STATISTICS 2 (A) TEST PAPER 9 : ANSWERS AND MARK SCHEME

1. (a) A discrete variable can only have certain values, usually integers — B1
   A continuous variable can take any value, often in a certain range — B1
   (b) X is continuous, but the calculator number is discrete, e.g. calculator cannot give 0·385721... — B1
   (c) Sketch : line from (0, 0) to (1, 1); on x-axis elsewhere — B2 — 5

2. (a) Quicker to use a sample, but it may be inaccurate — B1 B1
   (b) Catalogue of all videos in stock (c) All the separate videos — B1 B1
   (d) One particular sort, e.g. horror, may be unrepresentative — B2 — 6

3 (a) From tables, extreme 2·5% tails are given by $X \leq 3$ and $X \geq 13$, — M1 A1 A1
   so this is the critical region — A1
   (b) The bottom 5% tail is still given by $X \leq 3$: region is {0, 1, 2, 3} — M1 M1 A1 — 7

4 (a) Mean = 80 x 0·375 = 30, variance = 80 x 0·375 x 0·625 = 18·75 — M1 A1 M1 A1
   (b) $X \sim B(80, 0{\cdot}375) \approx N(30, 18{\cdot}75)$ — M1 A1
   $P(X > 40) = P(X > 40{\cdot}5) = P(Z > 10{\cdot}5/4{\cdot}33) = P(Z > 2{\cdot}42)$ — M1 A1 A1
   $= 1 - 0{\cdot}9922 = 0{\cdot}0078$ — M1 A1 — 11

5. (a) Mean = 300/100 = 3 Variance = $1222/100 - 3^2 = 3{\cdot}22$ — M1 A1 M1 A1
   (b) Mean ≈ Variance, and positive skewness — B1 B1
   (c) $H_0 : \lambda = 3$ and $H_1 : \lambda > 3$ — B1 B1
   Under H_0, no. of lorries in 15 minutes ~ Po(9) — M1 A1
   $P(X \geq 18) = 1 - 0{\cdot}995 = 0{\cdot}005 < 1\%$ so reject H_0 at 1% level, — M1 A1
   i.e. accept that mean has increased — A1 — 13

6. (a) No. of Cons ~ B(10, 0·35), so $P(X \geq 2) = 1 - 0{\cdot}086 = 0{\cdot}914$ — M1 A1 A1
   (b) No. of Cons or MRL ~ B(500, 0·37) ≈ N (185, 116·55), so — M1 A1
   $P(X > 200) = P(X > 199{\cdot}5) = P(Z > 14{\cdot}5/10{\cdot}79) = P(Z > 1{\cdot}34)$ — M1 A1 M1
   $= 1 - 0{\cdot}9099 = 0{\cdot}0901$ — A1 A1
   (c) No. of MRL ~ B(200, 0·02) ≈ Po(4) — M1 A1
   so $P(X \geq 5) = 1 - 0{\cdot}6288 = 0{\cdot}371$ — M1 A1
   (d) Binomial to Normal needs continuity correction, going from a — B1
   discrete to a continuous distribution — B1 — 16

7. (a) $k \int_0^1 x^2 - x^3 \, dx = 1$ $k \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = 1$ $k = 12$ — M1 A1
   Graph sketched : parabola, vertex upwards, through (0, 0), (1, 0) — B2
   (b) $E(X) = 12 \int_0^1 x^3 - x^4 \, dx = 12\left[\frac{1}{4} - \frac{1}{5}\right] = 0{\cdot}6$ — M1 A1 A1
   $\text{Var}(X) = 12 \int_0^1 x^4 - x^5 \, dx - 0.6^2 = 12\left[\frac{1}{5} - \frac{1}{6}\right] - 0{\cdot}36 = 0{\cdot}04$ — M1 A1 A1
   (c) $F(x) = 0 \ (x < 0)$, $F(x) = 4x^3 - 3x^4 \ (0 \leq x \leq 1)$, $F(x) = 1 \ (x > 1)$ — B1 M1 A1 B1
   (d) $P(x \leq 85\%) = F(0{\cdot}85) = 4(0{\cdot}85^3) - 3(0{\cdot}85^4) = 0{\cdot}89$, so cloud — M1 A1
   cover is ≤ 85% for 89% of the time — A1 — 17