Write your name here

| Surname | Other names |
|---|---|
| | |

# Statistics S1

## SWANASH

## A* Practice Paper

| Time: 1 hours 30 minutes | Paper - H | Year: 2017-2018 |
|---|---|---|

**The formulae that you may need to answer some questions are found at the end of this A star practice paper.**

**A student may use any basic scientific calculator except: facility for symbolic algebra manipulation, differentiation, integration, retrievable mathematical formulae.**

Total Marks

### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more/less space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

### Information

- The total mark for this paper is 75
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- Practicing many **Swanash A-star** papers will enhance your final exam grades

**1.**

(a)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| ***x*** | 1 | 5 | 8 | 4 | 2 | 6 | 7 |
| ***y*** | 3 | 11 | 17 | *m* | 5 | 13 | 15 |

**Table 1**

Given that in Table1 the product moment correlation coefficient, r, between the variables ***x*** and ***y*** is 1. Find the value of *m.*

**(3)**

(b)

| *x* | *y* |
|---|---|
| $2^{500}$ | 143027 |
| $-\sqrt{123}$ | $5^{250}$ |

**Table 2**

Find the *exact* value of product moment correlation coefficient, r, between the variables *x* and *y* in Table 2. Give a reason for your answer.

**(2)**

**Question 1 continued**

**(Total 5 marks)**

**2.**

| $X$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $P(X = x)$ | $d$ | $k$ | $2d$ | $2d$ | $k$ | $d$ |

(a) Write down $E(X + 5)$ to 1 *decimal place.*

**(1)**

(b) Calculate $Var(X + 5)$ to 2 *decimal places.*

**(5)**

(c) Six jam tins, of which two are defective and four are good, are to be tested one after another in random order until both defective jam tins are *identified*. The random variable $Y$ represents the number of jam tins that are tested to *identify* both defectives.

| $Y$ | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $P(Y = y)$ | $\frac{1}{15}$ | $p$ | $q$ | $r$ |

(i) Find the value of $p$

**(3)**

(ii) Find the value of $q$

**(5)**

**Question 2 continued**

**(Total 14 marks)**

**3.**

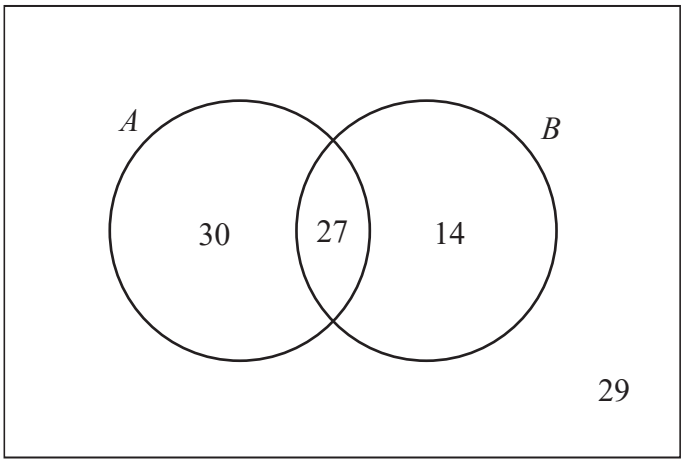



**Figure 1**

The Venn diagram in Figure 1 shows two events $A$ and $B$ for 100 adults.

(a) Draw a tree diagram to represent the above information.

**(4)**

The tree diagram in Figure 2 shows that some of the girls and boys play tennis whilst the remainder are engaged in other activities.

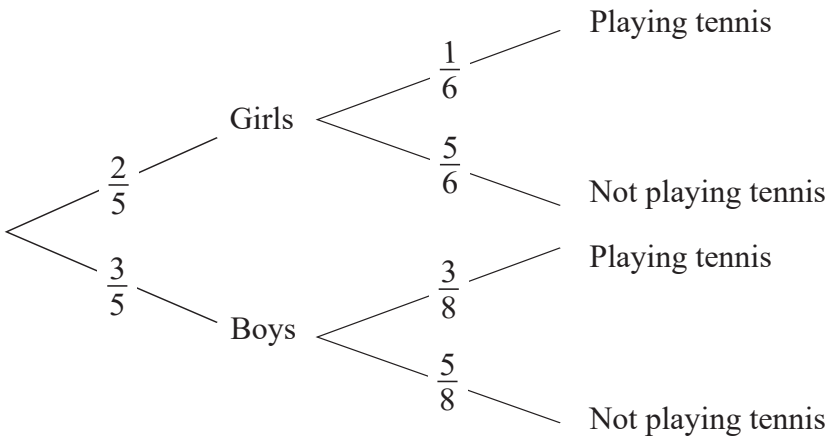



**Figure 2**

Let $G$, $B$ and $T$ denote the events that Girls, Boys and Tennis.

