

1. Briefly explain what is meant by
- (a) a statistical model, (2 marks)
 - (b) a sampling frame, (1 mark)
 - (c) a sampling unit. (1 mark)

2. (a) Explain what is meant by the critical region of a statistical test. (2 marks)
- (b) Under a hypothesis H_0 , an event A can happen with probability 4.2%. The event A does then happen. State, with justification, whether H_0 should be accepted or rejected at the 5% significance level. (2 marks)

3. (a) Briefly describe the main features of a binomial distribution. (2 marks)

I conduct an experiment by randomly selecting 10 cards, without replacement, from a normal pack of 52.

- (b) Explain why the distribution of X , the number of hearts obtained, is not $B(10, \frac{1}{4})$. (2 marks)

After making the appropriate adjustment to the experiment, which should be stated, so that the distribution is $B(10, \frac{1}{4})$, find

- (c) the probability of getting no hearts, (3 marks)
- (d) the probability of getting 4 or more hearts. (2 marks)
- (e) If the modified experiment is repeated 50 times, find the total number of hearts that you would you expect to have drawn. (2 marks)

4. A Geiger counter is observed in the presence of a radioactive source.

In 100 one-minute intervals, the number of counts recorded are as follows:

No of counts, X	0	1	2	3	4	5	6
Frequency	10	24	29	16	12	6	3

- (a) Find the mean and variance of this data, and show that it supports the idea that the random variable X is following a Poisson distribution. (5 marks)
- (b) Use a Poisson distribution with the mean found in part (a) to calculate, to 3 decimal places, the probability that more than 6 counts will be recorded in any particular minute. (4 marks)
- (c) Find the number of one-minute intervals, in the sample of 100, in which more than 6 counts would be expected. (2 marks)

5. A continuous random variable X has the cumulative distribution function

$$\begin{aligned} F(x) &= 0 & x < 2, \\ F(x) &= k(x-a)^2 & 2 \leq x \leq 6, \\ F(x) &= 1 & x \geq 6. \end{aligned}$$

- (a) Find the values of the constants a and k . **(4 marks)**
- (b) Show that the median of X is $2(1 + \sqrt{2})$. **(4 marks)**
- (c) Given that $X > 4$, find the probability that $X > 5$. **(6 marks)**
6. A small opinion poll shows that the Trendies have a 10% lead over the Oldies. The poll is based on a survey of 20 voters, in which the Trendies got 11 and the Oldies 9. The Oldies spokesman says that the result is consistent with a 10% lead for the Oldies, whilst the Trendies spokesperson says that this is impossible.
- (a) At the 5% significance level, test which is right, stating your null hypothesis carefully. **(6 marks)**
- (b) If it is indeed true that the Trendies are supported by 55% of the population, use a suitable approximation to find the probability that in a random sample of 200 voters they would obtain less than half of the votes. **(8 marks)**
7. A continuous random variable X has the probability density function

$$\begin{aligned} f(x) &= \frac{6x}{175} & 0 \leq x < 5, \\ f(x) &= \frac{6x(10-x)}{875} & 5 \leq x \leq 10, \\ f(x) &= 0 & \text{otherwise.} \end{aligned}$$

- (a) Verify that f is a probability density function. **(6 marks)**
- (b) Write down the probability that $X < 1$. **(2 marks)**
- (c) Find the cumulative distribution function of X , carefully showing how it changes for different domains. **(7 marks)**
- (d) Find the probability that $2 < X < 7$. **(2 marks)**