PURE MATHS 6 (A) TEST PAPER 9 : ANSWERS AND MARK SCHEME

1. Char. eqn. is $(\lambda - 2)(\lambda - 6) + 4 = 0$ $\quad \lambda^2 - 8\lambda + 16 = 0$ — M1 A1
$(\lambda - 4)^2 = 0$ $\quad \lambda = 4$ is the only eigenvalue — M1 A1
Eigenvector is such that $-x + 2y = 0$, so is any multiple of (2, 1) — M1 A1 — **6**

2. $3^2 = 9 > 6 = 3(2)$: true for $n = 2$ $\quad$ Assume true for $n = k$, so $3^k > 3k$ — B1 M1
$3^{k+1} = 3(3^k) > 3(3k) = 9k = 3k + 6k$ $\quad$ Now $k > 1$, so — M1 A1
$6k > 6$, so $3k + 6k > 3k + 6 > 3k + 3 = 3(k + 1)$: true for $n = k + 1$ — M1 A1 — **6**

3. $y = \text{arsinh}\, x = \ln [x + \sqrt{(1 + x^2)}]$ $\quad y(0) = \text{arsinh}\, 0 = 0$ — B1 B1
$dy/dx = [1 + x(1 + x^2)^{-1/2}]/[x + \sqrt{(1 + x^2)}]$ $\quad y'(0) = 1$ — M1 A1 A1
First non-zero term in expansion is x — A1 — **6**

4. $(z - i)/(z + 1) = (x + (y - 1)i)/((x + 1) + iy)$ — M1
$= [x(x + 1) + y(y - 1) + i(y - 1 - x)]/[(x + 1)^2 + y^2]$ — M1 A1
If arg $= \pi/4$, real part = imaginary part, i.e. $x^2 + x + y^2 - y = y - 1 - x$ — M1 A1
$x^2 + y^2 + 2x - 2y + 1 = 0$ $\quad$ Circle centre $(-1, 1)$, radius 1 — M1 A1 A1 — **8**

5. (a) Det $\mathbf{A} = 8 - 12 = -4$ — M1 A1

$$\mathbf{A}^{-1} = -\frac{1}{4}\begin{pmatrix} -8 & 0 & 4 \\ -6 & 0 & 2 \\ 3k+8 & -2 & -k-4 \end{pmatrix} = \begin{pmatrix} 2 & 0 & -1 \\ 1{\cdot}5 & 0 & -0{\cdot}5 \\ -2-3k/4 & 0{\cdot}5 & 1+k/4 \end{pmatrix}$$ — M1 M1 A1 A1

(b) (x, y, z) are found by pre-multiplying (3, 5, 7) by $\mathbf{A}^{-1}$, so — M1

$$x = -1, y = 1, z = \frac{7-k}{2}$$ — A1 A1 — **9**

6. (a) Need $\mathbf{r} - \mathbf{i} - \mathbf{j} = \lambda(3\mathbf{i} + 4\mathbf{k})$ e.g. $\mathbf{r} = \mathbf{i} + \mathbf{j}$, $\mathbf{r} = 4\mathbf{i} + \mathbf{j} + 4\mathbf{k}$ — M1 A1 A1
(b) $\mathbf{r} = \mathbf{i} + \mathbf{j} + t(3\mathbf{i} + 4\mathbf{k})$, or equivalent — M1 A1
(c) $\mathbf{r} = s(\mathbf{i} + \mathbf{j}) + t(4\mathbf{i} + \mathbf{j} + 4\mathbf{k})$ $\quad x = s + 4t,\ y = s + t,\ z = 4t$ — B1 M1 A1
$z/4 = (x - y)/3$ $\quad 4x - 4y - 3z = 0$ — M1 A1 — **10**

7. (a) $y''' = 2x - 2yy'$ $\quad y''(0) = -1,\quad y'''(0) = 2$ — M1 A1 A1
$y^{iv} = 2 - 2y'y' - 2yy''$ $\quad y^{iv}(0) = 2 - 2 + 2 = 2$ — M1 A1 A1
$y = 1 - x - x^2/2 + x^3/3 + x^4/12$ — M1 A1
(b) $y(0{\cdot}05) \approx 0{\cdot}9488$ — M1 A1
(c) $y_1 \approx h^2 y''_0 + 2y_0 - y_{-1} = 0{\cdot}05^2(-0{\cdot}8977) + 2(0{\cdot}9488) - 1$ — M1 M1 A1
$\approx 0{\cdot}895$ — A1 — **14**

8. (a) $\cos 4\theta + i \sin 4\theta = (\cos\theta + i\sin\theta)^4$ — M1
$= \cos^4\theta + 4i\cos^3\theta\sin\theta - 6\cos^2\theta\sin^2\theta - 4i\cos\theta\sin^3\theta + \sin^4\theta$ — M1 A1 A1
Comparing real and imag. parts yields given result for $\cos 4\theta$ — M1
$\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$ — A1
(b) Write $(\cos 4\theta)/(\sin 4\theta)$, divide by $\sin^4\theta$ top and bottom — M1 M1 A1
(c) If $\cot 4\theta = 0$, numerator = 0, so $\cot^2\theta = [6 \pm \sqrt{(36 - 4)}]/2$ — M1 M1
$= 3 \pm 2\sqrt{2}$, so $\cot\theta = \pm(3 \pm 2\sqrt{2})^{1/2}$ — A1 A1
$\pi/8$ is the smallest positive value of θ for which $\cot 4\theta = 0$ — M1
$\cot\theta$ is a decreasing function on $(0, \pi/2]$, so
$\cot \pi/8 = (3 + 2\sqrt{2})^{1/2}$ or $1 + \sqrt{2}$ — A1 A1 — **16**