(b) Draw a Venn diagram to represent the events $G$, $B$ and $T$. Calculate the probabilities associated with each region and write them in the appropriate places on the Venn diagram.

**(5)**

**Question 3 continued**

**(Total 9 marks)**

**4.** The marks, $x$, of 45 students randomly selected from those students who sat a mathematics examination in class A are shown in the stem and leaf diagram below.

| Mark | | Totals |
|---|---|---|
| 3 | 6 9 9 | (3) |
| 4 | 0 1 2 2 3 4 | (6) |
| 4 | 5 6 $p$ 7 8 | (5) |
| 5 | 0 2 3 3 4 4 | (6) |
| 5 | 5 5 6 7 7 9 | (6) |
| 6 | 0 0 0 0 1 3 4 $q$ 5 | (9) |
| 6 | 5 6 7 8 9 9 | (6) |
| 7 | 1 2 3 3 | (4) |

| Key | (3\|6 means 36) |
|---|---|

(a) For these students the interquartile range is 19. Find the value of $p$ and value of $q$

**(2)**

The marks of class B students who sat a mathematics examination are shown in the table below.

| marks | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| number of students | 10 | 14 | 10 | $2x$ | $x$ | 2 | 3 |

(b) Write down the value of $x$ if the median of these marks is 2.5

**(1)**

The study time on a typical week night, in minutes, of class C students are given below

| | | | |
|---|---|---|---|
| 200 | 90 | 310 | 260 |
| 160 | 70 | 250 | 40 |
| 180 | 140 | 220 | 80 |
| 250 | 210 | 320 | 100 |

(c) Construct a stem and leaf diagram to represent the study time for the class C students

**(4)**

**Question 4 continued**

**(Total 7 marks)**

**5.**

| $x$ | 2 | 5 | 4 | 7 | … | … | … | … |
|---|---|---|---|---|---|---|---|---|
| $y$ | 90 | 160 | 170 | 210 | … | … | … | … |
| $m = x + 10$ | 12 | 15 | 14 | 17 | … | … | … | … |
| $f = \frac{y}{10}$ | 9 | 16 | 17 | 21 | … | … | … | … |

(a) Given that for 8 pair (some values are missing) the equation of regression line $y$ on $x$ is

$y = 57.3 + 20.0\,x$

(i) Find the equation of the regression line $y$ on $m$

**(2)**

(ii) Write down the equation of the regression line $f$ on $m$

**(1)**

(b)

| $\boldsymbol{k}$ | 2 | 5 | 4 | 7 | 7 |
|---|---|---|---|---|---|
| $\boldsymbol{n}$ | 4 | $p$ | 6 | 2 | 3 |

The equation of the regression line for the above five set of data in the table is $\boldsymbol{n} = 7.2\overline{2} - 0.44\overline{4}\boldsymbol{k}$

(i) Find the exact value of $p$

**(4)**

(ii) When $\boldsymbol{k} = 5$ find the exact value of $\boldsymbol{n}$ using the above regression line

**(1)**

(iii) Given that the product moment correlation coefficient is − 0.298.

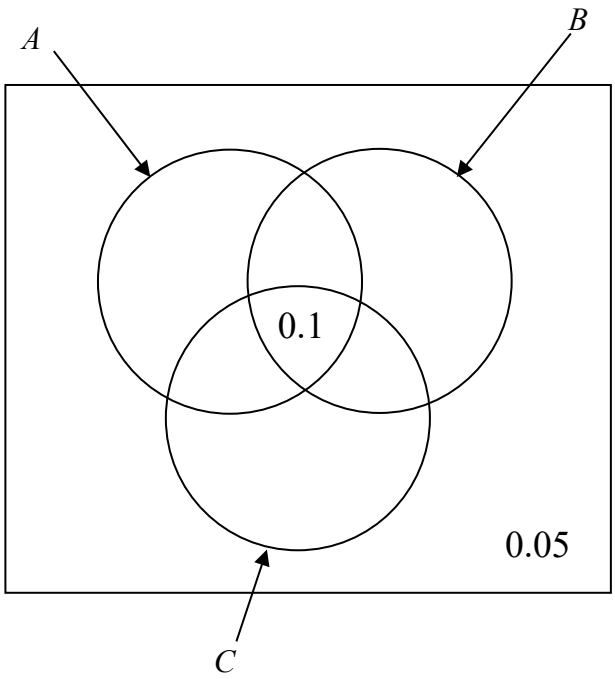
Explain why your predicted value of $\boldsymbol{n}$ in (ii) is different from the value of $p$ in (i)

