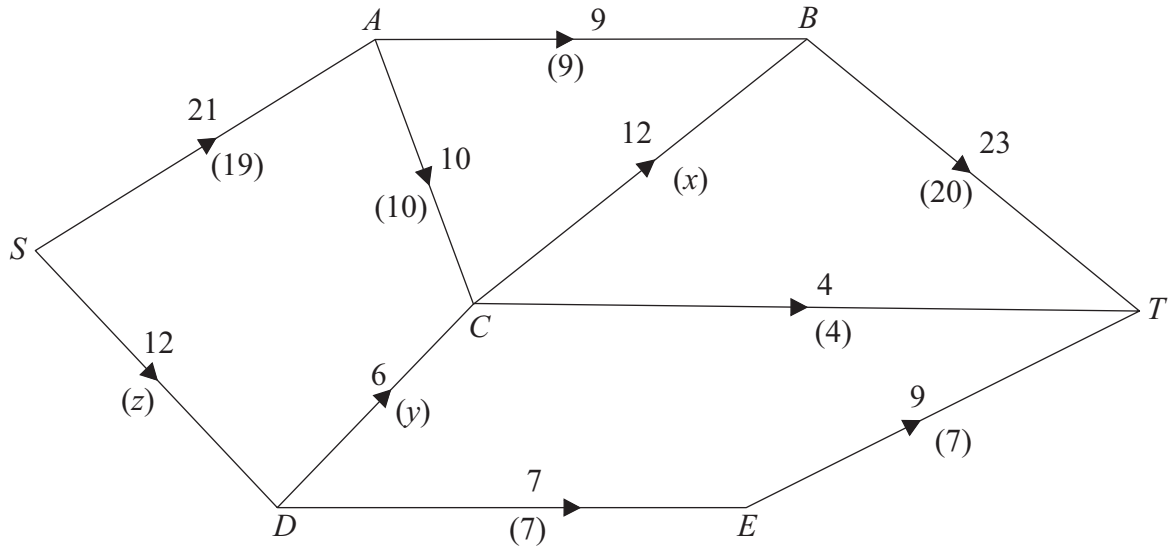


**Figure 1**

Figure 1 shows a directed, capacitated network where the number on each arc is its capacity. A possible flow is shown from $S$ to $T$ and the value in brackets on each arc is the flow in that arc.

(a) Find the values of $x$, $y$, and $z$.

**(3)**

(b) Find, by inspection, the maximal flow from $S$ to $T$ and verify that it is maximal.

**(2)**

**(Total 5 marks)**

**2.** A three-variable linear programming problem in $x$, $y$ and $z$ is to be solved. The objective is to maximise the profit $P$. The following initial tableau was obtained.

| Basic variable | $x$ | $y$ | $z$ | $r$ | $s$ | Value |
|---|---|---|---|---|---|---|
| $r$ | 2 | 0 | 4 | 1 | 0 | 80 |
| $s$ | 1 | 4 | 2 | 0 | 1 | 160 |
| $P$ | –2 | –8 | –20 | 0 | 0 | 0 |

(a) Taking the most negative number in the profit row to indicate the pivot column, perform one complete iteration of the simplex algorithm, to obtain tableau $T$. State the row operations that you use.

**(5)**

(b) Write down the profit equation shown in tableau $T$.

**(1)**

(c) State whether tableau $T$ is optimal. Give a reason for your answer.

**(1)**

**(Total 7 marks)**

**3.** Freezy Co. has three factories $A$, $B$ and $C$. It supplies freezers to three shops $D$, $E$ and $F$. The table shows the transportation cost in pounds of moving one freezer from each factory to each outlet. It also shows the number of freezers available for delivery at each factory and the number of freezers required at each shop. The total number of freezers required is equal to the total number of freezers available.

| | $D$ | $E$ | $F$ | Available |
|---|---|---|---|---|
| $A$ | 21 | 24 | 16 | 24 |
| $B$ | 18 | 23 | 17 | 32 |
| $C$ | 15 | 19 | 25 | 14 |
| Required | 20 | 30 | 20 | |

(a) Use the north-west corner rule to find an initial solution.

**(2)**

(b) Obtain improvement indices for each unused route.

**(5)**

(c) Use the stepping-stone method **once** to obtain a better solution and state its cost.

**(4)**

**(Total 11 marks)**

**4.**

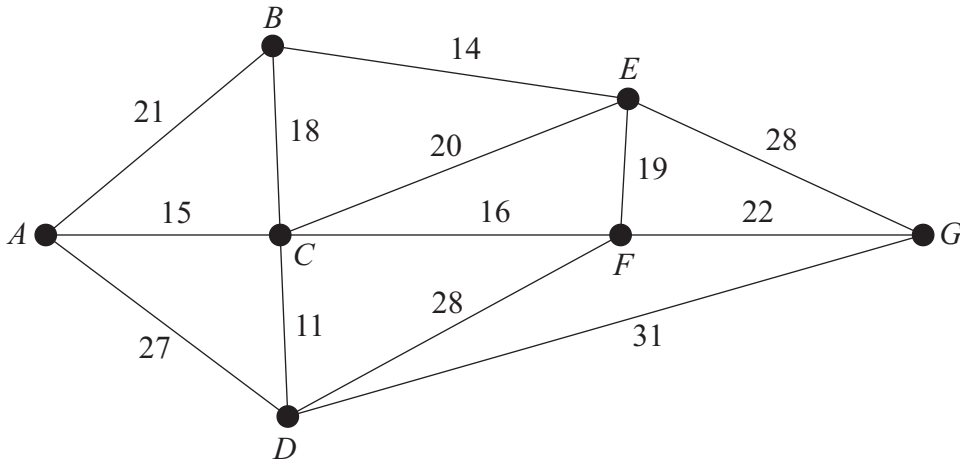


**Figure 2**

The network in Figure 2 shows the distances, in km, of the cables between seven electricity relay stations $A$, $B$, $C$, $D$, $E$, $F$ and $G$. An inspector needs to visit each relay station. He wishes to travel a minimum distance, and his route must start and finish at the same station.

By deleting $C$, a lower bound for the length of the route is found to be 129 km.

(a) Find another lower bound for the length of the route by deleting $F$. State which is the best lower bound of the two.

**(5)**

(b) By inspection, complete the table of least distances.

**(2)**

The table can now be taken to represent a complete network.

(c) Using the nearest-neighbour algorithm, starting at $F$, obtain an upper bound to the length of the route. State your route.

**(4)**

**(Total 11 marks)**

**5.** Three warehouses $W$, $X$ and $Y$ supply televisions to three supermarkets $J$, $K$ and $L$. The table gives the cost, in pounds, of transporting a television from each warehouse to each supermarket. The warehouses have stocks of 34, 57 and 25 televisions respectively, and the supermarkets require 20, 56 and 40 televisions respectively. The total cost of transporting the televisions is to be minimised.

| | $J$ | $K$ | $L$ |
|---|---|---|---|
| $W$ | 3 | 6 | 3 |
| $X$ | 5 | 8 | 4 |
| $Y$ | 2 | 5 | 7 |

Formulate this transportation problem as a linear programming problem. Make clear your decision variables, objective function and constraints.

**(Total 6 marks)**

---

**6.**

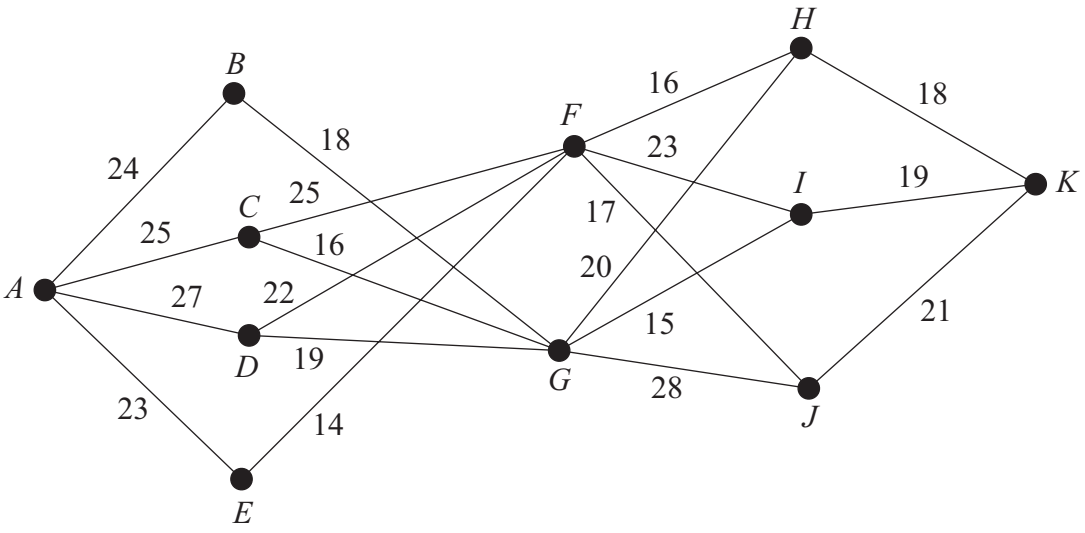


**Figure 3**

A maximin route is to be found through the network shown in Figure 3.

