GCE Examinations

Pure Mathematics
Module P6

Advanced Subsidiary / Advanced Level

Paper E

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 8 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Rosemary Smith & Shaun Armstrong

1. The point P represents a variable point $z = x + \mathrm{i}y$ in an Argand diagram where $x, y \in \mathbb{R}$.

Given that the locus of P is a circle with centre $-1 + \mathrm{i}$ and radius 2, find

(a) an equation of the circle in terms of z, **(2 marks)**

(b) the points on the locus of P which represent real numbers. **(3 marks)**

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2. Prove by induction that $2^n > 2n$ for all integers n, $n \geq 3$. **(6 marks)**

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3. *(a)* By using the series expansion for $\ln(1 + 2x)$ and the series expansion for e^x, or otherwise, and given that x is small, show that

$$\ln(1 + 2x) - 2x\mathrm{e}^{-x} \approx Ax^3,$$

and find the value of A. **(4 marks)**

(b) Hence find

$$\lim_{x \to 0}\left(\frac{\ln(1+2x) - 2x\mathrm{e}^{-x}}{x^3}\right).$$ **(2 marks)**

4.

$$\mathbf{A} = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 1 & -1 \\ -3 & 3 & 1 \end{pmatrix}.$$

(a) Show that $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ is an eigenvector of $\mathbf{A}$ and find the corresponding eigenvalue.

(2 marks)

(b) Prove that $\mathbf{A}$ has only one real eigenvalue, showing your working clearly. **(6 marks)**

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5. A transformation T from the z-plane to the w-plane is defined by

$$w = z^2$$

where $z = x + \mathrm{i}y$, $w = u + \mathrm{i}v$ and x, y, u and v are real.

(a) Show that T transforms the line $\operatorname{Im} z = 2$ in the z-plane onto a parabola in the w-plane and find an equation of the parabola, giving your answer in terms of u and v.
(5 marks)

The image in the w-plane of the half-line $\arg(z) = \dfrac{\pi}{4}$ is the half-line l.

(b) Find an equation of l. **(2 marks)**

The parabola and the half-line in the w-plane are represented on the same Argand diagram. Their point of intersection is represented by P.

(c) Find the complex number which is represented by P, giving your answer in the form $a + \mathrm{i}b$ where a and b are real.
(4 marks)

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6. It is given that y satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 + y\cos x \quad \text{and} \quad y = 1 \text{ at } x = 0.$$

(a) (i) Use the differential equation to find expressions for $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and $\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}$.

(ii) Hence, or otherwise, find y as a series in ascending powers of x up to and including the term in x^3.

(iii) Use your series to estimate the value of y at $x = -0.1$ **(10 marks)**

(b) Use the approximation $\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \dfrac{y_1 - y_{-1}}{2h}$ to estimate the value of y at $x = 0.1$
(3 marks)

Turn over

7. Referred to an origin O, the points A, B, C and D have coordinates (1, 1, 0), (3, 2, 5), (0, –1, –4) and (–2, –5, 0) respectively.

(a) Find, in the form $\mathbf{r.n} = p$, an equation of the plane Π passing through A, B and C.

(6 marks)

The line l passes through D and is perpendicular to Π.

(b) Find a vector equation of l. **(1 mark)**

The line l meets the plane Π at the point E.

(c) Find the coordinates of E. **(4 marks)**

The point F is the reflection of D in Π.

(d) Find the coordinates of F. **(2 marks)**

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8. The transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is represented by the matrix $\mathbf{M}$ where

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 3 & 1 \\ 2 & 2 & 0 \end{pmatrix}.$$

(a) Find $\mathbf{M}^{-1}$, showing your working clearly. **(6 marks)**

(b) Find the Cartesian equations of the line mapped by the transformation T onto the line with equations

$$\frac{x-1}{3} = \frac{y+1}{-3} = \frac{z}{4}.$$ **(7 marks)**

END