GCE Examinations

Advanced Subsidiary / Advanced Level

Pure Mathematics
Module P6

Paper C

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

Written by Rosemary Smith & Shaun Armstrong

P6 Paper C – Marking Guide

1.	*(a)*	$\frac{y_1 - y_0}{0.01} = e^{x_0}\cosh(2y_0 + x_0)$ $\therefore y_1 = 0.01\,e^{x_0}\cosh(2y_0 + x_0) + y_0$	M1 A1	
		$x_0 = 1, x_1 = 1.01; y_0 = 1$ $\therefore$ $y_1 = 1.2736... = 1.27$ (3sf)	A1	
	(b)	$\frac{y_1 - y_{-1}}{0.02} = e^{x_0}\cosh(2y_0 + x_0)$ $\therefore y_{-1} = y_1 - 0.02\,e^{x_0}\cosh(2y_0 + x_0)$	M1 A1	
		$x_{-1} = 0.99, x_0 = 1, x_1 = 1.01; y_0 = 1, y_1 = 1.2736..., y_{-1} = ?$		
		$\therefore$ $y_{-1} = 0.7263... = 0.726$ (3sf)	A1	**(6)**
2.	*(a)*	$\overrightarrow{AB} = -4\mathbf{i} + 3\mathbf{j} - \mathbf{k}$, $\overrightarrow{AC} = (a-2)\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}$	B1	
		$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 3 & -1 \\ a-2 & -6 & 2 \end{vmatrix}$		
		$= \mathbf{i}(6-6) - \mathbf{j}(-8 + a - 2) + \mathbf{k}[24 - 3(a-2)] = (10-a)\mathbf{j} + 3(10-a)\mathbf{k}$	M1 A2	
	(b)	area $= \frac{1}{2} \mid \overrightarrow{AB} \times \overrightarrow{AC} \mid = 4\sqrt{10}$	M1	
		$\therefore$ $\mid 10 - a \mid \times \sqrt{(1+9)} = 8\sqrt{10}$	M1	
		$\mid 10 - a \mid = 8$ so $a = 2$ or 18	A1	**(7)**
3.	*(a)*	$z^n + \frac{1}{z^n} = \cos n\theta + \mathrm{i}\sin n\theta + \cos(-n\theta) + \mathrm{i}\sin(-n\theta)$	M1	
		$= \cos n\theta + \mathrm{i}\sin n\theta + \cos n\theta - \mathrm{i}\sin n\theta = 2\cos n\theta$	A1	
	(b)	dividing by z^2 gives $5z^2 - 11z + 16 - \frac{11}{z} + \frac{5}{z^2} = 0$	M1	
		$\therefore$ $5(2\cos 2\theta) - 11(2\cos\theta) + 16 = 0$	M1	
		$5\cos 2\theta - 11\cos\theta + 8 = 0$	A1	
		$5(2\cos^2\theta - 1) - 11\cos\theta + 8 = 0$	M1	
		$10\cos^2\theta - 11\cos\theta + 3 = 0$		
		$(5\cos\theta - 3)(2\cos\theta - 1) = 0$	M1	
		$\therefore \cos\theta = \frac{3}{5}$ or $\frac{1}{2}$	A1	
		if $\cos\theta = \frac{3}{5}$, $\sin\theta = \pm\frac{4}{5}$; if $\cos\theta = \frac{1}{2}$, $\sin\theta = \pm\frac{\sqrt{3}}{2}$	M1	
		$\therefore z = \frac{3}{5} \pm \frac{4}{5}\mathrm{i}$ or $\frac{1}{2} \pm \frac{\sqrt{3}}{2}\mathrm{i}$	A1	**(10)**

4. *(a)* assume true for $n = k$ $\therefore$ $\mathbf{A}^k = \begin{pmatrix} 1 & k & \frac{1}{2}k(k+1) \\ 0 & 1 & k \\ 0 & 0 & 1 \end{pmatrix}$

$\therefore$ $\mathbf{A}^{k+1} = \begin{pmatrix} 1 & k & \frac{1}{2}k(k+1) \\ 0 & 1 & k \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1+k & 1+k+\frac{1}{2}k(k+1) \\ 0 & 1 & 1+k \\ 0 & 0 & 1 \end{pmatrix}$ M1 A1

$1 + k + \frac{1}{2}k(k + 1) = \frac{1}{2}(k + 1)(2 + k) = \frac{1}{2}(k + 1)[(k + 1) + 1]$ M1

$\therefore$ $\mathbf{A}^{k+1} = \begin{pmatrix} 1 & k+1 & \frac{1}{2}(k+1)[(k+1)+1] \\ 0 & 1 & k+1 \\ 0 & 0 & 1 \end{pmatrix}$ A1

$\therefore$ true for $n = k + 1$ if true for $n = k$

if $n = 1$ $\mathbf{A}^1 = \begin{pmatrix} 1 & 1 & \frac{1}{2}\times1\times2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ $\therefore$ true for $n = 1$ B1

$\therefore$ by induction true for $n \in \mathbb{Z}^+$ A1

(b) $\det \mathbf{A}^n = 1(1 - 0) - n(0 - 0) + \frac{1}{2}n(n + 1)(0 - 0) = 1$ M1 A1

matrix of cofactors: $\begin{pmatrix} 1 & 0 & 0 \\ -n & 1 & 0 \\ \frac{1}{2}n(n-1) & -n & 1 \end{pmatrix}$ $[n^2 - \frac{1}{2}n(n + 1) = \frac{1}{2}n(n - 1)]$ M1 A1

