# PURE MATHS 6 (A) TEST PAPER 8 : ANSWERS AND MARK SCHEME

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|---|---|---|---|
| 1. | Locus is $\lvert z-\mathrm{i}\rvert = 2\lvert z-2+\mathrm{i}\rvert$ | M1 A1 A1 | 3 |
| 2. | $y'(0.1) = 0{\cdot}1 + 0{\cdot}01 = 0{\cdot}11$ | M1 A1 | |
| | $y(0.2) \approx 2 \times 0{\cdot}1 \times 0{\cdot}11 + 0 = 0{\cdot}022$ | M1 A1 A1 | 5 |
| 3. | When $n = 1$, $7^{2n} - 5 = 49 - 5 = 44$, divisible by 4 | M1 A1 | |
| | Assume true for $n = k$, so let $7^{2k} - 5 = 4a$ | M1 | |
| | Then $7^{2(k+1)} - 5 = 49(7^{2k} - 5) + 245 - 5 = 49(4a) + 240$ | M1 A1 A1 | |
| | $= 4(49a + 60)$ which is a multiple of 4 | A1 | 7 |
| 4. | (a) Replacing $x$ by $x + x^2$ in $\ln(1+x)$ expansion gives | M1 | |
| | $x + x^2 - (x+x^2)^2/2 + (x+x^2)^3/3 = x + x^2/2 - 2x^3/3$ | M1 A1 M1 A1 | |
| | (b) $\ln(1/(1+x+x^2)) = -\ln(1+x+x^2) = -x - x^2/2 + 2x^3/3$ | M1 M1 A1 | 8 |
| 5. | (a) Direction is normal to plane, i.e. $\mathbf{i} + \mathbf{j} - 4\mathbf{k}$ | B1 | |
| | $\mathbf{r} = 4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k} + t(\mathbf{i} + \mathbf{j} - 4\mathbf{k})$ | M1 A1 | |
| | (b) $N$ is on $AB$ and in $\Pi$, so $(4+t \quad 3+t \quad 2-4t) \cdot (1 \quad 1 \quad -4) = 8$ | M1 M1 | |
| | $4 + t + 3 + t - 8 + 16t = 8 \qquad 18t = 9 \qquad t = 1/2$ | M1 A1 | |
| | $N$ is $(9/2,\ 7/2,\ 0)$ | A1 | |
| | (c) $AB = 2\,AN$, so $B$ is $(5, 4, -2)$ | M1 M1 A1 | 11 |
| 6. | (a) $y'' = yy' - 3e^{-x} \qquad y''' = yy'' + y'^2 + 3e^{-x}$ | B1 B1 | |
| | $y(0) = 2,\ y'(0) = 1,\ y''(0) = -1,\ y'''(0) = 2$ | M1 A1 | |
| | Series solution is $y = 2 + x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \ldots$ | M1 A1 | |
| | (b) $y(-1) = 3,\ y'(-1) = 2,\ y''(-1) = 6 - 3e$ | M1 A1 | |
| | Series solution is $y = 3 + 2(x+1) + \left(3 - \frac{3e}{2}\right)(x+1)^2 + \ldots$ | M1 A1 A1 | 11 |
| 7. | (a) Eigenvalues occur when $\det(\mathbf{M} - \lambda\mathbf{I}) = 0$, i.e. | M1 | |
| | $(-1-\lambda)(\lambda^2 - 3\lambda - 2) - 2(-\lambda - 1) = 0 \qquad (-1-\lambda)(\lambda^2 - 3\lambda - 4) = 0$ | M1 A1 | |
| | $(-1-\lambda)(\lambda+1)(\lambda-4) = 0 \qquad \lambda = -1$ or $\lambda = 4$ | M1 A1 (both) | |
| | When $\lambda = -1$, $y + z = 0$ and $2x + 4y + 2z = 0$ | M1 A1 | |
| | Eigenvectors of form $(1 \quad -1 \quad 1)$ | A1 | |
| | When $\lambda = 4$, $-5x + y + z = 0$, $2x - y + 2z = 0$, $y - 4z = 0$ | M1 A1 | |
| | $x = z,\ y = 4z \qquad$ Eigenvectors of form $(1 \quad 4 \quad 1)$ | A1 | |
| | (b) Lines have directions of eigenvectors: $x = -y = z$, $4x = y = 4z$ | B1 B1 | 13 |
| 8. | (a) $(\cos x + \mathrm{i}\sin x)^1 = \cos 1x + \mathrm{i}\sin 1x$, so true for $n = 1$ | B1 | |
| | Assume true for $n = k$: $(\cos x + \mathrm{i}\sin x)^k = \cos kx + \mathrm{i}\sin kx$ | M1 | |
| | Then $(\cos x + \mathrm{i}\sin x)^{k+1} = (\cos x + \mathrm{i}\sin x)(\cos kx + \mathrm{i}\sin kx)$ | A1 | |
| | $= (\cos x\cos kx - \sin x\sin kx) + \mathrm{i}(\cos x\sin kx + \sin x\cos kx)$ | A1 | |
| | $= \cos(kx + x) + \mathrm{i}\sin(kx + x) = \cos(k+1)x + \mathrm{i}\sin(k+1)x$ | A1 | |
| | so true for $n = k + 1$ Hence by induction, true for all $n$ | A1 | |
| | (b) Let $z = \cos x + \mathrm{i}\sin x$, so $1/z = \cos x - \mathrm{i}\sin x$ | B1 B1 | |
| | $(2\mathrm{i}\sin x)^5 = (z - 1/z)^5 = z^5 - 5z^3 + 10z - 10/z + 5/z^3 - 1/z^5$ | M1 A1 A1 | |
| | $32\mathrm{i}\sin^5 x = 2\mathrm{i}\sin 5x - 10\mathrm{i}\sin 3x + 20\mathrm{i}\sin x$, hence result | M1 A1 | |
| | (c) Integral $= 1/16\,[-(\cos 5x)/5 + 5(\cos 3x)/3 - 10\cos x]_0^{\pi}$ | M1 A1 | |
| | $= 1/16\,[2/5 - 10/3 + 20] = 16/15$ | M1 A1 | 17 |