1. (a) Briefly explain the difference between a one-tailed test and a two-tailed test. **(2 marks)**

   (b) State, with a reason, which type of test would be more appropriate to test the claim that this decade's average temperature is greater than the last decade's. **(2 marks)**

2. (a) Give one advantage and one disadvantage of

   (i) a sample survey, **(2 marks)**

   (ii) a census. **(2 marks)**

   (b) Suggest a situation in which each could be used. **(2 marks)**

3. A pharmaceutical company produces an ointment for earache that, in 80% of cases, relieves pain within 6 hours. A new drug is tried out on a sample of 25 people with earache, and 24 of them get better within 6 hours.

   (a) Test, at the 5% significance level, the claim that the new treatment is better than the old one. State your hypotheses carefully. **(6 marks)**

   A rival company suggests that the sample does not give a conclusive result;

   (b) Might they be right, and how could a more conclusive statement be achieved? **(3 marks)**

4. A centre for receiving calls for the emergency services gets an average of 3·5 emergency calls every minute. Assuming that the number of calls per minute follows a Poisson distribution,

   (a) find the probability that more than 6 calls arrive in any particular minute. **(3 marks)**

   Each operator takes a mean time of 2 minutes to deal with each call, and therefore seven operators are necessary to cope with the average demand.

   (b) Find how many operators are required for there to be a 99% probability that a call can be dealt with immediately. **(3 marks)**

   It is found from experience that a major disaster creates a surge of emergency calls. Taking the null hypothesis H_0 that there is no disaster,

   (c) find the number of calls that need to be received in one minute to disprove H_0 at the 0·1 % significance level. **(3 marks)**

5. The random variable X has a continuous uniform distribution on the interval $a \leq X \leq 3a$.

   (a) Without assuming any standard results, prove that μ, the mean value of X, is equal to $2a$ and derive an expression for σ^2, the variance of X, in terms of a. **(7 marks)**

   (b) Find the probability that $|X - \mu| < \sigma$ and compare this with the same probability when x is modelled by a Normal distribution with the same mean and variance. **(6 marks)**

6. Two people are playing darts. Peg hits points randomly on the circular board, whose radius is a. If the distance from the centre O of the point that she hits is modelled by the variable R,

   (a) explain why the cumulative distribution function $\mathrm{F}(r)$ is given by

$$\mathrm{F}(r) = 0 \qquad r < 0,$$
$$\mathrm{F}(r) = \frac{r^2}{a^2} \qquad 0 \le r \le a,$$
$$\mathrm{F}(r) = 1 \qquad r > a.$$

(4 marks)

   (b) By first finding the probability density function of R, show that the mean distance from O of the points that Peg hits is $\frac{2a}{3}$. **(7 marks)**

   Bob, a more experienced player, aims for O, and his points have a distance X from O whose cumulative distribution function is

$$\mathrm{F}(x) = 0,\ x < 0; \qquad \mathrm{F}(x) = \tfrac{x}{a}\left(2 - \tfrac{x}{a}\right),\ 0 \le x \le a; \qquad \mathrm{F}(x) = 1,\ x > a.$$

   (c) Find the probability density function of X, and explain why it shows that Bob is aiming for O. **(5 marks)**

7. In an orchard, all the trees are either apple or pear trees. There are four times as many apple trees as pear trees. Find the probability that, in a random sample of 10 trees, there are

   (a) equal numbers of apple and pear trees, **(3 marks)**

   (b) more than 7 apple trees. **(3 marks)**

   In a sample of 60 trees in the orchard,

   (c) find the expected number of pear trees. **(1 marks)**

   (d) Calculate the standard deviation of the number of pear trees and compare this result with the standard deviation of the number of apple trees. **(2 marks)**

   (e) Find the probability that exactly 35 in the sample of 60 trees are pear trees. **(4 marks)**

   (f) Find an approximate value for the probability that more than 15 of the 60 trees are pear trees. **(5 marks)**