Complete the table in the answer book, and hence find a maximin route.

**(Total 9 marks)**

---

**7.** Four salespersons $A$, $B$, $C$ and $D$ are to be sent to visit four companies 1, 2, 3 and 4. Each salesperson will visit exactly one company, and all companies will be visited.
Previous sales figures show that each salesperson will make sales of different values, depending on the company that they visit. These values (in £10 000s) are shown in the table below.

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Ann | 26 | 30 | 30 | 30 |
| Brenda | 30 | 23 | 26 | 29 |
| Connor | 30 | 25 | 27 | 24 |
| Dave | 30 | 27 | 25 | 21 |

(a) Use the Hungarian algorithm to obtain an allocation that **maximises** the sales. You must make your method clear and show the table after each stage.

**(11)**

(b) State the value of the maximum sales.

**(2)**

(c) Show that there is a second allocation that maximises the sales.

**(2)**

**(Total 15 marks)**

---

**8.** A two person zero-sum game is represented by the following pay-off matrix for player $A$.

| | I | II | III |
|---|---|---|---|
| I | 5 | 2 | 3 |
| II | 3 | 5 | 4 |

(a) Verify that there is no stable solution to this game.

**(3)**

(b) Find the best strategy for player $A$ and the value of the game to her.

**(8)**

**(Total 11 marks)**

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**TOTAL FOR PAPER: 75 MARKS**

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Paper Reference(s)

**6690/01**

# Edexcel GCE

## Decision Mathematics D2

## Advanced/Advanced Subsidiary

Sample Assessment Material

## Answer Book

**Do not return the question paper with the answer book.**

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Team Leader's use only

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**1.**

**Q1**

**(Total 5 marks)**

Leave blank

**2.**

| Basic variable | $x$ | $y$ | $z$ | $r$ | $s$ | Value |
|---|---|---|---|---|---|---|
| $r$ | 2 | 0 | 4 | 1 | 0 | 80 |
| $s$ | 1 | 4 | 2 | 0 | 1 | 160 |
| $P$ | –2 | –8 | –20 | 0 | 0 | 0 |

*You may not need to use all of these tableaux*

| Basic variable | $x$ | $y$ | $z$ | $r$ | $s$ | Value | Row operations |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |

| Basic variable | $x$ | $y$ | $z$ | $r$ | $s$ | Value | Row operations |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
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| Basic variable | $x$ | $y$ | $z$ | $r$ | $s$ | Value | Row operations |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |

**Q2**

**(Total 7 marks)**

Leave blank

**3.** *You may not wish to use all of these tables*

| | *D* | *E* | *F* |
|---|---|---|---|
| *A* | | | |
| *B* | | | |
| *C* | | | |

| | *D* | *E* | *F* |
|---|---|---|---|
| *A* | | | |
| *B* | | | |
| *C* | | | |

Leave blank

*You may not wish to use all of these tables*

| | *D* | *E* | *F* |
|---|---|---|---|
| *A* | | | |
| *B* | | | |
| *C* | | | |

| | *D* | *E* | *F* |
|---|---|---|---|
| *A* | | | |
| *B* | | | |
| *C* | | | |

| | *D* | *E* | *F* |
|---|---|---|---|
| *A* | | | |
| *B* | | | |
| *C* | | | |

**Q3**

**(Total 11 marks)**

Leave blank

**4.** (a)

(b) Table of least distances

| | *A* | *B* | *C* | *D* | *E* | *F* | *G* |
|---|---|---|---|---|---|---|---|
| *A* | – | 21 | 15 | 26 | 35 | 31 | 53 |
| *B* | 21 | – | 18 | 29 | 14 | | 42 |
| *C* | 15 | 18 | – | 11 | 20 | 16 | 38 |
| *D* | 26 | 29 | 11 | – | | 27 | 31 |
| *E* | 35 | 14 | 20 | | – | 19 | 28 |
| *F* | 31 | | 16 | 27 | 19 | – | 22 |
| *G* | 53 | 42 | 38 | 31 | 28 | 22 | – |

(c)

**Q4**

**(Total 11 marks)**

Leave blank

**5.**

**Q5**

**(Total 6 marks)**

Leave blank

**6.** *You may not wish to use all of these lines*

| Stage | State | Action | Value |
|---|---|---|---|
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**Maximin route:** _____

_____

_____

_____

**Q6**

**(Total 9 marks)**

Leave blank

**7.** *You may not wish to use all of these lines*

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Ann | | | | |
| Brenda | | | | |
| Connor | | | | |
| Dave | | | | |

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Ann | | | | |
| Brenda | | | | |
| Connor | | | | |
| Dave | | | | |

Leave blank

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Ann | | | | |
| Brenda | | | | |
| Connor | | | | |
| Dave | | | | |

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Ann | | | | |
| Brenda | | | | |
| Connor | | | | |
| Dave | | | | |

**Q7**

**(Total 15 marks)**

Leave blank

**8.**

| | I | II | III |
|---|---|---|---|
| I | 5 | 2 | 3 |
| II | 3 | 5 | 4 |

Leave blank

Q8

**(Total 11 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**

# C Sample mark schemes

## Notes on Marking Principles

**1. Types of mark**

M marks: method marks
A marks: accuracy marks
B marks: unconditional accuracy marks (independent of M marks)

**2. Abbreviations**

cao – correct answer only
ft – follow through
isw – ignore subsequent working
SC: special case
oe – or equivalent (and appropriate)
dep – dependent
indep - independent

**3. No working**

If no working is shown then correct answers normally score full marks.

If no working is shown then incorrect (even though nearly correct) answers score no marks.

**4. With working**

If there is a wrong answer indicated on the answer line always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.

If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.

Any case of suspected misread loses A (and B) marks on that part, but can gain the M marks.

If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.

If there is a choice of methods shown, then no marks should be awarded, unless the answer on the answer line makes clear the method that has been used.

If there is no answer on the answer line then check the working for an obvious answer.

**5. Follow through marks**

Follow through marks which involve a single stage calculation can be awarded without working since you can check the answer yourself, but if ambiguous do not award.

Follow through marks which involve more than one stage of calculation can only be awarded on sight of the relevant working, even if it appears obvious that there is only one way you could get the answer given.

**6. Ignoring subsequent work**

It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct

It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.

Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

**7. Probability**

Probability answers must be given a fractions, percentages or decimals. If a candidate gives a decimal equivalent to a probability, this should be written to at least 2 decimal places (unless tenths).

Incorrect notation should lose the accuracy marks, but be awarded any implied method marks.

If a probability answer is given on the answer line using both incorrect and correct notation, award the marks.

If a probability fraction is given then cancelled incorrectly, ignore the incorrectly cancelled answer.

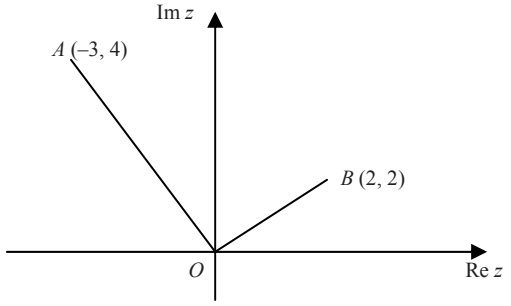
**8. Linear equations**

Full marks can be gained if the solution alone is given on the answer line, or otherwise unambiguously indicated in working (without contradiction elsewhere). Where the correct solution only is shown substituted, but not identified as the solution, the accuracy mark is lost but any method marks can be awarded.