$\therefore$ $(\mathbf{A}^n)^{-1} = \begin{pmatrix} 1 & -n & \frac{1}{2}n(n-1) \\ 0 & 1 & -n \\ 0 & 0 & 1 \end{pmatrix}$ A1 **(11)**

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5. *(a)* let $y = \arccos x$ $\therefore$ $\cos y = x$

$-\sin y \dfrac{\mathrm{d}y}{\mathrm{d}x} = 1$ M1

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-1}{\sin y} = \dfrac{-1}{\sqrt{1-x^2}}$ $\therefore$ $\mathrm{f}'(x) = \dfrac{-1}{(1-x^2)^{\frac{1}{2}}}$ M1 A1

(b) $\mathrm{f}''(x) = \frac{1}{2}(-2x)(1 - x^2)^{-\frac{3}{2}} = \dfrac{-x}{(1-x^2)^{\frac{3}{2}}}$ M1 A1

$\therefore$ $(1 - x^2)\mathrm{f}''(x) - x\mathrm{f}'(x) = (1 - x^2) \times \dfrac{-x}{(1-x^2)^{\frac{3}{2}}} + \dfrac{x}{(1-x^2)^{\frac{1}{2}}}$

$= \dfrac{-x}{(1-x^2)^{\frac{1}{2}}} + \dfrac{x}{(1-x^2)^{\frac{1}{2}}} = 0$ A1

(c) $(1 - x^2)\mathrm{f}'''(x) - 2x\mathrm{f}''(x) - x\mathrm{f}''(x) - \mathrm{f}'(x) = 0$ M1 A1

$\mathrm{f}(0) = \frac{\pi}{2}$, $\mathrm{f}'(0) = -1$, $\mathrm{f}''(0) = 0$, $\mathrm{f}'''(0) = -1$ A1

$\therefore$ $\mathrm{f}(x) = \frac{\pi}{2} - 1x + 0 - 1(\frac{1}{3!})x^3 + \ldots = \frac{\pi}{2} - x - \frac{1}{6}x^3 + \ldots$ M1 A1 **(11)**

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6. *(a)* $\begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & -\lambda & 1 \\ 1 & 1 & 2-\lambda \end{vmatrix} = 0$ M1

$(2-\lambda)[-\lambda(2-\lambda)-1] + 1[-(2-\lambda)-1] + 1(-1+\lambda) = 0$ A1

$(2-\lambda)(\lambda^2 - 2\lambda - 1) - (2-\lambda) - 1 - 1 + \lambda = 0$ M1

$(2-\lambda)(\lambda^2 - 2\lambda - 1) - (2-\lambda) - (2-\lambda) = 0$

$(2-\lambda)(\lambda^2 - 2\lambda - 1 - 2) = (2-\lambda)(\lambda^2 - 2\lambda - 3) = 0$ A1

$(2-\lambda)(\lambda - 3)(\lambda + 1) = 0$ M1

$\therefore\ \lambda = 2$ is an eigenvalue, also $\lambda = -1$ or 3 A2

(b) $\lambda = 2,\ \begin{pmatrix} 0 & -1 & 1 \\ -1 & -2 & 1 \\ 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ M1

$-y + z = 0\ \therefore\ y = z;\quad x + y = 0\ \therefore\ x = -y$ M1 A1

$\therefore$ eigenvector $k\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ A1

(c) $\mathbf{P} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{3}} & \frac{2}{\sqrt{6}} & 0 \\ \frac{-1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ M1 A2 **(14)**

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7. *(a)* $|\,x + \mathrm{i}y + 1 + 8\mathrm{i}\,| = 3\,|\,x + \mathrm{i}y + 1\,|$

$(x+1)^2 + (y+8)^2 = 9(x+1)^2 + 9y^2$ M1 A1

$y^2 + 16y + 64 = 8(x+1)^2 + 9y^2$ M1

$8(x+1)^2 + 8y^2 - 16y - 64 = 0$

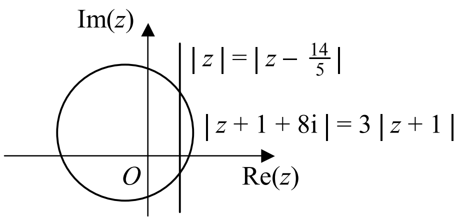
$(x+1)^2 + y^2 - 2y - 8 = 0$

$(x+1)^2 + (y-1)^2 - 1 - 8 = 0$ M1

$(x+1)^2 + (y-1)^2 = 9$ A1

$\therefore$ circle, centre $-1 + \mathrm{i}$, radius 3 A2

(b) $|\,z\,| = |\,z - \frac{14}{5}\,|$ is perp. bisector of 0 and $\frac{14}{5}$ i.e. $\mathrm{Re}(z) = \frac{7}{5}$ B1



B1 B2

(c) intersect when $x = \frac{7}{5}$ M1

$\therefore\ (\frac{7}{5} + 1)^2 + (y-1)^2 = 9$ A1

$(y-1)^2 = 9 - \frac{144}{25} = \frac{81}{25}$ M1

$\therefore\ y = 1 \pm \frac{9}{5}$ A1

intersect at $\frac{7}{5} - \frac{4}{5}\mathrm{i}$ and $\frac{7}{5} + \frac{14}{5}\mathrm{i}$ A1 **(16)**

Total **(75)**

Performance Record – P6 Paper C

Question no.	1	2	3	4	5	6	7	Total
Topic(s)	step-by-step soln. of diff. eqn.	vectors	complex nos., De Moivre's	proof by induction, matrices	Maclaurin series	matrices, eigenvals., diag'lise	complex loci	
Marks	6	7	10	11	11	14	16	75
Student								