GCE Examinations

Advanced Subsidiary / Advanced Level

Pure Mathematics

Module P4

Paper C

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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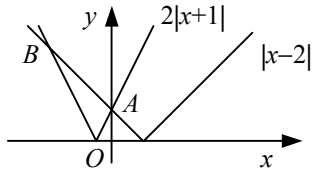
P4 Paper C – Marking Guide

1.



B2

by inspection, at A $x = 0$ B1
at B $-(x-2) = -2(x+1)$ giving $x = -4$ M1 A1
using graphs, require $-4 < x < 0$ A1 **(6)**

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2. *(a)* $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ M1 A1

$x.vx.(v + x\dfrac{dv}{dx}) = x^2 + (vx)^2 \quad \therefore xv\dfrac{dv}{dx} = 1$ M1 A1

$\int v\ dv = \int \dfrac{1}{x}\ dx$ M1

$\frac{1}{2}v^2 = \ln|x| + c$ A1

giving e.g. $y^2 = x^2(k + \ln x^2)$ A1

(b) $x = 1, y = 2 \quad \therefore k = 4$ M1
$x > 0 \ \therefore y^2 = x^2(4 + 2\ln x) = 2x^2(\ln x + 2)$ A1 **(9)**

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3. *(a)* series sum $= \sum_{r=1}^{n} (2r)^3 = 8 \times \sum_{r=1}^{n} r^3 = 8 \times \frac{1}{4}n^2(n+1)^2 = 2n^2(n+1)^2$ M2 A1

(b) series sum $= \sum_{r=1}^{2n} r^3 - 2 \times \sum_{r=1}^{n} (2r)^3$ M1 A1

$= \frac{1}{4}(2n)^2(2n+1)^2 - 2 \times 2n^2(n+1)^2$ M1

$= n^2[(2n+1)^2 - 4(n+1)^2]$ A1

giving $-n^2(4n+3)$ M1 A1 **(9)**

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4. aux. eqn. $m^2 - 6m + 9 = 0$ M1
$(m-3)^2 = 0 \quad \therefore m = 3$ A1
C.F. $y = (A + Bx)e^{3x}$ A1
P.I. try $y = Cx^2e^{3x}$ M1
$\dfrac{dy}{dx} = 3Cx^2e^{3x} + 2Cxe^{3x}, \quad \dfrac{d^2y}{dx^2} = 9Cx^2e^{3x} + 12Cxe^{3x} + 2Ce^{3x}$ M1 A2
$\therefore 9Cx^2e^{3x} + 12Cxe^{3x} + 2Ce^{3x} - 18Cx^2e^{3x} - 12Cxe^{3x} + 9Cx^2e^{3x} = 2e^{3x}$ M1
giving $2Ce^{3x} = 2e^{3x} \quad \therefore C = 1$ A1
gen. soln. $y = (A + Bx)e^{3x} + x^2e^{3x} = e^{3x}(A + Bx + x^2)$ A1 **(10)**

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5. *(a)* $f(1) = 0.443$; $f(1.5) = -11.1$
f cont. over interval, change of sign ∴ root — M1 A1

(b) $f'(x) = 2 - \sec^2 x$ — M1

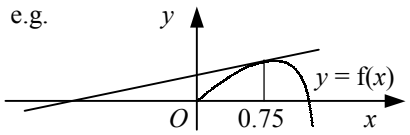
$x_{n+1} = x_n - \dfrac{2x_n - \tan x_n}{2 - \sec^2 x_n}$ — A1

$x_0 = 1.25$, $x_1 = 1.1868$, $x_2 = 1.1670$, $x_3 = 1.1656$, $x_4 = 1.1656$ — M1 A1

$\alpha = 1.17$ (2dp) — A1

$f(1.165) = 0.00248$; $f(1.175) = -0.0432$
change of sign ∴ root ∴ accurate to 2dp — M1 A1

(c) e.g.



B1

N-R uses intersec. of tangent at initial value with x-axis as next approx. — B1
at $x = 0.75$ this tangent is further away from root — B1 **(12)**

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6. *(a)* $z = 15 - 9i + iw$ ∴ $45 - 27i + 3iw + w = 14$ — M1
$w(1 + 3i) = -31 + 27i$ — M1
$\therefore w = \frac{-31+27i}{1+3i} \times \frac{1-3i}{1-3i} = \frac{50+120i}{10} = 5 + 12i$ — M1 A1
$3z = 14 - (5 + 12i) = 9 - 12i$ ∴ $z = 3 - 4i$ — M1 A1

(b) $(p + iq)^2 = 3 - 4i$; p, q real — M1
$p^2 + 2pqi - q^2 = 3 - 4i$ — M1
$\therefore p^2 - q^2 = 3$; $2pq = -4$ — A1
e.g. sub. for q giving $(p^2 - 4)(p^2 + 1) = 0$ — M1 A1
$p^2 = -1$ (no solns) or 4 ∴ $p = \pm 2$ — M1
sub. in giving $(2 - i)$ and $(-2 + i)$ — A1 **(13)**

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7. *(a)* $4 \sin 2\theta = 4 \cos \theta$
$4 \cos \theta (2 \sin \theta - 1) = 0$ — M1
$\sin \theta = \frac{1}{2}$ or $\cos \theta = 0$ giving $\theta = \frac{\pi}{6}$ or $\frac{\pi}{2}$ — M1 A1
at P, $\theta = \frac{\pi}{6}$ ∴ P is $(2\sqrt{3}, \frac{\pi}{6})$ — M1 A1

(b) area $= \frac{1}{2}\int_0^{\frac{\pi}{6}} (4 \sin 2\theta)^2 \, d\theta + \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (4 \cos \theta)^2 \, d\theta$ — M1 A1

$= 4\int_0^{\frac{\pi}{6}} 1 - \cos 4\theta \, d\theta + 4\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 1 + \cos 2\theta \, d\theta$ — M1 A2

$= 4[\theta - \frac{1}{4}\sin 4\theta]_0^{\frac{\pi}{6}} + 4[\theta + \frac{1}{2}\sin 2\theta]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ — M1 A2

$= 4[(\frac{\pi}{6} - \frac{\sqrt{3}}{8}) - (0) + (\frac{\pi}{2}) - (\frac{\pi}{6} + \frac{\sqrt{3}}{4})]$ — M1 A1

$= 4[\frac{\pi}{2} - \frac{3\sqrt{3}}{8}] = 2\pi - \frac{3}{2}\sqrt{3}$ — A1 **(16)**

Total **(75)**

Performance Record – P4 Paper C

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	inequality	1st order diff. eqn	series	2nd order diff. eqn	numerical methods	complex nos.	polar coords	
Marks	6	9	9	10	12	13	16	75
Student								