1. Given that $x > 0$, prove by induction that, for all integers $n \geq 1$,

$$(1 + x)^n \geq 1 + nx.$$

**(5 marks)**

2. Find the first three terms in the Maclaurin series for

$$\sin\left(x + \frac{\pi}{6}\right).$$

**(5 marks)**

3. Find the fourth roots of $8(1 + i\sqrt{3})$, giving your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. **(7 marks)**

4. If $\mathbf{M} = \begin{pmatrix} x & 0 & 1 \\ x & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$, where $x \neq 0$, find $\mathbf{M}^{-1}$ in terms of $x$. **(7 marks)**

5. (a) Differentiate $\sec x \tan x$ with respect to $x$. **(2 marks)**

   (b) Find the first two non-zero terms in the series expansion of $\sec x$ in ascending powers of $x$. **(4 marks)**

   (c) Given that the third term in the expansion of $\sec x$ is $\frac{5}{24}x^4$, deduce the first three non-zero terms in the expansion of $\ln(\sec x + \tan x)$. **(4 marks)**

6. (a) Given that $\frac{dy}{dx} = x^2 y$ and that $y = 1{\cdot}2$ when $x = 0{\cdot}1$, take $h = 0{\cdot}1$ in the approximation $\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_0}{h}$ to estimate the value of $y$ when $x = 0{\cdot}2$. **(5 marks)**

   (b) Solve the differential equation by an exact method and hence find the true value of $y$ when $x = 0{\cdot}2$. **(6 marks)**

7. The points $A\,(2, -1, 3)$, $B\,(3, -2, 2)$ and $C\,(4, -2, 1)$ lie in a plane $\Pi$.

   (a) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **(2 marks)**

   (b) Find a vector which is normal to the plane $\Pi$. **(3 marks)**

   (c) Write the equation of $\Pi$ in the form $\mathbf{r} \cdot \hat{\mathbf{n}} = p$, where $\hat{\mathbf{n}}$ is a unit vector.
   State the significance of the constant $p$. **(5 marks)**

   (d) Find, in radians to 3 significant figures, the acute angle between $\Pi$ and the plane with equation $\mathbf{r} \cdot (\mathbf{i} - \mathbf{j} + 2\mathbf{k}) = -1$. **(5 marks)**

8. A transformation $T$ from the $z$-plane to the $w$-plane is defined by the equation

$$w = \frac{z + \mathrm{i}}{z - 2}.$$

   (a) Find the image under $T$ of the complex number $z = 4 - 2\mathrm{i}$. **(2 marks)**

   (b) Find the complex number which is mapped by $T$ to $w = 1 - \mathrm{i}$. **(5 marks)**

   Given that $z = x + \mathrm{i}y$ and $w = u + \mathrm{i}v$,

   (c) show that the image of the line $y = -x$ under $T$ is a circle and give its equation in terms of $u$ and $v$. **(8 marks)**