GCE Examinations

Advanced Subsidiary / Advanced Level

Pure Mathematics
Module P4

Paper D

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

1. *(a)* $(3 - i)$ must also be root   B1

     $\therefore [x - (3 + i)][x - (3 - i)] = x^2 - 6x + 10$ is a factor   M1 A1

     (b) linear factor is $(3x + \frac{k}{10})$ M1 A1

         $\therefore$ coeff. of $x = 30 - \frac{6k}{10} = 42$ M1
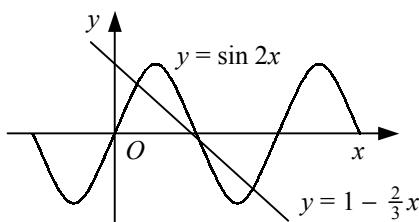
         giving $k = -20$ A1   **(7)**

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2. $\frac{x}{x-1} - \frac{2}{3-x} > 0 \;\therefore\; \frac{x^2-x-2}{(x-1)(x-3)} > 0$ M1 A1

$\frac{(x-2)(x+1)}{(x-1)(x-3)} > 0 \;\therefore\;$ critical values are $-1, 1, 2, 3$ M1 A1

considering change of sign of factors gives

$x < -1$ or $1 < x < 2$ or $x > 3$ M2 A2   **(8)**

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3. $\frac{dy}{dx} + 4xy = 3x \;\therefore\;$ int. fac. $= e^{\int 4x\,dx} = e^{2x^2}$ M1 A1

$e^{2x^2} \frac{dy}{dx} + 4xy\, e^{2x^2} = 3x\, e^{2x^2}$

$\frac{d}{dx}(y\, e^{2x^2}) = 3x\, e^{2x^2}$ M1

$y\, e^{2x^2} = \int 3x\, e^{2x^2}\, dx = \frac{3}{4} e^{2x^2} + c$ M1 A1

$x = 0, y = \frac{1}{2} \;\therefore\; c = -\frac{1}{4}$ M1 A1

$\therefore y = \frac{1}{4}(3 - e^{-2x^2})$ A1   **(8)**

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4. *(a)* $\frac{3r+4}{r(r+1)(r+2)} \equiv \frac{A}{r} + \frac{B}{r+1} + \frac{C}{r+2}$ M1

     giving $A = 2, B = -1, C = -1 \;\therefore\; \frac{3r+4}{r(r+1)(r+2)} \equiv \frac{2}{r} - \frac{1}{r+1} - \frac{1}{r+2}$ M1 A1

     (b) $\sum_{r=1}^{n} (\frac{2}{r} - \frac{1}{r+1} - \frac{1}{r+2})$

     $= (\frac{2}{1} + \frac{2}{2} + \frac{2}{3} + ... + \frac{2}{n}) - (\frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n} + \frac{1}{n+1}) - (\frac{1}{3} + ... + \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2})$ M1 A2

     $= \frac{2}{1} + \frac{2}{2} - \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+1} - \frac{1}{n+2}$ M1

     $= \frac{5}{2} - \frac{2}{n+1} - \frac{1}{n+2}$ A1

     giving $\frac{n(5n+9)}{2(n+1)(n+2)}$ M1 A1   **(10)**

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5. *(a)* $\frac{dy}{dx} = 2ax + b \quad \frac{d^2y}{dx^2} = 2a$ M1 A1

     $2a + 4ax + 2b + 10ax^2 + 10bx + 10c = 5x^2 - 13x + 1$ M1

     giving $a = \frac{1}{2}, b = -\frac{3}{2}, c = \frac{3}{10}$ M1 A1

     (b) aux. eqn. $m^2 + 2m + 10 = 0$ M1

             $m = -1 \pm 3i$ M1 A1

     gen. soln. $y = e^{-x}(A \cos 3x + B \sin 3x) + \frac{1}{2}x^2 - \frac{3}{2}x + \frac{3}{10}$ A2   **(10)**

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6. *(a)*

$y = \sin 2x$

$y = 1 - \frac{2}{3}x$

B2

3 pts of intersec. $\therefore$ 3 solutions   B1

(b) (i) f(2.5) = −0.292; f(3) = 0.721
f cont. over interval, change of sign $\therefore$ root   M1 A1

(ii) $\alpha \approx 2.5 + \frac{0.292258}{0.292258+0.720585} \times 0.5 = 2.64$ (2dp)   M1 A2

(iii) f(2.635) = −0.0919; f(2.645) = −0.0744
no change of sign $\therefore$ no root $\therefore$ not correct to 2dp   M1 A1

(c) $f'(x) = \frac{2}{3} + 2\cos 2x$ M1

$x_{n+1} = x_n - \dfrac{\frac{2}{3}x_n + \sin 2x_n - 1}{\frac{2}{3} + 2\cos 2x_n}$ A1

$x_1 = 0.399956$, $x_2 = 0.407755$, $x_3 = 0.407798$ M1 A1
root = 0.408 (3sf)   A1   **(15)**

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7. *(a)* require $\dfrac{d(r\sin\theta)}{d\theta} = 0$ M1

$r\sin\theta = a\sin\theta(1 - \cos\theta) = a(\sin\theta - \frac{1}{2}\sin 2\theta)$ M1

$\therefore \dfrac{d(r\sin\theta)}{d\theta} = a(\cos\theta - \cos 2\theta)$ A1

giving $2\cos^2\theta - \cos\theta - 1 = (2\cos\theta + 1)(\cos\theta - 1) = 0$ M1

$\therefore \cos\theta = -\frac{1}{2}, 1$ so $\theta = (0), \frac{2\pi}{3}, \frac{4\pi}{3}$ M1 A1

$\therefore P\left(\frac{3}{2}a, \frac{2\pi}{3}\right), Q\left(\frac{3}{2}a, \frac{4\pi}{3}\right)$ A1

(b) area bounded by *C* and lines *OP* and *OQ*

$= \frac{1}{2}\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} a^2(1 - \cos\theta)^2\,d\theta = \frac{1}{2}a^2\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} 1 - 2\cos\theta + \cos^2\theta\,d\theta$ M1 A1

giving $\frac{1}{4}a^2\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} 3 - 4\cos\theta + \cos 2\theta\,d\theta$ M1 A1

$= \frac{1}{4}a^2[3\theta - 4\sin\theta + \frac{1}{2}\sin 2\theta]_{\frac{2\pi}{3}}^{\frac{4\pi}{3}}$ A1

giving $\frac{1}{8}a^2(4\pi + 9\sqrt{3})$ M1 A1

area of triangle $OPQ = \frac{1}{2} \times \frac{3}{2}a \times \frac{3}{2}a \times \sin\frac{2\pi}{3} = \frac{9}{16}a^2\sqrt{3}$ M1 A1

shaded area $= \frac{1}{8}a^2(4\pi + 9\sqrt{3}) - \frac{9}{16}a^2\sqrt{3} = \frac{1}{16}a^2(8\pi + 9\sqrt{3})$ A1   **(17)**

Total   **(75)**

Performance Record – P4 Paper D

Question no.	1	2	3	4	5	6	7	Total
Topic(s)	complex nos.	inequality	1st order diff. eqn	series	2nd order diff. eqn	numerical methods	polar coords	
Marks	7	8	8	10	10	15	17	75
Student								