GCE Examinations

Advanced Subsidiary / Advanced Level

Pure Mathematics
Module P4

Paper E

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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P4 Paper E – Marking Guide

1. *(a)* $w = \frac{10+5i}{2-i} \times \frac{2+i}{2+i} = \frac{15+20i}{5} = 3 + 4i$ M1 A2

(b) let $z = x + iy$ $\therefore x + iy + 2(x - iy) = 3 + 4i$ M1

$\therefore 3x = 3, -y = 4$ M1 A1

$x = 1, y = -4$ $\therefore z = 1 - 4i$ A1 **(7)**

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2. $\sum_{r=0}^{n} (r+1)(r+2) = \sum_{r=1}^{n+1} r(r+1) = \sum_{r=1}^{n+1} (r^2 + r)$ M1 A2

$= \frac{1}{6}(n+1)(n+2)(2n+3) + \frac{1}{2}(n+1)(n+2)$ M1 A1

$= \frac{1}{6}(n+1)(n+2)[(2n+3)+3]$ M1

$= \frac{1}{6}(n+1)(n+2)[2n+6]$

$= \frac{1}{3}(n+1)(n+2)(n+3)$ A1 **(7)**

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3. $\frac{dy}{dx} - y = x$ $\therefore$ int. fac. $= e^{\int -1\,dx} = e^{-x}$ M1 A1

$\therefore e^{-x}\frac{dy}{dx} - ye^{-x} = xe^{-x}$ M1

$\frac{d}{dx}(ye^{-x}) = xe^{-x}$

$ye^{-x} = \int xe^{-x}\,dx$ A1

$u = x, u' = 1; v' = e^{-x}, v = -e^{-x}$ M1

$ye^{-x} = -xe^{-x} - \int -e^{-x}\,dx$ A1

$ye^{-x} = -xe^{-x} - e^{-x} + c$ A1

$y = ce^{x} - x - 1$

$x = 0, y = 0$ $\therefore c = 1$ M1

$\therefore y = e^{x} - x - 1$ A1 **(9)**

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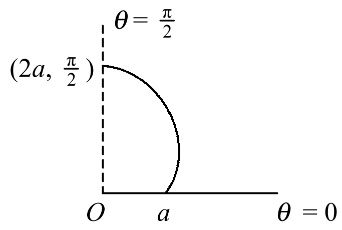
4. *(a)*



B2

(b) require $\frac{d(r\cos\theta)}{d\theta} = 0$ M1

$r\cos\theta = a\cos\theta(1 + \sin\theta) = a(\cos\theta + \cos\theta\sin\theta)$ A1

$\therefore \frac{d(r\cos\theta)}{d\theta} = a[-\sin\theta + \cos\theta(\cos\theta) + \sin\theta(-\sin\theta)]$ M1

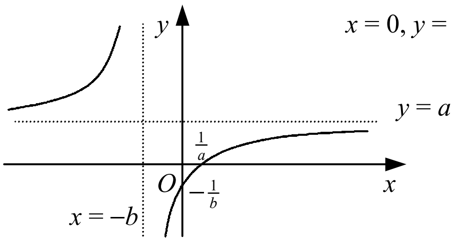
$\therefore -\sin\theta + (1 - \sin^2\theta) - \sin^2\theta = 0$

$2\sin^2\theta + \sin\theta - 1 = 0$ A1

$(2\sin\theta - 1)(\sin\theta + 1) = 0$ M1

$\therefore \sin\theta = \frac{1}{2}, -1$ A1

$0 \le \theta \le \frac{\pi}{2}$ $\therefore \theta = \frac{\pi}{6}$ giving $(\frac{3}{2}a, \frac{\pi}{6})$ M1 A1 **(10)**