**9. Parts of questions**

Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another.

| Question number | | Scheme | Marks |
|---|---|---|---|
| | | | |
| **1.** | (a) | $f'(x) = 3x^2 - 6x + 5$ | M1A1 (2) |
| | (b) | $f(1.4) = -0.136$ | B1 |
| | | $f'(1.4) = 2.48$ | B1ft |
| | | $x_0 = 1.4,\ x_1 = 1.4 - \dfrac{-0.136}{2.48}$ | M1 |
| | | $= 1.455$ (3 dpl) | A1 (4) |
| | | | **(6 marks)** |
| **2.** | (a) | $\begin{pmatrix} a & 4 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 \\ 2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 8 & a & a+8 \\ 2 & -1 & 1 \end{pmatrix}$ | M1 A1 A1 (3) |
| | (b) | $\det \mathbf{A} = a - (-4) = a + 4$ | B1 (1) |
| | (c) | Area of $R = 2$ | B1 |
| | | Area of $R' = 18$ | |
| | | Area scale factor is $9 = a + 4$ | M1 |
| | | $\therefore a = 5$ | A1 (3) |
| | | | **(7 marks)** |
| **3.** | (a) | $\mathbf{R}^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ | M1 A1 (2) |
| | (b) | Rotation of 90°, clockwise (about (0,0)) | B1, B1 (2) |
| | (c) | Rotation of 45° clockwise | B1ft (1) |
| | | | **(5 marks)** |
| **4.** | | End points: (4, –8) and (5, 2) | B1 |
| | | $\dfrac{\alpha - 4}{8} = \dfrac{5 - \alpha}{2}$ (or equiv.) | M1 |
| | | $\alpha = 4.8$ | A1 (3) |
| | | | **(3 marks)** |

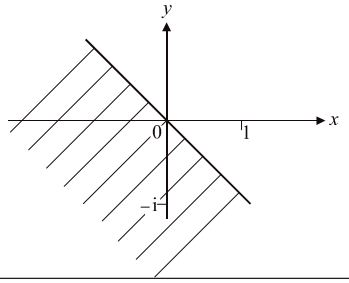
| Question number | | Scheme | Marks |
|---|---|---|---|
| **5.** | (a) | $\sum_{r=1}^{n}\left(r^2-r-1\right)=\sum_{r=1}^{n}r^2-\sum_{r=1}^{n}r-\sum_{r=1}^{n}1$ | M1 |
| | | $\sum_{r=1}^{n}1=n$ | B1 |
| | | $\sum_{r=1}^{n}\left(r^2-r-1\right)=\frac{n}{6}(n+1)(2n+1)-\frac{1}{2}n(n+1)-n$ | M1 |
| | | $=\frac{n}{6}\left(2n^2-8\right)$ | M1 A1 |
| | | $=\frac{1}{3}(n-2)n(n+2)$ (✱) | A1 (6) |
| | (b) | $\sum_{r=10}^{40}\left(r^2-r-1\right)=\sum_{r=1}^{40}\left(r^2-r-1\right)-\sum_{r=1}^{9}\left(r^2-r-1\right)$ | M1 |
| | | $=\frac{1}{3}\times3\times40\times42-\frac{1}{3}\times7\times9\times11=1449$ | M1 A1 (3) |
| | | | **(9 marks)** |
| **6.** | (a) | $\lvert z\rvert=\sqrt{\left(3^2+4^2\right)}=5$ | M1 A1 (2) |
| | (b) | $\arg z=\pi-\arctan\frac{4}{3}=2.21$ | M1 A1 (2) |
| | (c) | $w=\frac{-14+2\mathrm{i}}{-3+4\mathrm{i}}=\frac{(-14+2\mathrm{i})(-3-4\mathrm{i})}{(-3+4\mathrm{i})(-3-4\mathrm{i})}$ | M1 |
| | | $=\frac{(42+8)+\mathrm{i}(-6+56)}{9+16}$ | A1 A1 |
| | | $=\frac{50+50\mathrm{i}}{25}=2+2\mathrm{i}$ | A1 (4) |
| | (d) |  | B1 B1 (2) |
| | | | **(10 marks)** |

| Question number | | Scheme | Marks |
|---|---|---|---|
| **7.** | (a) | $a = 4$ | B1 (1) |
| | (b) | $y = 2a^{\frac{1}{2}}x^{\frac{1}{2}} \Rightarrow y' = a^{\frac{1}{2}}x^{-\frac{1}{2}}$ $y =$ and attempt $y'$ | M1 |
| | | $y' = \frac{1}{t}$ sub $x = 4t^2$ | M1 |
| | | Tangent is $y - 8t = \frac{1}{t}\left(x - 4t^2\right)$ | M1 |
| | | $yt = x + 4t^2$ (✱) | A1cso (4) |
| | (c) | $x = -4$ | B1ft |
| | | $15t = -4 + 4t^2$ Substitute (–4, 15) | M1 |
| | | $4t^2 - 15t - 4 = 0$ | |
| | | $(4t + 1)(t - 4) = 0$ Attempt to solve | M1 |
| | | $t = 4$ or $-\frac{1}{4}$ | A1 |
| | | $A = (64, 32)$ $B = (\frac{1}{4}, -2)$ M for attempt $A$ or $B$ | M1 A1 A1 (7) |
| | | | **(12 marks)** |
| **8.** | (a) | 1 + 2i | B1 (1) |
| | (b) | $(x - 1 + 2\text{i})(x - 1 - 2\text{i})$ are factors of f($x$) | M1 |
| | | so $x^2 - 2x + 5$ is a factor of f ($x$) | M1 A1 |
| | | f($x$) = ($x^2$ – 2$x$ + 5)(2$x$ – 1) | M1 A1ft |
| | | Third root is $\frac{1}{2}$ | A1 (6) |
| | (c) | $p = 10 + 2$ | M1 |
| | | = 12 | A1 (2) |
| | | | **(9 marks)** |

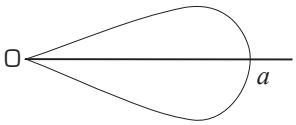
| Question number | Scheme | Marks |
|---|---|---|
| **9.** (a) | $\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}^1 = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$, for $n = 1$, $\begin{pmatrix} n+1 & n \\ -n & 1-n \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$ | B1 |
| | $\therefore$ true for $n = 1$ | |
| | Assume true for $n = k$, | |
| | $\begin{pmatrix} k+1 & k \\ -k & 1-k \end{pmatrix}\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 2k+2-k & k+1-0 \\ -2k-1+k & -k+0 \end{pmatrix}$ | M1 A2/1/0 |
| | $= \begin{pmatrix} (k+1)+1 & k+1 \\ -(k+1) & 1-(k+1) \end{pmatrix}$ | M1 A1 |
| | $\therefore$ true for $n = k + 1$ if true for $n = k$ | |
| | $\therefore$ true for $n \in \mathbb{Z}^+$ by induction | A1<br>(7) |
| (b) | $f(1) = 4 + 6 - 1 = 9 = 3 \times 3$ | B1 |
| | $\therefore$ true for $n = 1$ | |
| | Assume true for $n = k$, $f(k) = 4^k + 6k - 1$ is divisible by 3 | |
| | $f(k + 1) = 4^{k+1} + 6(k + 1) - 1$ | M1 A1 |
| | $= 4\times4^k + 6(k + 1) - 1$ | A1 |
| | $f(k + 1) - f(k) = 3\times4^k + 6$ | M1 |
| | $\therefore$ $f(k + 1) = 3(4^k + 2) - f(k)$ which is divisible by 3 | A1 |
| | $\therefore$ true for $n = k + 1$ if true for $n = k$ | |
| | $\therefore$ true for $n \in \mathbb{Z}^+$ by induction | A1<br>(7)<br>**(14 marks)** |

