PURE MATHS 6 (A) TEST PAPER 5 : ANSWERS AND MARK SCHEME

1.	(a)	Horizontal line at $y = -1/2$	M1 A1	
	(b)	$\text{Im}(z) = -1/2$	B1	
	(c)	If line is tangent to circle with centre at 2, then $k = 2{\cdot}5$	M1 A1	5
2.	(a)	$e^{-x} = 1 - x + x^2/2 - x^3/6 + x^4/24$	B2	
	(b)	$e^{-x}(\sin x + \cos x) \approx (1 - x + x^2/2)(1 + x - x^2/2)$	M1 A1	
		$= 1 + x - x^2/2 - x - x^2 + x^3/2 = 1 - x^2$	M1 A1 A1	7
3.	(a)	Char. eqn. is $\det(\mathbf{M} - \lambda\mathbf{I}) = 0$	M1	
		$(2-\lambda)(\lambda^2 - 3\lambda + 2 - 1) - 1(2-\lambda) = 0$	A1 A1	
		$(\lambda - 2)(\lambda^2 - 3\lambda) = 0 \qquad \lambda(\lambda-2)(\lambda-3) = 0$	M1 A1	
		Eigenvalues of **M** are 0, 2, 3	A1	
	(b)	0 is an eigenvalue, so **M** is singular ($\det \mathbf{M} = 0$)	B1	7
4.	(a)	$(-3\mathbf{i} + 2\mathbf{j} + \mathbf{k}) \times (-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) = 5\mathbf{i} + 7\mathbf{j} + \mathbf{k}$	M1 A1 A1 A1	
	(b)	Volume $= \lvert \mathbf{a} \times \mathbf{b} \,.\, \mathbf{c} \rvert = \lvert 5 + 28 - 2 \rvert = 31$	M1 M1 A1	7
5.	(a)	$y''' = -3xy' - 3y \qquad y^{iv} = -3xy'' - 6y'$	M1 M1 A1	
		$y''(0) = 0 \qquad y'''(0) = -3 \qquad y^{iv}(0) = 6$	A1 A1	
		$y = 1 - x - x^3/2 + x^4/4$	M1 A1	
	(b)	$y(0.1) \approx 1 - 0{\cdot}1 - 0{\cdot}0005 + 0{\cdot}000025 = 0{\cdot}8995$ (to 4 d.p.)	M1 A1 A1	10
6.	(a)	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	B2	
	(b)	Eigenvalues of **N** are 1 and 2	B1 B1	
	(c)	(x, y) maps to $(x, 2y)$, so transformation is a stretch by a	B1	
		factor of 2, parallel to the y-axis	B1	
	(d)	$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 2 & 0 \end{pmatrix}$	M1 A1	
	(e)	Use $(\mathbf{MN})^{-1} = \mathbf{N}^{-1}\mathbf{M}^{-1}$	B2	10
7.	(a)	$\mathbf{r} = (-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) + s(3\mathbf{i} - 4\mathbf{k}) + t(4\mathbf{i} + 3\mathbf{j} - 5\mathbf{k})$, or equivalent	M1 A1 A1	
	(b)	$x = -2 + 3s + 4t, \quad y = 1 + 3t, \quad z = 3 - 4s - 5t$	M1 A1	
		$4x + 3z = 1 + t, \quad y = 1 + 3(4x + 3z - 1) \qquad 12x - y + 9z = 2$	M1 M1 A1	
	(c)	Direction is $(12 \quad -1 \quad 9)$, so equation of line is	B1	
		$(\mathbf{r} - (-2\mathbf{i} + \mathbf{j} + 3\mathbf{k})) \times (12\mathbf{i} - \mathbf{j} + 9\mathbf{k}) = 0$	M1 A1	
	(d)	Π is $\mathbf{r} \,.\, (12\mathbf{i} - \mathbf{j} + 9\mathbf{k}) = 2$, so distance $= 2/\sqrt{226}$	B1 M1 A1	14
8.	(a)	$\cos 6\theta + i \sin 6\theta = (\cos\theta + i\sin\theta)^6 = \cos^6\theta + 6i\cos^5\theta\sin\theta$	M1	
		$- 15\cos^4\theta\sin^2\theta - 20i\cos^3\theta\sin^3\theta + 15\cos^2\theta\sin^4\theta + 6i\cos\theta\sin^5\theta - \sin^6\theta$	A1 A1	
		$\sin 6\theta = 6\cos^5\theta\sin\theta - 20\cos^3\theta\sin^3\theta + 6\cos\theta\sin^5\theta$	M1 A1	
		$= \sin\theta\cos\theta(6\cos^4\theta - 20\cos^2\theta(1 - \cos^2\theta) + 6(1 - \cos^2\theta)^2)$	M1	
		$= \sin\theta\cos\theta(32\cos^4\theta - 32\cos^2\theta + 6)$	A1	
	(b)	$(\sin 6\theta)/(\sin 2\theta)$		
		$= \sin\theta\cos\theta(32\cos^4\theta - 32\cos^2\theta + 6)/(2\sin\theta\cos\theta)$	M1 A1	
		$= 16\cos^4\theta - 16\cos^2\theta + 3$	A1	
	(c)	$\sin 2\theta = 0$, i.e. $\theta = \pi/2$, or $(\sin 6\theta)/(\sin 2\theta) = -1$	B1	
		$16\cos^4\theta - 16\cos^2\theta + 4 = 0 \quad 4(2\cos^2\theta - 1)(2\cos^2\theta - 1) = 0$	M1 A1	
		$\cos\theta = \pm 1/\sqrt{2} \qquad \theta = \pi/4$ or $3\pi/4$ or $\pi/2$	M1 A1	15