STATISTICS 2 (A) TEST PAPER 8 : ANSWERS AND MARK SCHEME

1. (a) One-tailed : is a parameter greater (or less) than a given value? B1
   Two-tailed : is a parameter different from a given value? B1
   (b) One-tailed, as testing for 'warmer' rather than 'different' B1 B1 **4**

2. (a) (i) Sample is quick, does not use all population, but may be unreliable B2
   (ii) Census is accurate, but may be slow and very expensive B2
   (b) Sample : e.g. lifetime of light bulbs B1
   Census : e.g. government statistics B1 **6**

3. (a) $X \sim B(25, p)$ $H_0 : p = 0{\cdot}8$ $H_1 : p > 0{\cdot}8$ B1 B1
   Assuming H_0, P(24 or more people recovering within 6 hours) =
   $P(X \le 1)$ in $B(25, 0{\cdot}2) = 0{\cdot}0274 < 5\%$ so reject H_0 at 5% level M1 M1 A1 A1
   (b) Yes : at 1% level, do not reject H_0, i.e. new drug is no better M1 A1
   Do more tests, to get more conclusive answer B1 **9**

4. (a) $X \sim Po(3.5)$ $P(X > 6) = 1 - 0{\cdot}9347 = 0{\cdot}0653$ B1 M1 A1
   (b) $P(X \le 8) = 99{\cdot}01\%$, so the centre must be able to cope with B1
   8 calls, and therefore needs 16 operators M1 A1
   (c) $P(X > 10) = 0{\cdot}1\%$, $P(X > 11) = 0{\cdot}03\%$, so need 11 calls M1 A1 A1 **9**

5. (a) $f(x) = \frac{1}{2a}$, $a < x \le 3a$. $E(X) = \int_a^{3a} \frac{x}{2a}\,dx = \left[\frac{x^2}{4a}\right]_a^{3a} = \frac{8a^2}{4a} = 2a$ B1 M1 A1 A1
   $E(X^2) = \int_a^{3a} \frac{x^2}{2a}\,dx = \left[\frac{x^3}{6a}\right]_a^{3a} = \frac{13a^2}{3}$ $\text{Var}(X) = \frac{a^2}{3}$ M1 A1 A1
   (b) $P(|X - \mu| < \sigma) = P(|X - 2a| < \frac{a}{\sqrt{3}}) = \frac{1}{2a} \times 2\frac{a}{\sqrt{3}} = 0{\cdot}577$ M1 A1 A1
   Normal : $P(|X - \mu| < \sigma) = P(|Z| < 1) = 2(0{\cdot}3413) = 0{\cdot}683$ M1 A1 A1 **13**

6. (a) Must land on board, so $F(r) = 0$ $(r < 0)$, $F(r) = 1$ $(r > a)$ B1
   By definition, $F(r) = P(X < r) = \frac{\pi r^2}{\pi a^2} = \frac{r^2}{a^2}$ $(0 \le r \le a)$ M1 A1 A1
   (b) $f(r) = F'(r) = \frac{2r}{a^2}$ $(0 \le r \le a)$; $f(r) = 0$ otherwise M1 A1 M1 B1
   $E(R) = \int_0^a \frac{2r^2}{a^2}\,dr = \frac{2}{a^2}\left[\frac{r^3}{3}\right]_0^a = \frac{2a}{3}$ M1 A1 A1
   (c) $f(x) = F'(x) = \frac{2}{a} - \frac{2x}{a^2}$ $(0 \le x \le a)$; $f(x) = 0$ otherwise M1 A1 B1
   $f(x)$ decreases from $x = 0$ to $x = a$, so more likely to land near O M1 A1 **16**

7. (a) No. of pears is $B(10, 0{\cdot}2)$ B1
   $P(X = 5) = 0{\cdot}9936 - 0{\cdot}9672 = 0{\cdot}0264$ M1 A1
   (b) $P(X < 3) = P(X \le 2) = 0{\cdot}678$ M1 A1 A1
   (c) $E(X) = 60 \times 0{\cdot}2 = 12$ B1
   (d) $\sqrt{(12 \times 0{\cdot}8)} = \sqrt{9{\cdot}6} = 3{\cdot}10$ B1
   Same answer for s.d. of apples (just interchange 0·2 and 0·8) B1
   (e) In $B(60, 0{\cdot}2)$, $P(X = 15) = {}^{60}C_{15}(0{\cdot}2)^{15}(0{\cdot}8)^{45} = 0{\cdot}0759$ B1 M1 A1 A1
   (f) $B(60, 0{\cdot}2) \approx N(12, 9{\cdot}6)$ $P(X > 15) = P(X > 15{\cdot}5)$ B1 M1 A1
   $= P(Z > 3{\cdot}5/\sqrt{9{\cdot}6}) = P(Z > 1{\cdot}13) = 0{\cdot}129$ M1 A1 **18**

