GCE Examinations

# Pure Mathematics
# Module P4

Advanced Subsidiary / Advanced Level

## Paper H

Time: 1 hour 30 minutes

*Instructions and Information*

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

*Advice to Candidates*

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

*Written by Shaun Armstrong & Chris Huffer*

**1.** *(a)* Given that

$$f(r) = r!\,,$$

show that

$$f(r + 1) - f(r) = r \times r!$$ **(2 marks)**

*(b)* Hence find

$$\sum_{r=1}^{n} (r \times r!)$$ **(4 marks)**

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**2.** *(a)* Given that

$$y = \frac{2x}{x^2 + 9},$$

express $x$ in terms of $y$. **(5 marks)**

*(b)* Hence prove that for all real values of $x$

$$-\frac{1}{a} \le \frac{2x}{x^2 + 9} \le \frac{1}{a},$$

where $a$ is a positive integer which you should find. **(3 marks)**

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**3.** Find the general solution of the differential equation

$$x\frac{dy}{dx} + xy = 1 - y,$$

giving your answer in the form $y = f(x)$. **(9 marks)**

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**4.**

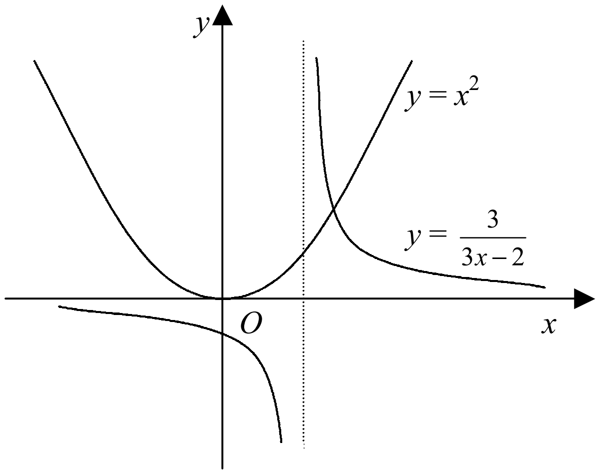


Fig. 1

Figure 1 shows part of the curves $y = x^2$ and $y = \dfrac{3}{3x-2}$.

The curves meet at the point with $x$-coordinate $\alpha$.

*(a)* Find the integer $N$ such that $\dfrac{N}{10} < \alpha < \dfrac{N+1}{10}$. **(4 marks)**

*(b)* Use interval bisection on the interval found in part *(a)* to find the value of $\alpha$ correct to 2 decimal places. **(5 marks)**

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**5.** Given that

$$f(z) \equiv z^4 - 4z^3 + kz^2 - 4z + 13,$$

where $k$ is a real constant, and that $z = \mathrm{i}$ is a solution of the equation $f(z) = 0$,

*(a)* show that $k = 14$, **(3 marks)**

*(b)* find all solutions of the equation $f(z) = 0$. **(7 marks)**

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***Turn over***

**6.** The shape of a company logo is to be the region enclosed by the curve with polar equation

$$r^2 = a^2 \sin 2\theta, \quad 0 \le \theta \le \tfrac{\pi}{2}.$$

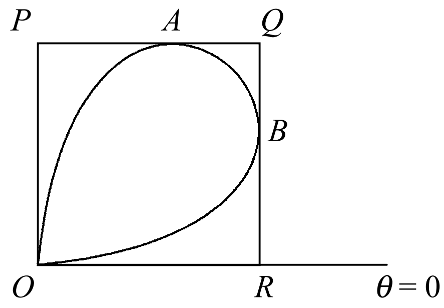


Fig. 2

A sign in the shape of the logo is to be made by cutting the area enclosed by the curve from a square sheet of metal *OPQR* where *O* is the pole and *R* lies on the initial line, $\theta = 0$, as shown in Figure 2. *PQ* and *QR* are tangents to the curve, parallel and perpendicular to the initial line respectively, at the points *A* and *B* on the curve.

*(a)* Find the value of $\theta$ at the point *A*. **(7 marks)**

*(b)* Show that the area of *OPQR* is $\dfrac{3\sqrt{3}}{8}a^2$. **(3 marks)**

*(c)* Find the area of the metal sheet which is not used. **(5 marks)**

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**7.** Given that $x = ke^{-t}$ satisfies the differential equation

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 8e^{-t},$$

*(a)* find the value of *k*. **(3 marks)**

*(b)* Hence find the solution of the differential equation for which $x = 1$ and $\dfrac{dx}{dt} = 3$ at $t = 0$.

**(8 marks)**

The maximum value of *x* occurs when $t = T$.

*(c)* Show that the maximum value of *x* is $\frac{40}{27}$ and find the value of *T*. **(7 marks)**

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**END**