| Question number | Scheme | Marks |
|---|---|---|
| **1.** | $\frac{x}{x-3} > \frac{1}{x-2} \Rightarrow \frac{x}{x-3} - \frac{1}{x-2} > 0$ | |
| | $\Rightarrow \frac{x^2-3x+3}{(x-3)(x-2)} > 0$ | M1 A1 |
| | Numerator always positive | B1 |
| | Critical points of denominator $x = 2, x = 3$ | B1 |
| | $x < 2 : \text{den} = (-\text{ve})(-\text{ve}) = +\text{ve}$ | |
| | $2 < x < 3 : \text{den} = (-\text{ve})(+\text{ve}) = -\text{ve}$ | |
| | $3 < x : \text{den} = (+\text{ve})(+\text{ve}) = +\text{ve}$ | M1 A1 |
| | Set of values $x < 2$ and $x > 3$ $\{x : x < 2\} \cup \{x : x > 3\}$ | A1 (7) |
| **2.** (a) | $\frac{(r+1)^2 - r^2}{r^2(r+1)^2} = \frac{2r+1}{r^2(r+1)^2}$ | M1 A1 (2) |
| (b) | $\sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2} = \sum_{r=1}^{n} \left( \frac{1}{r^2} - \frac{1}{(r+1)^2} \right)$ | M1 |
| | $= \left( \frac{1}{1^2} - \frac{1}{2^2} \right) + \left( \frac{1}{2^2} - \frac{1}{3^2} \right) + \ldots \left( \frac{1}{n^2} - \frac{1}{(n+1)^2} \right)$ | M1 |
| | $= 1 - \frac{1}{(n+1)^2}$ | A1 (3) |
| **3.** (a) | $w = \frac{z-\text{i}}{z+1} \Rightarrow w(z+1) = (z-\text{i})$ | |
| | $z(w-1) = -\text{i} - w$ | |
| | $z = \frac{-\text{i}-w}{w-1}$ | M1 A1 |
| | $\lvert z \rvert = 1 \Rightarrow \left\lvert \frac{-\text{i}-w}{w-1} \right\rvert = 1$ | |
| | i.e. $\lvert w-1 \rvert = \lvert w+\text{i} \rvert$ | M1 A1 (4) |
| (b) | $\lvert z \rvert \leq 1 \Rightarrow \lvert w+\text{i} \rvert \leq \lvert w-1 \rvert$ | |
| |   | B1 (line) B1 (shading) (2) |

| Question number | Scheme | Marks |
|---|---|---|
| **4.** | $\frac{d^2y}{dx^2} + y\frac{dy}{dx} = x,\ y = 0,\ \frac{dy}{dx} = 2 \text{ at } x = 1$ | |
| | $\frac{d^2y}{dx^2} = 0 + 1 = 1$ | B1 |
| | Differentiating with respect to $x$ | |
| | $\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2} = 1$ | M1 A1 |
| | $\left.\frac{d^3y}{dx^3}\right\|_{x=1} = -(2)^2 + 0 + 1 = -3$ | A1 |
| | By Taylor's Theorem | |
| | $y = 0 + 2(x-1) + \frac{1}{2!}1(x-1)^2 + \frac{1}{3!}(-3)(x-1)^3$ | M1 A1 |
| | $= 2(x-1) + \frac{1}{2}(x-1)^2 - \frac{1}{2}(x-1)^3$ | A1 (7) |
| **5.** (a) | $\frac{dS}{dt} - (0.1)S = t$ | M1 |
| | Integrating factor $e^{-\int(0.1)dt} = e^{-(0.1)t}$ | |
| | $\frac{d}{dt}\left[Se^{-(0.1)t}\right] = te^{-(0.1)t}$ | A1 A1 |
| | $\therefore Se^{-(0.1)t} = \int te^{-(0.1)t}dt$ | |
| | $= -10te^{-(0.1)t} - 100e^{-(0.1)t} + C$ | M1 A1 |
| | $S = Ce^{(0.1)t} - 10t - 100$ | A1 (6) |
| (b) | $S = 200 \text{ at } t = 0$ | |
| | $\Rightarrow 200 = C - 100 \quad \text{i.e. } C = 300$ | |
| | $S = 300e^{(0.1)t} - 10t - 100$ | M1 A1 |
| | At $t = 10,\ S = 300e - 100 - 100$ | |
| | $= 615.484\ 55$ | M1 |
| | Assets £615 million | A1ft (4) |

| Question number | Scheme | Marks |
|---|---|---|
| **6.** (a) |  | B1 (Shape) B1 (Labels) (2) |
| (b) | Tangent parallel to initial line when $y = r\sin\theta$ is stationary | |
| | Consider therefore $\frac{\mathrm{d}}{\mathrm{d}\theta}\left(a^2\cos 2\theta\sin^2\theta\right)$ | |
| | $= -2\sin 2\theta\sin^2\theta + \cos 2\theta(2\sin\theta\cos\theta)$ | |
| | $= 0$ | M1A1 |
| | $2\sin\theta[\cos 2\theta\cos\theta - \sin 2\theta\sin\theta] = 0$ | |
| | $\sin\theta \neq 0 \Rightarrow \cos 3\theta = 0 \Rightarrow \theta = \frac{\pi}{6}$ or $\frac{-\pi}{6}$ | M1A1 |
| | Coordinates of the points $\left(\frac{1}{\sqrt{2}}a,\ \frac{\pi}{6}\right)\left(\frac{1}{\sqrt{2}}a,\ \frac{-\pi}{6}\right)$ | A1A1 (6) |
| (c) | $\text{Area} = \frac{1}{2}\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} r^2\,\mathrm{d}\theta = \frac{1}{2}a^2\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}}\cos 2\theta\,\mathrm{d}\theta$ | M1A1 |
| | $= \frac{1}{2}a^2\left[\frac{\sin 2\theta}{2}\right]_{\frac{-\pi}{4}}^{\frac{\pi}{4}} = \frac{a^2}{4}\left[1-(-1)\right] = \frac{a^2}{2}$ | M1A1 (4) |

| Question number | | Scheme | Marks |
|---|---|---|---|
| 7. | (a) | (i) $x = e^t$, $\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = e^{-t}\frac{dy}{dt}$ | M1A1 |
| | | $\left(\frac{dx}{dt} = e^t\right)$ | |
| | | (ii) $\frac{d^2y}{dx^2} = \frac{dt}{dx}\frac{d}{dt}\left[e^{-t}\frac{dy}{dt}\right]$ | M1 |
| | | $= e^{-t}\left[-e^{-t}\frac{dy}{dt} + e^{-t}\frac{d^2y}{dt^2}\right]$ | A1 |
| | | $= e^{-2t}\left[\frac{d^2y}{dt^2} - \frac{dy}{dt}\right]$ | A1 (5) |
| | (b) | $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3$ | |
| | | $e^{2t}e^{-2t}\left[\frac{d^2y}{dt^2} - \frac{dy}{dt}\right], -2e^te^{-t}\frac{dy}{dt} + 2y = e^{3t}$, | M1A1,A1 |
| | | $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = e^{3t}$ | (3) |
| | (c) | Auxiliary equation $m^2 - 3m + 2 = 0$ | |
| | | $(m-1)(m-2) = 0$ | |
| | | Complementary function $y = Ae^t + Be^{2t}$ | M1A1 |
| | | Particular integral $= \frac{e^{3t}}{3^2 - (3\times 3) + 2} = \frac{1}{2}e^{3t}$ | M1A1 |
| | | General solution $y = Ae^t + Be^{2t} + \frac{1}{2}e^{3t}$ | |
| | | $= Ax + Bx^2 + \frac{1}{2}x^3$ | M1A1ft (6) |

| Question number | | Scheme | Marks |
|---|---|---|---|
| **8.** | (a) | $z^p + \frac{1}{z^p} = e^{ip\theta} + \frac{1}{e^{ip\theta}}$ | |
| | | $= \left(e^{ip\theta} + e^{-ip\theta}\right)$ | |
| | | $= 2\cos p\theta$ | M1A1 (2) |
| | (b) | By De Moivre if $z = e^{i\theta}$ | |
| | | $z^p + \frac{1}{z^p} = 2\cos p\theta$ | |
| | | $p = 1: \ (2\cos\theta)^4 = \left(z + \frac{1}{z}\right)^4$ | M1A1 |
| | | $= z^4 + 4z^3.\frac{1}{z} + 6z^2\frac{1}{z^2} + 4z.\frac{1}{z^3} + \frac{1}{z^4}$ | A1 |
| | | $= \left(z^4 + \frac{1}{z^4}\right) + 4\left(z^2 + \frac{1}{z^2}\right) + 6$ | |
| | | $= 2\cos 4\theta + 8\cos 2\theta + 6$ | M1A1 |
| | | $\therefore \cos^4\theta = \frac{1}{8}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{3}{8}$ | A1ft (6) |
| | (c) | $V = \pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} y^2\,dx = \pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^4 x\,dx$ | |
| | | $= \pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1}{8}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{3}{8}\right)d\theta$ | M1A1ft |
| | | $= \pi\left[\frac{1}{32}\sin 4\theta + \frac{1}{4}\sin 2\theta + \frac{3}{8}\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$ | M1A1ft |
| | | $= \frac{3}{8}\pi^2$ | M1A1 (6) |

