GCE Examinations

Pure Mathematics
Module P4

Advanced Subsidiary / Advanced Level

Paper E

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Shaun Armstrong & Chris Huffer

1. The complex number w is given by $w = \dfrac{10+5\text{i}}{2-\text{i}}$.

(a) Express w in the form $a + \text{i}b$ where a and b are real. **(3 marks)**

(b) Using your answer to part *(a)* find the complex number z such that

$$z + 2z^* = w.$$ **(4 marks)**

---

2. Show that

$$\sum_{r=0}^{n} (r+1)(r+2) = \tfrac{1}{3}(n+1)(n+2)(n+3).$$ **(7 marks)**

---

3. Find the equation of the curve which passes through the origin and for which

$$\frac{\text{d}y}{\text{d}x} = x + y,$$

giving your answer in the form $y = \text{f}(x)$. **(9 marks)**

---

4. The curve C has the polar equation

$$r = a(1 + \sin\theta), \qquad 0 \le \theta \le \tfrac{\pi}{2}.$$

(a) Sketch the curve C. **(2 marks)**

(b) Find the polar coordinates of the point on the curve where the tangent to the curve is perpendicular to the initial line $\theta = 0$.

(8 marks)

---

5. *(a)* Find, in terms of a and b, the equations of the asymptotes to the curve with equation

$$y = \frac{ax-1}{x+b},$$

where a and b are positive constants. **(3 marks)**

(b) Sketch the curve

$$y = \frac{ax-1}{x+b},$$

showing the coordinates of any points of intersection with the coordinate axes.

(3 marks)

(c) Hence, or otherwise, find the set of values of x for which

$$\left|\frac{3x-1}{x+2}\right| < 2.$$ **(6 marks)**

---

6. *(a)* Show that the equation $e^x - 4\sin x = 0$ has a root, α, in the interval [0, 1] and a root, β, in the interval [1, 1.5].

(3 marks)

(b) Using the Newton-Raphson method with an initial value of $x = 0.5$, find α correct to 2 decimal places.

(5 marks)

(c) Use linear interpolation once between the values $x = 1$ and $x = 1.5$ to find an approximate value for β, giving your answer correct to 1 decimal place.

(3 marks)

(d) Determine whether or not your answer to part *(c)* gives the value of β correct to 1 decimal place.

(2 marks)

Turn over

7. *(a)* Given that y is a function of t and that $x = t^{\frac{1}{2}}$, where $x > 0$, show that

(i) $\dfrac{dy}{dx} = 2t^{\frac{1}{2}}\dfrac{dy}{dt}$,

(ii) $\dfrac{d^2y}{dx^2} = 2\dfrac{dy}{dt} + 4t\dfrac{d^2y}{dt^2}$. **(6 marks)**

(b) Use your answers to part *(a)* to show that the substitution $x = t^{\frac{1}{2}}$ transforms the differential equation

$$\frac{1}{x^2}\frac{d^2y}{dx^2} + \left(\frac{4}{x} - \frac{1}{x^3}\right)\frac{dy}{dx} + 3y = 3x^2 + 5 \qquad \text{(I)}$$

into the differential equation

$$4\frac{d^2y}{dt^2} + 8\frac{dy}{dt} + 3y = 3t + 5.$$ **(4 marks)**

(c) Hence find the general solution of differential equation (I). **(7 marks)**

END