GCE Examinations

Pure Mathematics Module P4

Advanced Subsidiary / Advanced Level

Paper D

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Shaun Armstrong & Chris Huffer

1. The function f is defined by

$$f(x) \equiv 3x^3 + kx^2 + 42x + k,$$

where k is an integer.

Given that $(3 + i)$ is a root of the equation $f(x) = 0$,

(a) find a quadratic factor of $f(x)$, **(3 marks)**

(b) find the value of k. **(4 marks)**

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2. Find the set of values of x for which

$$\frac{x}{x-1} > \frac{2}{3-x}. \quad \textbf{(8 marks)}$$

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3. Given that $y = \frac{1}{2}$ when $x = 0$, solve the differential equation

$$\frac{dy}{dx} - 3x + 4xy = 0,$$

giving your answer in the form $y = f(x)$. **(8 marks)**

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4. *(a)* Express $\dfrac{3r+4}{r(r+1)(r+2)}$ in partial fractions. **(3 marks)**

(b) Hence, show that

$$\sum_{r=1}^{n} \frac{3r+4}{r(r+1)(r+2)} = \frac{n(5n+9)}{2(n+1)(n+2)}. \quad \textbf{(7 marks)}$$

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5. *(a)* Find the values of a, b and c such that $y = ax^2 + bx + c$ satisfies the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = 5x^2 - 13x + 1. \qquad \textbf{(5 marks)}$$

(b) Hence, find the general solution of this differential equation. **(5 marks)**

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6. $f(x) \equiv \frac{2}{3}x + \sin 2x - 1, \ x \in \mathbb{R}$.

(a) By sketching the graphs of $y = \sin 2x$ and $y = 1 - \frac{2}{3}x$ on the same diagram, find the number of solutions to the equation $f(x) = 0$.

(3 marks)

(b) (i) Show that one root, α, of the equation $f(x) = 0$ lies in the interval (2.5, 3).

(ii) Use one application of the method of linear interpolation on this interval to find an approximate value for α, giving your answer correct to 2 decimal places.

(iii) Determine whether or not your answer to part (ii) gives the value of α correct to 2 decimal places.

(7 marks)

(c) Use the Newton-Raphson method with a starting value of $x = 0.5$ to find another root of the equation $f(x) = 0$ correct to 3 significant figures.

(5 marks)

Turn over

7.

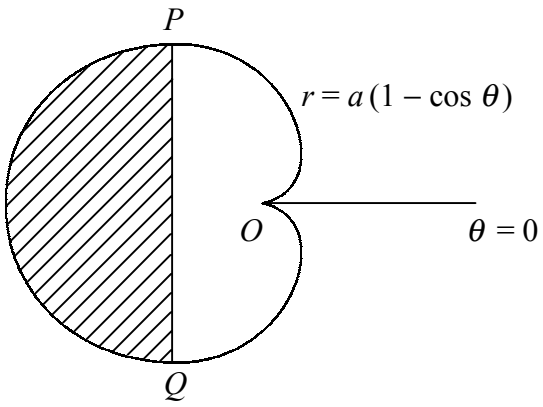

Fig. 1

Figure 1 shows the curve C with polar equation

$$r = a(1 - \cos\theta), \qquad 0 \leq \theta < 2\pi,$$

where a is a positive constant.

At the points P and Q the tangents to the curve are parallel to the initial line $\theta = 0$.

(a) Find the polar coordinates of P and Q. **(7 marks)**

The shaded region is bounded by the curve C and the straight line PQ.

(b) Show that the area of the shaded region is $\frac{1}{16}a^2(8\pi + 9\sqrt{3})$. **(10 marks)**

END