## 6669/01: Further Pure Mathematics FP3

| Question number | Scheme | Marks |
|---|---|---|
| **1.** | $\begin{vmatrix} (7-\lambda) & 6 \\ 6 & (2-\lambda) \end{vmatrix} = 0$ | |
| | $(7-\lambda)(2-\lambda)-36=0$ | M1 A1 |
| | $\lambda^2-9\lambda+14-36=0$ | |
| | $\lambda^2-9\lambda-22=0$ | |
| | $(\lambda-11)(\lambda+2)=0 \Rightarrow \lambda_1=-2, \lambda_2=11$ | M1 A1 (4) |
| **2.** | $9\left(\frac{e^x+e^{-x}}{2}\right)-6\left(\frac{e^x-e^{-x}}{2}\right)=7$ | B1 |
| | $3e^{2x}-14e^x+15=0$ | M1 A1 |
| | $(3e^x-5)(e^x-3)=0 \qquad e^x=\frac{5}{3}, \quad e^x=3$ | M1 A1 |
| | $x=\ln\frac{5}{3} \qquad x=\ln 3$ | A1 (6) |
| **3.** | $s=\int_0^{2\pi}\left[\dot{x}^2+\dot{y}^2\right]^{\frac{1}{2}}dt$ | |
| | $\frac{dx}{dt}=\dot{x}=a(1-\cos t); \quad \frac{dy}{dt}=\dot{y}=a\sin t$ | M1 A1; A1 |
| | $s=\int_0^{2\pi}a\left[(1-\cos t)^2+\sin^2 t\right]^{\frac{1}{2}}dt=a\int_0^{2\pi}\left[2-2\cos t\right]^{\frac{1}{2}}dt$ | |
| | $=2a\int_0^{\frac{\pi}{2}}\sin\left(\frac{t}{2}\right)dt, \quad =-4a\left[\cos\left(\frac{t}{2}\right)\right]_0^{2\pi}=8a$ | M1 A1 A1ft (6) |

| Question number | Scheme | Marks |
|---|---|---|
| **4.** | $x = 2\sinh t$ | B1 |
| | $\sqrt{\left(x^2+4\right)} = \left(4\sinh^2 t + 4\right)^{\frac{1}{2}} = 2\cosh t$ | |
| | $\mathrm{d}x = 2\cosh t\,\mathrm{d}t$ | |
| | $I = \int \sqrt{\left(x^2+4\right)}\mathrm{d}x = 4\int \cosh^2 t\,\mathrm{d}t$ | M1A1 |
| | $= 2\int (\cosh 2t + 1)\,\mathrm{d}t$ | |
| | $= \sinh 2t + 2t + c$ | M1 A1 |
| | $= \frac{1}{2}x\sqrt{\left(x^2+4\right)} + 2\operatorname{arsinh}\left(\frac{x}{2}\right) + c$ | M1 A1ft (7) |
| **5.** (a) | $y = \arcsin x$ | |
| | $\Rightarrow \sin y = x$ | M1 |
| | $\cos y \frac{\mathrm{d}y}{\mathrm{d}x} = 1$ | |
| | $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$ | M1 A1 (3) |
| (b) | $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -\frac{1}{2}\left(1-x^2\right)^{\frac{-3}{2}}(-2x)$ | |
| | $= x\left(1-x^2\right)^{\frac{-3}{2}}$ | M1 A1 |
| | $\left(1-x^2\right)\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - x\frac{\mathrm{d}y}{\mathrm{d}x} = \left(1-x^2\right)x\left(1-x^2\right)^{\frac{-3}{2}} - x\left(1-x^2\right)^{-\frac{1}{2}} = 0$ | M1 A1 (4) |

| Question number | | Scheme | Marks |
|---|---|---|---|
| 6. | (a) | $I_n = \int_0^{\frac{\pi}{2}} x^n \sin x \, dx$ | |
| | | $= \left[x^n(-\cos x)\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} nx^{n-1}(-\cos x)dx$ | M1 A1 |
| | | $= 0 + n\left\{\left[x^{n-1}\sin x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}}(n-1)x^{n-2}\sin x \, dx\right\}$ | A1 |
| | | $= n\left[\left(\frac{\pi}{2}\right)^{n-1} - (n-1)I_{n-2}\right]$ | |
| | | So $I_n = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2}$ | A1 (4) |
| | (b) | $I_3 = 3\left(\frac{\pi}{2}\right)^2 - 3.2I_1$ | |
| | | $I_1 = \int_0^{\frac{\pi}{2}} x\sin x \, dx \quad = \left[x(-\cos x)\right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x \, dx$ | M1 |
| | | $= \left[\sin x\right]_0^{\frac{\pi}{2}} = 1$ | A1 |
| | | $I_3 = (3)\left(\frac{\pi}{2}\right)^2 - 6 = \frac{3\pi^2}{4} - 6$ | M1 A1 (4) |

| Question number | Scheme | Marks |
|---|---|---|
| **7.** (a) | $\mathbf{A}(x) = \begin{pmatrix} 1 & x & -1 \\ 3 & 0 & 2 \\ 1 & 1 & 0 \end{pmatrix}$ | |
| | Cofactors $\begin{pmatrix} -2 & 2 & 3 \\ -1 & 1 & x-1 \\ 2x & -5 & -3x \end{pmatrix}$ | M1 A1<br>A1<br>A1 |
| | Determinant $= 2x - 3 - 2 = 2x - 5$ | M1 A1 |
| | $\mathrm{A}^{-1}(x) = \dfrac{1}{2x-5}\begin{pmatrix} -2 & -1 & 2x \\ 2 & 1 & -5 \\ 3 & (x-1) & -3x \end{pmatrix}$ | M1 A1ft (8) |
| (b) | $\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \mathrm{B}^{-1}\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = \dfrac{1}{1}\begin{pmatrix} -2 & -1 & 6 \\ 2 & 1 & -5 \\ 3 & 2 & -9 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ | M1 A1ft |
| | $= (17, -13, -24)$ | M1 A1 (4) |

| Question number | Scheme | Marks |
|---|---|---|
| **8.** (a) | $\overrightarrow{AB} = (-1,\ 3,-1);\ \overrightarrow{AC} = (-1,3,1).$ | M1 A1 |
| | $\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 3 & -1 \\ -1 & 3 & 1 \end{vmatrix}$ | |
| | $= \mathbf{i}(3+3)+\mathbf{j}(1+1)+\mathbf{k}(-3+3)$ | |
| | $= 6\mathbf{i}+2\mathbf{j}$ | M1 A1<br>A1 |
| | Area of $\Delta ABC = \frac{1}{2}\left\lvert \overrightarrow{AB}\times\overrightarrow{AC}\right\rvert$ | |
| | $= \frac{1}{2}\sqrt{36+4} = \sqrt{10}$ square units | M1 A1ft (7) |
| (b) | Volume of tetrahedron $= \frac{1}{6}\left\lvert \overrightarrow{AD}.\left(\overrightarrow{AB}\times\overrightarrow{AC}\right)\right\rvert$ | |
| | $= \frac{1}{6}\lvert -12+8\rvert$ | |
| | $= \frac{2}{3}$ cubic units | M1 A1 (2) |
| (c) | Unit vector in direction $\overrightarrow{AB}\times\overrightarrow{AC}$ i.e. perpendicular to plane containing $A$, $B$, and $C$ is | |
| | $\mathbf{n} = \frac{1}{\sqrt{40}}(6\mathbf{i}+2\mathbf{j}) = \frac{1}{\sqrt{10}}(3\mathbf{i}+\mathbf{j})$ | M1 |
| | $p = \left\lvert \mathbf{n}\cdot\overrightarrow{AD}\right\rvert = \frac{1}{\sqrt{10}}\lvert(3\mathbf{i}+\mathbf{j})\cdot(-2\mathbf{i}+4\mathbf{j})\rvert$ | |
| | $= \frac{1}{\sqrt{10}}\lvert -6+4\rvert = \frac{2}{\sqrt{10}}$ units. | M1 A1 (3) |