5.	*(a)*	$y = \dfrac{ax + ab - ab - 1}{x + b} = a - \dfrac{ab + 1}{x + b}$ $\therefore$ asymptotes are $x = -b$ and $y = a$	M1 A2	
	(b)	 $x = 0, y = -\frac{1}{b}$; $y = 0, x = \frac{1}{a}$	B1 B2	
	(c)	$3x - 1 = 2(x + 2)$ gives $x = 5$	M1 A1	
		$-(3x - 1) = 2(x + 2)$ gives $x = -\frac{3}{5}$	M1 A1	
		considering curve below x-axis reflected in x-axis gives $-\frac{3}{5} < x < 5$	M1 A1	**(12)**
6.	*(a)*	let $f(x) = e^x - 4\sin x$, $f(0) = 1$; $f(1) = -0.648$;		
		f cont. over interval, change of sign $\therefore$ root in interval $[0, 1]$	M1 A1	
		$f(1) = -0.648$; $f(1.5) = 0.492$		
		f cont. over interval, change of sign $\therefore$ root in interval $[1, 1.5]$	A1	
	(b)	$f'(x) = e^x - 4\cos x$, $x_{n+1} = x_n - \dfrac{e^{x_n} - 4\sin x_n}{e^{x_n} - 4\cos x_n}$	M1 A2	
		giving $\alpha = 0.37$ (2dp)	M1 A1	
	(c)	$\beta \approx 1 + \frac{0.64760}{0.64760 + 0.49171} \times 0.5 = 1.284... = 1.3$ (1dp)	M1 A2	
	(d)	$f(1.25) = -0.306$; $f(1.35) = -0.0455$		
		no change of sign $\therefore$ no root $\therefore$ not correct to 1dp	M1 A1	**(13)**
7.	*(a)*	(i) $\dfrac{dx}{dt} = \frac{1}{2}t^{-\frac{1}{2}}$ $\therefore$ $\dfrac{dt}{dx} = 2t^{\frac{1}{2}}$	M1	
		$\dfrac{dy}{dx} = \dfrac{dy}{dt} \cdot \dfrac{dt}{dx} = 2t^{\frac{1}{2}}\dfrac{dy}{dt}$	M1 A1	
		(ii) $\dfrac{d^2y}{dx^2} = \dfrac{dt}{dx}\left(2t^{\frac{1}{2}}\dfrac{d^2y}{dt^2} + t^{-\frac{1}{2}}\dfrac{dy}{dt}\right)$	M1 A1	
		$= 2t^{\frac{1}{2}}\left(2t^{\frac{1}{2}}\dfrac{d^2y}{dt^2} + t^{-\frac{1}{2}}\dfrac{dy}{dt}\right) = 2\dfrac{dy}{dt} + 4t\dfrac{d^2y}{dt^2}$	A1	
	(b)	$\dfrac{1}{t}\left(2\dfrac{dy}{dt} + 4t\dfrac{d^2y}{dt^2}\right) + \left(4t^{-\frac{1}{2}} - t^{-\frac{3}{2}}\right)2t^{\frac{1}{2}}\dfrac{dy}{dt} + 3y = 3t + 5$	M1 A1	
		$2t^{-1}\dfrac{dy}{dt} + 4\dfrac{d^2y}{dt^2} + 8\dfrac{dy}{dt} - 2t^{-1}\dfrac{dy}{dt} + 3y = 3t + 5$	M1	
		giving $4\dfrac{d^2y}{dt^2} + 8\dfrac{dy}{dt} + 3y = 3t + 5$	A1	
	(c)	aux. eqn. $4m^2 + 8m + 3 = 0$	M1	
		$(2m + 1)(2m + 3) = 0$; $m = -\frac{1}{2}, -\frac{3}{2}$ C.F. $y = Ae^{-\frac{1}{2}t} + Be^{-\frac{3}{2}t}$	A1	
		for P.I. try $y = at + b$ $\therefore$ $\dfrac{dy}{dt} = a$, $\dfrac{d^2y}{dt^2} = 0$	M1	
		so $4(0) + 8a + 3(at + b) = 3t + 5$	M1	
		$\therefore$ $3a = 3$; $8a + 3b = 5$ giving $a = 1, b = -1$	A1	
		gen. soln. $y = Ae^{-\frac{1}{2}t} + Be^{-\frac{3}{2}t} + t - 1$	M1	
		$\therefore$ gen. soln. of (I): $y = Ae^{-\frac{1}{2}x^2} + Be^{-\frac{3}{2}x^2} + x^2 - 1$	A1	**(17)**
			Total	**(75)**

Performance Record – P4 Paper E

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	complex nos.	series	1st order diff. eqn	polar coords	inequality	numerical methods	2nd order diff. eqn	
Marks	7	7	9	10	12	13	17	75
Student								