PURE MATHS 6 (A) TEST PAPER 10 : ANSWERS AND MARK SCHEME

1. $\lambda = -1$ makes second and third rows identical, so matrix is singular
(Or expand determinant and solve to get $\lambda = -1$) — M3 A1 — **4**

2. (a) Derivation of result : derivative at $(x_0, y_0) \approx$ gradient of chord joining (x_0, y_0) to (x_1, y_1). — M1 A1 A1

(b) $\left(\frac{dy}{dx}\right)_0 = -1 \approx \frac{y_1 - y_0}{0 \cdot 1}$ $y_1 \approx -0{\cdot}1 + 1 = 0{\cdot}9$ — M1 A1

$\left(\frac{dy}{dx}\right)_1 = -0{\cdot}5 \approx \frac{y_2 - y_1}{0 \cdot 1}$ $y_2 \approx -0{\cdot}05 + 0{\cdot}9 = 0{\cdot}85$ — M1 A1 A1 — **8**

3. (a) $|x + i(y - 1)| = 2|x + 1 + iy|$ $x^2 + (y-1)^2 = 4[(x+1)^2 + y^2]$ — M1 A1
$x^2 + y^2 - 2y + 1 = 4x^2 + 4y^2 + 8x + 4$ $3x^2 + 3y^2 + 8x + 2y + 3 = 0$ — M1 A1
$(x + 4/3)^2 + (y + 1/3)^2 = 8/9$ Circle centre $(-4/3, -1/3)$, radius $(2\sqrt{2})/3$ — M1 M1 A1 A1
(b) $w = z + 4/3 + i/3$ — B2 — **10**

4. (a) $(\cos x + i \sin x)^1 = \cos 1x + i \sin 1x$, so true for $n = 1$ — B1
Assume true for $n = k$: $(\cos x + i \sin x)^k = \cos kx + i \sin kx$ — M1
Then $(\cos x + i \sin x)^{k+1} = (\cos x + i \sin x)(\cos kx + i \sin kx)$ — A1
$= (\cos x \cos kx - \sin x \sin kx) + i(\cos x \sin kx + \sin x \cos kx)$ — A1
$= \cos (kx + x) + i \sin (kx + x) = \cos (k+1)x + i \sin (k+1)x$ — A1
so true for $n = k + 1$ Hence by induction, true for all n — A1
(b) $\cos 4\theta + i \sin 4\theta = -1/2 - i\sqrt{3}/2$ $4\theta = 4\pi/3 + 2n\pi$ — B1 M1 A1
$\theta = \pi/3,\ 5\pi/6,\ 4\pi/3,\ 11\pi/6$ — A1 A1 — **11**

5. (a) First three derivatives are $3e^{3x}$, $9e^{3x}$, $27e^{3x}$ — M1 A1 (all)
nth derivative seems to be $3^n e^{3x}$ — B1
Certainly true for $n = 1$ Assume true for $n = k$ — B1 M1
$\frac{d}{dx}(3^k e^{3x}) = 3^k(3e^{3x}) = 3^{k+1}e^{3x}$, so true for $n = k + 1$, etc. — M1 A1
(b) $e^{3x} = 1 + 3x + \frac{9x^2}{2} + \frac{9x^3}{2}$ $\ln(1 + 2x) = 2x - 2x^2 + \frac{8x^3}{3}$ — B1 B1
(c) Multiplying, get $e^{3x}\ln(1 + 2x) = 2x + 4x^2 + \frac{17x^3}{3} + \ldots$ — M1 A1 A1 A1 — **13**

6. (a) $\overrightarrow{AB} = -6\mathbf{i} + \mathbf{j} - 4\mathbf{k} = \overrightarrow{DC}$, so $ABCD$ is a parallelogram — M1 A1
(b) $(-6\mathbf{i} + \mathbf{j} - 4\mathbf{k}) \times (4\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}) = 21\mathbf{i} + 14\mathbf{j} - 28\mathbf{k}$ — M1 A1 A1
(c) $\mathbf{r} \cdot (21\mathbf{i} + 14\mathbf{j} - 28\mathbf{k}) = c'$ $\mathbf{r} \cdot (3\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}) = c$ — M1 M1
Let $\mathbf{r} = \overrightarrow{OA}$, so $c = -4$ $\mathbf{r} \cdot (3\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}) = -4$ — A1 A1
(d) Length $= |-4| / |3\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}| = 4/\sqrt{29}$ — M1 A1
(e) Volume $= 1/3 \times |\overrightarrow{AB} \times \overrightarrow{AD}| \times$ height $= 1/3 \times 7 \times \sqrt{29} \times 4/\sqrt{29} = 28/3$ — M1 M1 A1 — **14**

7. (a) $P' = (-1, 4, 3)$ (b) For Q', $2x - y = 5$, $2y - z = -3$ — B1; M1
$-x + 2z = 0$ $y = -1, z = 1, x = 2$ $Q' = (2, -1, 1)$ — M1 A1
(c) Det $(\mathbf{M} - c\mathbf{I}) = 0$: $(2 - \lambda)^3 - 1 = 0$ λ real : $2 - \lambda = 1$ $\lambda = 1$ — M1 M1 A1
For eigenvector $(x \quad y \quad z)$, get $x - y = 0$, $y - z = 0$, $z - x = 0$ — M1 A1
E.v.s of form $(1 \quad 1 \quad 1)$ For magnitude 1, $(1/\sqrt{3} \quad 1/\sqrt{3} \quad 1/\sqrt{3})$ — A1 A1
(d) $ST(P) = S(P') = \begin{pmatrix} 2 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}\begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} -5 \\ 9 \\ 2 \end{pmatrix}$ — M1 M1 A1
P maps to $(-5, 9, 2)$ — A1 — **15**

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