| Question number | Scheme | Marks |
|---|---|---|
| **9.** (a) | $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ | |
| | $\frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0$ | M1 A1 |
| | $\frac{dy}{dx} = \frac{2x}{a^2}\frac{b^2}{2y} = \frac{b^2}{a^2}\frac{a\sec\theta}{b\tan\theta} = \frac{b}{a\sin\theta}$ | M1 A1 |
| | Gradient of normal is then $-\frac{a}{b}\sin\theta$ | |
| | Equation of normal: $(y - b\tan\theta) = -\frac{a}{b}\sin\theta(x - a\sec\theta)$ | |
| | $ax\sin\theta + by = (a^2 + b^2)\tan\theta$ | M1 A1 (6) |
| (b) | M: A normal cuts $x = 0$ at $y = \frac{(a^2 + b^2)}{b}\tan\theta$ | M1 A1 |
| | B normal cuts $y = 0$ at $x = \frac{a^2 + b^2}{a\sin\theta}\tan\theta$ | |
| | $= \frac{(a^2 + b^2)}{a\cos\theta}$ | A1 |
| | Hence M is $\left[\frac{(a^2 + b^2)}{2a}\sec\theta, \frac{(a^2 + b^2)}{2b}\tan\theta\right]$ | M1 |
| | Eliminating $\theta$ | M1 |
| | $\sec^2\theta = 1 + \tan^2\theta$ | |
| | $\left[\frac{2aX}{a^2 + b^2}\right]^2 = 1 + \left[\frac{2bY}{a^2 + b^2}\right]^2$ | A1 |
| | $4a^2X^2 - 4b^2Y^2 = [a^2 + b^2]^2$ | A1 (7) |

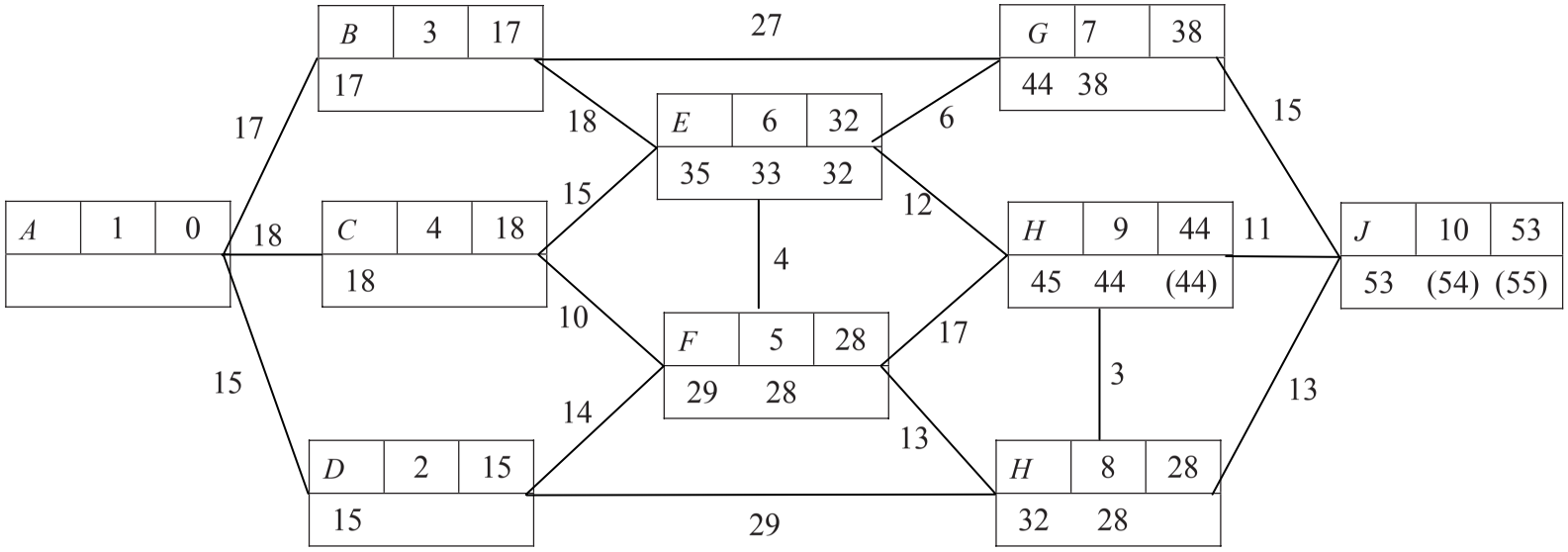
  

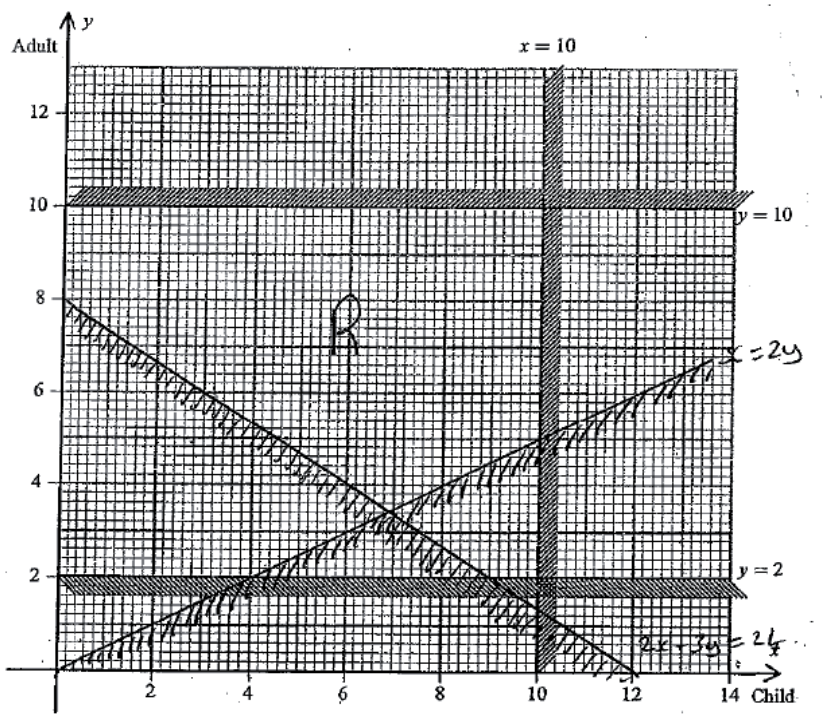
| Question Number | Scheme | Marks |
|---|---|---|
| **1.** | $\left[\frac{1+10}{2}\right]$ = 6 Nicky reject top of list | M1 |
| | $\left[\frac{7+10}{2}\right]$ = 9 Trevor reject bottom of list | A1 |
| | $\left[\frac{7+8}{2}\right]$ = 8 Steve reject bottom of list | A1 |
| | [7] = 7 Preety reject<br>Nigel not in list | A1<br>(4) |
| **2.** (a) | G – 3 = J – 4 = L – 5 | M1 |
| | Change status: G = 3 – J = 4 – L = 5 | A1 |
| | Improved matching: E = 2<br>G = 3<br>J = 4<br>L = 5 | B1 (3) |
| (b) | e.g George and Yi Wen may both only be assigned to 3 | B1 (1) |
| (c) | Y – 3 = G – 2 = E – 4 = J – 1 | M1 |
| | Change status: Y = 3 – G = 2 – E = 4 –J = 1 | A1 |
| | Complete Matching E = 4<br>G = 2<br>J = 1<br>L = 5<br>Y = 3 | A1 (3)<br>**(7 marks)** |
| **3.** (a) | (i) *FH*, *AD*, *DE*, *CE*, (not *DC*), $\begin{pmatrix} BC \\ EG \end{pmatrix}$, (not *AC*), *CF*, *HI*, (not *FI*), *IJ* | M1 A1 A1 (3) |
| | (ii) *AD*, *DE*, *EC*, $\begin{pmatrix} BC \\ EG \end{pmatrix}$, *CF*, *FH*, *HI*, *IJ* stop | M1 A1 A1 (3) |
| (b) | Start off the tree with *AB* and *FI*, then apply Kruskal | M1 A1 (2)<br>**(8 marks)** |

