GCE Examinations

Advanced Subsidiary / Advanced Level

Pure Mathematics
Module P4

Paper A

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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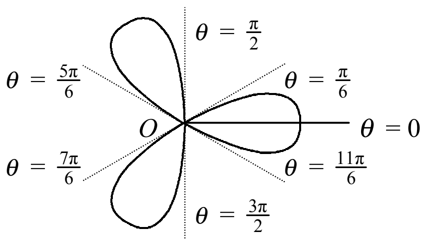
P4 Paper A – Marking Guide

1.	*(a)*	$f(1) = 1 - 5 + 17 - 13 = 0 \quad \therefore (z-1)$ is a factor	B1	
	(b)	$f(z) = (z-1)(z^2 - 4z + 13)$	M1 A1	
		quad. gives $z = \frac{4 \pm \sqrt{16-52}}{2}$	M1	
		$\therefore$ roots are $1,\ 2-3i,\ 2+3i$	A2	**(6)**
2.		$\frac{dy}{dx} + \frac{3}{x}y = \frac{e^x}{x^3}$	M1	
		$\therefore$ int. fac. $= e^{\int \frac{3}{x}\,dx} = e^{3\ln\lvert x\rvert} = x^3$	M1 A1	
		$x^3\frac{dy}{dx} + 3x^2y = e^x$		
		$\frac{d}{dx}(x^3y) = e^x$	M1	
		$x^3y = \int e^x\ dx = e^x + c \quad \therefore y = \frac{e^x + c}{x^3}$	M1 A1	**(6)**
3.	*(a)*	$\frac{1}{r(r+1)} \equiv \frac{A}{r} + \frac{B}{r+1}$ giving $A = 1, B = -1$		
		$\therefore \frac{1}{r(r+1)} \equiv \frac{1}{r} - \frac{1}{r+1}$	M1 A1	
	(b)	$\sum_{r=3}^{35} (\frac{1}{r} - \frac{1}{r+1}) = (\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \ldots + \frac{1}{35}) - (\frac{1}{4} + \frac{1}{5} + \ldots + \frac{1}{35} + \frac{1}{36})$	M1 A1	
		$= \frac{1}{3} - \frac{1}{36} = \frac{11}{36}$	M1 A1	**(6)**
4.		$\frac{(x-3)^2}{x+1} - 2 < 0 \quad \therefore \frac{x^2 - 8x + 7}{x+1} < 0$	M1 A1	
		$\frac{(x-1)(x-7)}{x+1} < 0 \quad \therefore$ critical values are $-1, 1, 7$	M1 A1	
		considering change of sign of factors gives		
		$x < -1$ or $1 < x < 7$	M1 A2	**(7)**
5.	*(a)*		B3	
	(b)	area $= 6 \times \frac{1}{2}\int_0^{\frac{\pi}{6}} (a\cos 3\theta)^2\ d\theta = 3a^2\int_0^{\frac{\pi}{6}} \cos^2 3\theta\ d\theta$	M1 A1	
		$= \frac{3}{2}a^2\int_0^{\frac{\pi}{6}} 1 + \cos 6\theta\ d\theta$	M1	
		$= \frac{3}{2}a^2[\theta + \frac{1}{6}\sin 6\theta]_0^{\frac{\pi}{6}}$	A1	
		giving $\frac{1}{4}\pi a^2$	M1 A1	**(9)**

6. *(a)* 2 solutions — B1

(b) f(0) = −1; f(1) = 1.64
f cont. over interval, change of sign ∴ root — M1 A1

(c) $f'(x) = e^x + 2\sin x$ — M1

$x_{n+1} = x_n - \dfrac{e^{x_n} - 2\cos x_n}{e^{x_n} + 2\sin x_n}$ — A1

giving 0.54 (2dp) — M1 A1

(d) f(0.535) = −0.0131; f(0.545) = 0.0144
change of sign ∴ root ∴ correct to 2dp — M1 A1

(e) f(−2) = 0.967629; f(−1) = −0.712725

$\beta \approx -2 + \frac{0.967629}{0.967629+0.712725} \times 1$ — M1 A1

= −1.42 (2dp) — A1 **(12)**

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7. *(a)* $\frac{A}{1-i} + \frac{B}{2+i} = 6 \;\therefore\; A(2+i) + B(1-i) = 6(1-i)(2+i)$ — M1

giving $2A + B + i(A - B) = 18 - 6i$ — A1

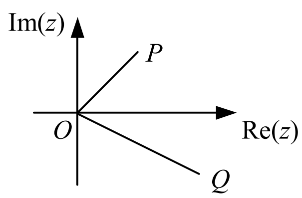
∴ $2A + B = 18$

$A - B = -6$ — M1 A1

giving $A = 4$, $B = 10$ — M1 A1

(b) $z = \frac{4}{1-i} \times \frac{1+i}{1+i} = 2 + 2i$ — M1 A1

$w = \frac{10}{2+i} \times \frac{2-i}{2-i} = 4 - 2i$ — A1



B2

(c) $z - w = (2 + 2i) - (4 - 2i) = -2 + 4i$ — M1

$|z - w| = \sqrt{(4 + 16)} = \sqrt{20} = 2\sqrt{5}$ — M1 A1 **(14)**

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8. *(a)* $\frac{dx}{dt} = -p\sin t + q\cos t$, $\frac{d^2x}{dt^2} = -p\cos t - q\sin t$ — M1 A1

$-p\cos t - q\sin t - 4p\sin t + 4q\cos t + 3p\cos t + 3q\sin t = \sin t$ — M1

$2p + 4q = 0$

$2q - 4p = 1$ — A1

giving $p = -\frac{1}{5}$, $q = \frac{1}{10}$ — M1 A1

(b) aux. eqn. $m^2 + 4m + 3 = 0$ — M1

$(m + 1)(m + 3) = 0 \;\therefore\; m = -1, -3$ — A1

C.F. $x = Ae^{-t} + Be^{-3t}$ — A1

gen. soln. $x = Ae^{-t} + Be^{-3t} - \frac{1}{5}\cos t + \frac{1}{10}\sin t$

$\therefore\; \frac{dx}{dt} = -Ae^{-t} - 3Be^{-3t} + \frac{1}{5}\sin t + \frac{1}{10}\cos t$ — M1 A1

$t = 0, x = 1, \frac{dx}{dt} = \frac{1}{2}$ gives $1 = A + B - \frac{1}{5}$ and $\frac{1}{2} = -A - 3B + \frac{1}{10}$ — M1

solve simul. $A = 2$, $B = -\frac{4}{5}$ — M1 A1

soln. $x = 2e^{-t} - \frac{4}{5}e^{-3t} - \frac{1}{5}\cos t + \frac{1}{10}\sin t$ — A1 **(15)**

Total **(75)**

Performance Record – P4 Paper A

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	Total
Topic(s)	complex nos.	1st order diff. eqn	series	inequality	polar coords	numerical methods	complex nos.	2nd order diff. eqn	
Marks	6	6	6	7	9	12	14	15	75
Student									