**(1)**

**Question 5 continued**

**(Total 9 marks)**

**6.**




Given that,

P($C \cap B$) = 0.3,

P($A' \cap B' \cap C$ ) = 0.15,

P($A \cup B'$ ) = 0.7

P($A \cap B \cap C'$ ) = 0.2,

P($C \cup A'$ ) = 0.6

(a) Complete the above Venn diagram.

**(6)**

(b) Find P($A$)

**(1)**

(c) Find P($A|B$)

**(2)**

(d) Hence, show that $A$ and $B$ are independent events.

**(1)**

Leave blank

**Question 6 continued**

**(Total 10 marks)**

**7.** The times, in seconds, taken by *Mr.Swarna* to run 400 metres on nine successive days were

53.2, 55.7, 54.2, 52.7, 53.6, 56.8, 54.0, 53.7, 59.8

(a) Show that the mean of the times is 54.9 seconds to 1 decimal place.

**(2)**

Given that $\Sigma x^2 = 27122.59$

(b) Calculate the standard deviation of the times.

**(3)**

(c) The times, in seconds, taken by *Mr.Swarna* to run 200 metres on nine successive days have a mean of 25 and range of 4.9. Explain why none of his time could be as high as 30.

**(2)**

The mean and standard deviation of the times, in seconds, taken by *Mr.Swarna* to run 200 metres are 25 and 2 respectively. If each time scaled by adding 1 minute and then reducing the time by a further 5%.

(d) Find the mean and standard deviation of the scaled times.

**(4)**

**Question 7 continued**

**(Total 11 marks)**

**8.**

Two electric devices D1 and D2 have life-length in hours of $X$ and $Y$ respectively. Where,

$$X \sim \mathrm{N}(\mu, 49) \text{ and } Y \sim \mathrm{N}(\mu, 169)$$

(a) If $\mathrm{P}(X < 110) = \mathrm{P}(Y > 70)$ then find $\mu$

**(3)**

(b) Find the probability that a device D1 will last for at least 90 hours.

**(2)**

(c) Find the probability that a device D1 which has already operated for 90 hours will last at most another 15 hours.

**(3)**

(d) Third device D3 has life-length in hours of $M$. Where,

$$M \sim \mathrm{N}(80, 100)$$

If, $\mathrm{P}(M < 78 + \sqrt{10}) = \mathrm{P}(M > b)$. Find the *exact* value of $b$ **(2)**

**Question 8 continued**

**(Total 10 marks)**

***TOTAL FOR PAPER: 75 MARKS***

# Formulae for Statistics S1

### *Probability*

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)\, P(B|A)$$

### *Continuous distributions*

| Distribution of $X$ | Mean | Variance |
|---|---|---|
| Normal $N(\mu, \sigma^2)$ | $\mu$ | $\sigma^2$ |

### *Correlation and regression*

For a set of $n$ pairs of values $(x_i, y_i)$

$$S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$$

$$S_{yy} = \Sigma(y_i - \bar{y})^2 = \Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}$$

$$S_{xy} = \Sigma(x_i - \bar{x})(y_i - \bar{y}) = \Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}$$

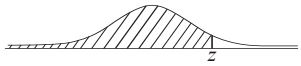
The product moment correlation coefficient is

$$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\{\Sigma(x_i - \bar{x})^2\}\{\Sigma(y_i - \bar{y})^2\}}} = \frac{\Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}}{\sqrt{\left(\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\right)\left(\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}\right)}}$$

The regression coefficient of $y$ on $x$ is $b = \frac{S_{xy}}{S_{xx}} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2}$

Least squares regression line of $y$ on $x$ is $y = a + bx$ where $a = \bar{y} - b\bar{x}$