| Question Number | | Scheme | Marks |
|---|---|---|---|
| **4.** | (a) | E.g:<br>650 431 245 643 **455** 710 234 162 452 **134**<br>650 643 710 **455** 431 245 234 162 452 **134**<br>650 710 643 **455** 431 **245** 452 **234** **162** 134<br>**710** 650 643 455 431 452 **245** 234 162 134<br>**710** 650 643 455 **452** 431 245 234 162 134 | M1 A1<br>A1 ft<br>A1 ft<br>A1<br>(5) |
| | (b) | Bin 1 710 + 245 Bin 3 643 + 162 + 134 Bin 5 431<br>Bin 2 650 + 234 Bin 4 455 + 452 | M1 A1<br>A1 A1(ft) (4) |
| | (c) | $\frac{4116}{1000}$ = 4.116 5 bins needed optimal | M1 A1(ft) (2)<br>**(11 marks)** |
| **5.** | (a) | e.g. Each edge contributes 2 to the sum of degree, hence this sum must be even.<br>Therefore there must be an even (or zero) number of vertices of odd degree<br>Hence there cannot be an odd number of vertices of odd degree | B2, 1, 0<br>(2) |
| | (b) | $CD + FH = 200 + 220 = 420$<br>$CF + DH = 180 + 380 = 560$<br>$CH + DF = 400 + 160 = 560$<br>Repeat $CA$, $AD$ and $FH$ | M1 A1<br>A1<br>A1 (4) |
| | (c) | Length = 4180 + 420 = 4600 m | B1 (ft) (1)<br>**(9 marks)** |

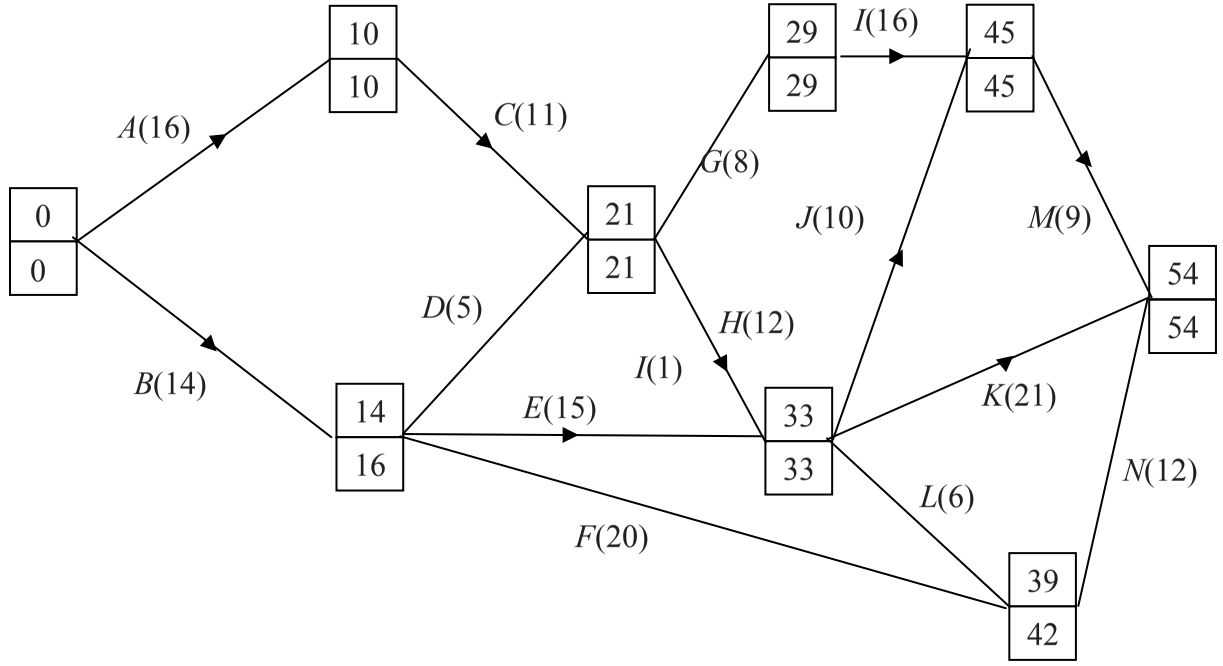
  

## 6689/01: Decision Mathematics D1

| Question Number | Scheme | Marks |
|---|---|---|
| **6.** (a) |  | M1<br>A1<br>A1 ft<br>A1 ft |
| | Route: *A C F E G J* | |
| | Length: 53 km | A1 (5) |
| (b) | General explanation - trace back from *J*<br>- Include arc *XY* if *Y* is already on path and if difference in trial labels equals length of arc. | |
| | Specific explanation 53 – 15 = 38 *GJ*<br>38 – 6 = 32 *EG*<br>32 – 4 = 28 *FE*<br>28 – 10 = 18 *CF*<br>18 – 18 = 0 *AC* | B 2ft 1ft (2) |
| (c) | Eg *A D F E G J* or *A C E G J*; length 54 km | B1; B1 ft ( 2) |
| | | **(9 marks)** |

| Question Number | | Scheme | | | | Marks | |
|---|---|---|---|---|---|---|---|
| 7. | (a) | To show a strict inequality | | | | B1 | (1) |
| | (b) | There must be fewer than 10 children | | | | B1 | |
| | | There must be between 2 and 10 adults inclusive | | | | B2, 1, 0 | (3) |
| | (c) | $2x + 3y \geq 24$ | | | | B1 | |
| | | $x \leq 2y$ | | | | B1 | (2) |
| | (d) | Diagram 1<br>  <br> | | | | B1 ft ($2x + 3y = 24$)<br>B1 ft ($x = 2y$)<br>B1 ft (shading) | |
| | (e) | Minimum | 0 Children | 8 Adults | - 8 Passengers | B1<br>M1 A1 | (4) |
| | | Maximum | 9 Children | 10 Adults | - 19 Passengers | B1 B1 | (4) |
| | | | | | | | **(14 marks)** |

| Question Number | | Scheme | Marks |
|---|---|---|---|
| 8. | (a) |  | M1 A1 (2)<br>M1 A1 (2) |
| | (b) | **A** $G-I-M$, $H-K$<br>**C** | A1 (1) |
| | (c) | Float on $D = 21 - 5 - 14 = 2$<br>Float on $F = 42 - 20 - 14 = 8$ | B1 ft<br>M1 A1 ft (3) |
| | (d) | Gantt Chart | B4 (4) |
| | (e) | Day 15: $C$<br>Day 25: $G, H, E, F$ | B1<br>B2, 1, 0 (3) |
| | | | **(15 marks)** |

**1.** (a) (By conservation of flow at $B$, $C$ and $D$) — B3, 2ft 1ft 0

$x = 11$ $\quad y = 5$ $\quad z = 12$

$(\sqrt{x} - 6)$ $\quad (\sqrt{y} + 7)$ — (3)

(b) Flow is 31 — B1

(max flow = min cut), cut through $AB$, $AC$ and $SD$ — B1 (2)

**(5 marks)**

**2.** (a)

| b.v | $x$ | $y$ | $z$ | $r$ | $s$ | Value | Raw ops |
|---|---|---|---|---|---|---|---|
| $z$ | $\frac{1}{2}$ | 0 | 1 | $\frac{1}{4}$ | 0 | 20 | $R_1 \div 4$ |
| $s$ | 0 | 4 | 0 | $-\frac{1}{2}$ | 1 | 120 | $R_2 - 2R_1$ |
| $P$ | 8 | −8 | 0 | 5 | 0 | 400 | $R_3 + 20R_1$ |

