1. Find the set of values of x for which

$$\left|\frac{1}{x}\right| \le 2x + 1.$$ **(4 marks)**

2. Use suitable summation formulae to evaluate $\displaystyle\sum_{r=1}^{25} (r+2)^3$.   **(5 marks)**

3. One root of the equation $x^2 + bx + c = 0$, where b and c are real, has modulus 3 and argument $\dfrac{\pi}{3}$.

    (a) State the modulus and argument of the other root of the equation.   **(3 marks)**

    (b) Find the values of b and c.   **(4 marks)**

4. Find, in the form $P = f(T)$, the general solution of the differential equation

$$\frac{dP}{dT} = 2T - \frac{P}{2}.$$ **(8 marks)**

5. $f(x) = 2\sin\dfrac{x}{3} - e^{-x}$.

    (a) Show that the equation $f(x) = 0$ has a root in the interval $(0, 1)$ and another root in the interval $(9, 10)$.   **(2 marks)**

    (b) Use linear interpolation once on $(9, 10)$ to find an approximation to the root in this interval, correct to 2 decimal places.   **(3 marks)**

    (c) Use the Newton-Raphson process twice, with $x = 0.5$ as the starting value, to find a better estimate of the root in $(0, 1)$. Give your answer to a suitable degree of accuracy.   **(5 marks)**

6. $w = 15 - 8i$ and $z = -4 - 3i$.

    (a) Express in the form $a + bi$, where a and b are real, each of the complex numbers

    (i) $w - z$,   (ii) wz,   (iii) $\dfrac{w}{z}$.   **(5 marks)**

    (b) Show w and z on an Argand diagram, and describe the geometric relationship between w, z and $w - z$.   **(3 marks)**

    (c) Find a pair of real numbers r and s such that $r(w - z) + swz$ is real.   **(3 marks)**

7. Find the solution of the differential equation

$$\frac{d^2y}{dx^2} + 9\frac{dy}{dx} - 22y = 121x^2 + 11x$$

for which $y = 1$ when $x = 0$ and $\dfrac{dy}{dx} = 8$ when $x = 0$. **(14 marks)**

8. The curve C has polar equation

$$r = a\sin 2\theta, \quad 0 \le \theta \le \frac{\pi}{2},$$

where a is a positive constant.

(a) Sketch the curve C. **(2 marks)**

(b) Find the polar coordinates of the point on C at which the tangent is parallel to the initial line. Give the values of θ in radians to 3 significant figures. **(7 marks)**

(c) Find the area of the region contained by the curve C. **(7 marks)**