## THE NORMAL DISTRIBUTION FUNCTION




| $z$ | $\Phi(z)$ | $z$ | $\Phi(z)$ | $z$ | $\Phi(z)$ | $z$ | $\Phi(z)$ | $z$ | $\Phi(z)$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 0.5000 | 0.50 | 0.6915 | 1.00 | 0.8413 | 1.50 | 0.9332 | 2.00 | 0.9772 |
| 0.01 | 0.5040 | 0.51 | 0.6950 | 1.01 | 0.8438 | 1.51 | 0.9345 | 2.02 | 0.9783 |
| 0.02 | 0.5080 | 0.52 | 0.6985 | 1.02 | 0.8461 | 1.52 | 0.9357 | 2.04 | 0.9793 |
| 0.03 | 0.5120 | 0.53 | 0.7019 | 1.03 | 0.8485 | 1.53 | 0.9370 | 2.06 | 0.9803 |
| 0.04 | 0.5160 | 0.54 | 0.7054 | 1.04 | 0.8508 | 1.54 | 0.9382 | 2.08 | 0.9812 |
| 0.05 | 0.5199 | 0.55 | 0.7088 | 1.05 | 0.8531 | 1.55 | 0.9394 | 2.10 | 0.9821 |
| 0.06 | 0.5239 | 0.56 | 0.7123 | 1.06 | 0.8554 | 1.56 | 0.9406 | 2.12 | 0.9830 |
| 0.07 | 0.5279 | 0.57 | 0.7157 | 1.07 | 0.8577 | 1.57 | 0.9418 | 2.14 | 0.9838 |
| 0.08 | 0.5319 | 0.58 | 0.7190 | 1.08 | 0.8599 | 1.58 | 0.9429 | 2.16 | 0.9846 |
| 0.09 | 0.5359 | 0.59 | 0.7224 | 1.09 | 0.8621 | 1.59 | 0.9441 | 2.18 | 0.9854 |
| 0.10 | 0.5398 | 0.60 | 0.7257 | 1.10 | 0.8643 | 1.60 | 0.9452 | 2.20 | 0.9861 |
| 0.11 | 0.5438 | 0.61 | 0.7291 | 1.11 | 0.8665 | 1.61 | 0.9463 | 2.22 | 0.9868 |
| 0.12 | 0.5478 | 0.62 | 0.7324 | 1.12 | 0.8686 | 1.62 | 0.9474 | 2.24 | 0.9875 |
| 0.13 | 0.5517 | 0.63 | 0.7357 | 1.13 | 0.8708 | 1.63 | 0.9484 | 2.26 | 0.9881 |
| 0.14 | 0.5557 | 0.64 | 0.7389 | 1.14 | 0.8729 | 1.64 | 0.9495 | 2.28 | 0.9887 |
| 0.15 | 0.5596 | 0.65 | 0.7422 | 1.15 | 0.8749 | 1.65 | 0.9505 | 2.30 | 0.9893 |
| 0.16 | 0.5636 | 0.66 | 0.7454 | 1.16 | 0.8770 | 1.66 | 0.9515 | 2.32 | 0.9898 |
| 0.17 | 0.5675 | 0.67 | 0.7486 | 1.17 | 0.8790 | 1.67 | 0.9525 | 2.34 | 0.9904 |
| 0.18 | 0.5714 | 0.68 | 0.7517 | 1.18 | 0.8810 | 1.68 | 0.9535 | 2.36 | 0.9909 |
| 0.19 | 0.5753 | 0.69 | 0.7549 | 1.19 | 0.8830 | 1.69 | 0.9545 | 2.38 | 0.9913 |
| 0.20 | 0.5793 | 0.70 | 0.7580 | 1.20 | 0.8849 | 1.70 | 0.9554 | 2.40 | 0.9918 |
| 0.21 | 0.5832 | 0.71 | 0.7611 | 1.21 | 0.8869 | 1.71 | 0.9564 | 2.42 | 0.9922 |
| 0.22 | 0.5871 | 0.72 | 0.7642 | 1.22 | 0.8888 | 1.72 | 0.9573 | 2.44 | 0.9927 |
| 0.23 | 0.5910 | 0.73 | 0.7673 | 1.23 | 0.8907 | 1.73 | 0.9582 | 2.46 | 0.9931 |
| 0.24 | 0.5948 | 0.74 | 0.7704 | 1.24 | 0.8925 | 1.74 | 0.9591 | 2.48 | 0.9934 |
| 0.25 | 0.5987 | 0.75 | 0.7734 | 1.25 | 0.8944 | 1.75 | 0.9599 | 2.50 | 0.9938 |
| 0.26 | 0.6026 | 0.76 | 0.7764 | 1.26 | 0.8962 | 1.76 | 0.9608 | 2.55 | 0.9946 |
| 0.27 | 0.6064 | 0.77 | 0.7794 | 1.27 | 0.8980 | 1.77 | 0.9616 | 2.60 | 0.9953 |