M1 A1 M1 A1 ft A1 ft (5)

(b) $P + 8x - 8y + 5r = 400$ — B1 ft (1)

(c) Not optimal since there is a negative number in the profit row — B1 ft (1)

**(7 marks)**

**3.** (a)

| | $D$ | $E$ | $F$ |
|---|---|---|---|
| $A$ | 20 | 4 | |
| $B$ | | 26 | 6 |
| $C$ | | | 14 |

M1 A1 (2)

(b) $S_A = 0$ $\quad S_B = -1$ $\quad S_c = 7$

$D_D = 21$ $\quad D_E = 24$ $\quad D_F = 18$ — M1

$I_{13} = I_{AF} = 16 - 0 - 18 = -2$ — A1

$I_{21} = I_{BD} = 18 + 1 - 21 = -2$ — M1

$I_{31} = I_{CD} = 15 - 7 - 21 = -13$ * — A1 ft

$I_{32} = I_{CE} = 19 - 7 - 24 = -12$ — A1 ft (5)

(c) Eg $CD(+) \rightarrow AD(-) \rightarrow AE(+) \rightarrow BE(-) \rightarrow BF(+) \rightarrow CF(-)$ $\theta = 14$ — M1 A1 ft

| | $D$ | $E$ | $F$ |
|---|---|---|---|
| $A$ | 6 | 18 | |
| $B$ | | 12 | 20 |
| $C$ | 14 | | |

Cost £1384 — A1 ft A1 (4)

**(11 marks)**

| Question number | | Scheme | Marks |
|---|---|---|---|
| **4.** | (a) | Deleting $F$ leaves r.s.t | M1 |
| | | r.s.t length = 86<br>So lower bound = 86 + 16 + 19 = 121 | A1<br><br>M1 A1 (4) |
| | | Best $LB$ is 129 by deleting $C$ | B1 ft (1) |
| | (b) | Add 33 to $BF$ and $FB$ | B1 |
| | | Add 31 to $DE$ and $ED$ | B1 (2) |
| | (c) | Tour, visits each vertex, order correct using table of least distances. | M1 A1 |
| | | e.g $F\ C\ D\ A\ B\ E\ G\ F$ (actual route $F\ C\ D\ C\ A\ B\ E\ G\ F$) | A1 |
| | | Upper bound of 138 km | A1 (4)<br>**(11 marks)** |
| **5.** | | Let $x_{ij}$ be number of units transported from $i$ to $j$<br>Where $i \in \{W, X, Y\}$ and $j \in \{J, K, L\}$<br>Warehouse Supermarket | B1 |
| | | Objective minimise "c" = $3x_{WJ} + 6x_{WK} + 3x_{WL} + 5x_{XJ} + 8x_{XK} + 4x_{XL} + 2x_{YJ} + 5x_{YK} + 7x_{YL}$ | B1 |
| | | Subject to $x_{WJ} + x_{WK} + x_{WL} = 34$ | M1 A1 |
| | | $x_{XJ} + x_{XK} + x_{XL} = 57$ | |
| | | $x_{YJ} + x_{YK} + x_{YL} = 25$ | A1 |
| | | $x_{WJ} + x_{XJ} + x_{YJ} = 20$ | |
| | | $x_{WK} + x_{XK} + x_{YK} = 56$ | |
| | | $x_{WL} + x_{XL} + x_{YL} = 40$ | |
| | | $x_{ij} \geq 0 \quad \forall$ i $\in \{W, X, Y\}$ and $j \in \{J, K, L\}$ | B1<br>**(6 marks)** |

| Question number | Scheme | Marks |
|---|---|---|
| 6. | | |

| Stage | State | Action | Value | Marks |
|---|---|---|---|---|
| 1 | *H* | *HK* | 18 * | |
| | *I* | *IK* | 19 * | M1 A1 |
| | *J* | *JK* | 21 * | |
| 2 | *F* | **FH** | min(16, 18) = 16 | M1 A1 A1 |
| | | *FI* | min (23, 19) = 19 * | |
| | | *FJ* | min(17, 21) = 17 | |
| | *G* | *GH* | min(20, 18) = 18 | |
| | | *GI* | min(15, 19) = 15 | A1 |
| | | *GJ* | min(28, 21) = 21 * | |
| 3 | *B* | *BG* | min(18, 21) = 18 * | M1 A1 ft |
| | *C* | *CF* | min(25, 19) = 19 * | |
| | | *CG* | min(16, 21) = 16 | |
| | *D* | *DF* | min(22, 19) = 19 * | |
| | | *DG* | min(19, 21) = 19 * | |
| | *E* | *EF* | min(14, 19) = 14 * | |
| 4 | *A* | *AB* | min(24, 18) = 18 | A1 ft |
| | | *AC* | min(25, 19) = 19 * | |
| | | *AD* | min(27, 19) = 19 * | |
| | | *AE* | min(23, 14) = 14 | |

Routes *A C F I K*, or *A D F I K* or *A D G J K* — A1 ft

**(9 marks)**

| Question number | | Scheme | Marks |
|---|---|---|---|
| 7. | (a) | To maximise, subtract all entries from $n \geq 30$ | M1 |
| | | e.g $\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & 5 & 3 & 6 \\ 0 & 3 & 5 & 9 \end{bmatrix}$ | |
| | | Minimise uncovered element is 1 So $\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 6 & 3 & 0 \\ 0 & 4 & 2 & 5 \\ 0 & 2 & 4 & 8 \end{bmatrix}$ | M1 A2ft 1ft 0 |
| | | | M1 |
| | | min. el = 2 $\begin{bmatrix} 7 & 0 & 0 & 2 \\ 0 & 4 & 1 & 0 \\ 0 & 2 & 0 & 5 \\ 0 & 0 & 2 & 8 \end{bmatrix}$ min. el = 2 $\begin{bmatrix} 7 & 0 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 2 & 6 \end{bmatrix}$ | A2 ft 1 ft 0 |
| | | $A-2 \quad B-4 \quad C-3 \quad D-1$ | |
| | | $A-3 \quad B-4 \quad C-1 \quad D-2$ | M1 A1 ft (2) |
| | (b) | £1160 000 | B2, 1, 0 (2) |
| | (c) | Gives other solution | M1 A1 ft (2) |
| | | | **(15 marks)** |

| Question number | Scheme | Marks |
|---|---|---|
| **8.** (a) | | M1 A1 |

| | I | II | III | |
|---|---|---|---|---|
| I | 5 | 2 | 3 | Min 2 |
| II | 3 | 5 | 4 | Min 3 ← max |
| | Max 5 | 5 | 4 | |
| | | | min | |

Since $3 \neq 4$ not stable — A1 (3)

(b) Let $A$ play I with probability $p$

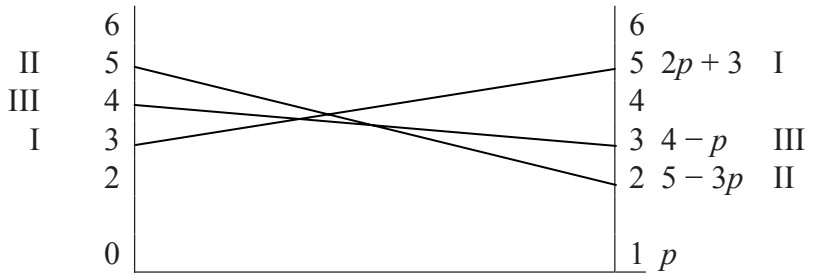
Let $A$ play II with probability $(1 - p)$

If $B$ plays I $A$'s gain are $5p + 3(1 - p) = 2p + 3$

If $B$ plays II $A$'s gain are $2p + 5(1 - p) = 5 - 3p$ — M1 A1 (2)

If $B$ plays III $A$'s gain are $3p + 4(1 - p) = 4 - p$

A 2, 1, 0 (2)




Intersection of $2p + 3$ and $4 - p$ $\quad p = \frac{1}{3}$ — M1 A1ft

A should play I of time and II of time; value (to $A$) = 3 — A1 ft A1 ft (2)

**(15 marks)**

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