PURE MATHS 6 (A) TEST PAPER 3 : ANSWERS AND MARK SCHEME

1.	True when $n = 1$: $a - 1$ divides $a - 1$ Suppose $a^k - 1 = m(a-1)$	B1 M1	
	$a^{k+1} - 1 = a(a^k - 1) + a - 1 = (am + 1)(a - 1)$, so $a - 1$ divides $a^{k+1} - 1$	M1 A1 A1	5
2.	(a) (i) mod. = 1, arg. = $7\pi/12$ (ii) mod. = 4, arg. = $-\pi/12$	B1 B1; B1 B1	
	(b) Points shown Perpendicular bisector drawn	B1 B2	7
3.	$\cos(0-\pi) = -1$ $\quad -\sin(0-\pi) = 0$ $\quad -\cos(0-\pi) = 1$	M1 A1 A1	
	$\sin(0-\pi) = 0$ $\quad \cos(0-\pi) = -1$	M1 A1	
	First 3 terms are $-1 + \frac{1}{2}(x-\pi)^2 + \frac{1}{24}(x-\pi)^4$	M1 A1 A1	8
4.	(a) Grad. at $(x_0, y_0) \approx$ grad. of chord from (x_{-1}, y_{-1}) to (x_1, y_1)	M1 A2	
	(b) $y'(1) \approx 3$ $\quad y(1{\cdot}1) \approx 0{\cdot}2(3) + 0{\cdot}73 = 1{\cdot}33$	B1 M1 A1	
	$y'(1{\cdot}1) \approx 3.75$ $\quad y(1{\cdot}2) \approx 0{\cdot}2(3{\cdot}75) + 1 = 1{\cdot}75$	M1 A1 M1 A1	10
5.	(a) $\begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & -\lambda & 1 \\ 1 & 1 & 2-\lambda \end{vmatrix} = 0 \quad \begin{vmatrix} 2-\lambda & 0 & 1 \\ -1 & 1-\lambda & 1 \\ 1 & 3-\lambda & 2-\lambda \end{vmatrix} = 0$	M1 M1	
	$(2-\lambda)(\lambda^2 - 3\lambda + 2 - 3 + \lambda) + 1(\lambda - 3 - 1 + \lambda) = 0$	M1 A1	
	$(2-\lambda)(\lambda^2 - 2\lambda - 1) + 2(\lambda - 2) = 0 \quad (2-\lambda)(\lambda^2 - 2\lambda - 3) = 0$	M1	
	$(2-\lambda)(\lambda + 1)(\lambda - 3) = 0 \quad \lambda = -1, 2, 3$	A1 A1	
	(b) $D = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ or equivalent	M1 A1	
	(c) $A^{-1} = -\frac{1}{6}\begin{pmatrix} -1 & 3 & -1 \\ 3 & 3 & -3 \\ -1 & -3 & -1 \end{pmatrix}$	M1 A1 A1	
	Eigenvalues of A^{-1} are $-1, 1/2, 1/3$	B2	14
6.	(a) $6 - 2 - c = p,\ 9 - 2c - 2 = p \quad 4 - c = 7 - 2c$	M1	
	$c = 3,\ p = 1$	A1 A1	
	(b) $\mathbf{r}.(3\mathbf{i} - 2\mathbf{j} - \mathbf{k}) = 1 \quad d = 1/\sqrt{(9+4+1)} = 1/\sqrt{14}$	M1 A1 A1	
	(c) Let N be foot of perpendicular from O to plane		
	$\overrightarrow{ON} = 3t\mathbf{i} - 2t\mathbf{j} - t\mathbf{k} \quad \mathbf{r}.\overrightarrow{ON} = 1$, so $9t + 4t + t = 1$	M1 M1	
	$t = 1/14$, so point is $(3/7, -2/7, -1/7)$	A1 A1	
	(d) $\overrightarrow{OA} \times \overrightarrow{AB} = -7\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$	M1 A1 A1	
	Area = $1/2\ \lvert \overrightarrow{OA} \times \overrightarrow{AB} \rvert = 1/2\sqrt{(49 + 25 + 9)} = (\sqrt{83})/2$	M1 A1	15
7.	(a) $x = \cos y \quad dx/dy = -\sin y$	B1 B1	
	(b) $dy/dx = -1/\sin y = -1/\sqrt{(1-x^2)} \quad [= -(1-x^2)^{-1/2}]$	M1 A1	
	$d^2y/dx^2 = 1/2\,(1-x^2)^{-3/2}\,.\,-2x = -x(1-x2)^{-3/2}$	M1 A1	
	$d^3y/dx^2 = -(1-x^2)^{-3/2} + 3x/2\,(1-x^2)^{-5/2}.\,-2x$	M1 M1	
	$= -3x^2(1-x^2)^{-5/2} - (1-x^2)^{-3/2}$	A1	
	(c) $y(0) = \pi/2 \quad y(0) = -1 \quad y''(0) = 0 \quad y'''(0) = -1$	M1 A1 A1	
	Series is $\frac{\pi}{2} - x - \frac{x^3}{6}$	M1 A1	
	(d) Putting $x = 0{\cdot}6$, $y = \pi/2 - 0{\cdot}6 - 0{\cdot}036 = 0{\cdot}93$ (to 2 d.p.)	M1 A1	16