| 0.28 | 0.6103 | 0.78 | 0.7823 | 1.28 | 0.8997 | 1.78 | 0.9625 | 2.65 | 0.9960 |
| 0.29 | 0.6141 | 0.79 | 0.7852 | 1.29 | 0.9015 | 1.79 | 0.9633 | 2.70 | 0.9965 |
| 0.30 | 0.6179 | 0.80 | 0.7881 | 1.30 | 0.9032 | 1.80 | 0.9641 | 2.75 | 0.9970 |
| 0.31 | 0.6217 | 0.81 | 0.7910 | 1.31 | 0.9049 | 1.81 | 0.9649 | 2.80 | 0.9974 |
| 0.32 | 0.6255 | 0.82 | 0.7939 | 1.32 | 0.9066 | 1.82 | 0.9656 | 2.85 | 0.9978 |
| 0.33 | 0.6293 | 0.83 | 0.7967 | 1.33 | 0.9082 | 1.83 | 0.9664 | 2.90 | 0.9981 |
| 0.34 | 0.6331 | 0.84 | 0.7995 | 1.34 | 0.9099 | 1.84 | 0.9671 | 2.95 | 0.9984 |
| 0.35 | 0.6368 | 0.85 | 0.8023 | 1.35 | 0.9115 | 1.85 | 0.9678 | 3.00 | 0.9987 |
| 0.36 | 0.6406 | 0.86 | 0.8051 | 1.36 | 0.9131 | 1.86 | 0.9686 | 3.05 | 0.9989 |
| 0.37 | 0.6443 | 0.87 | 0.8078 | 1.37 | 0.9147 | 1.87 | 0.9693 | 3.10 | 0.9990 |
| 0.38 | 0.6480 | 0.88 | 0.8106 | 1.38 | 0.9162 | 1.88 | 0.9699 | 3.15 | 0.9992 |
| 0.39 | 0.6517 | 0.89 | 0.8133 | 1.39 | 0.9177 | 1.89 | 0.9706 | 3.20 | 0.9993 |
| 0.40 | 0.6554 | 0.90 | 0.8159 | 1.40 | 0.9192 | 1.90 | 0.9713 | 3.25 | 0.9994 |
| 0.41 | 0.6591 | 0.91 | 0.8186 | 1.41 | 0.9207 | 1.91 | 0.9719 | 3.30 | 0.9995 |
| 0.42 | 0.6628 | 0.92 | 0.8212 | 1.42 | 0.9222 | 1.92 | 0.9726 | 3.35 | 0.9996 |
| 0.43 | 0.6664 | 0.93 | 0.8238 | 1.43 | 0.9236 | 1.93 | 0.9732 | 3.40 | 0.9997 |
| 0.44 | 0.6700 | 0.94 | 0.8264 | 1.44 | 0.9251 | 1.94 | 0.9738 | 3.50 | 0.9998 |
| 0.45 | 0.6736 | 0.95 | 0.8289 | 1.45 | 0.9265 | 1.95 | 0.9744 | 3.60 | 0.9998 |
| 0.46 | 0.6772 | 0.96 | 0.8315 | 1.46 | 0.9279 | 1.96 | 0.9750 | 3.70 | 0.9999 |
| 0.47 | 0.6808 | 0.97 | 0.8340 | 1.47 | 0.9292 | 1.97 | 0.9756 | 3.80 | 0.9999 |
| 0.48 | 0.6844 | 0.98 | 0.8365 | 1.48 | 0.9306 | 1.98 | 0.9761 | 3.90 | 1.0000 |
| 0.49 | 0.6879 | 0.99 | 0.8389 | 1.49 | 0.9319 | 1.99 | 0.9767 | 4.00 | 1.0000 |
| 0.50 | 0.6915 | 1.00 | 0.8413 | 1.50 | 0.9332 | 2.00 | 0.9772 | | |

## PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

The values $z$ in the table are those which a random variable $Z \sim N(0, 1)$ exceeds with probability $p$; that is, $P(Z > z) = 1 - \Phi(z) = p$.

| $p$ | $z$ | $p$ | $z$ |
|---|---|---|---|
| 0.5000 | 0.0000 | 0.0500 | 1.6449 |
| 0.4000 | 0.2533 | 0.0250 | 1.9600 |
| 0.3000 | 0.5244 | 0.0100 | 2.3263 |

| $p$ | $z$ | $p$ | $z$ |
|---|---|---|---|
| 0.2000 | 0.8416 | 0.0050 | 2.5758 |
| 0.1500 | 1.0364 | 0.0010 | 3.0902 |
| 0.1000 | 1.2816 | 0.0005 | 3.2905 |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