1. Find the set of values of x for which $\frac{2x}{x+2} > 1$. **(5 marks)**

2. Find all the solutions of the equation $x^3 - 4x^2 + 8x = 0$. **(5 marks)**

3. It is required to solve the equation $e^{\sin x} - 3x = 0$.
   (a) Show that this equation has a root between 0 and 1. **(2 marks)**
   (b) Taking $x = 0{\cdot}5$ as a first approximation, use the Newton-Raphson method once to obtain a second approximation to a root of this equation, to 3 decimal places. **(4 marks)**

4. (a) Show that $\sum_{r=4}^{n} (r^2 - 6r + 5) = \sum_{t=1}^{n-3} (t^2 - 4)$. **(3 marks)**
   (b) Hence or otherwise find an expression for this sum in terms of n, giving your answer in factorised form. **(5 marks)**

5. A curve passes through the point (0, 1). Its gradient $\frac{dy}{dx}$ at the point (x, y) satisfies the relationship $3\frac{dy}{dx} = y + e^x$
   (a) Use an integrating factor to solve this differential equation, expressing y in terms of x. **(8 marks)**
   (b) Sketch the curve for $0 \le x \le 2$. **(3 marks)**

6. The complex numbers z_1 and z_2 are given by

$$z_1 = 2 - 3i, \quad z_2 = a + bi.$$

   It is also given that $z_1 z_2 = 8 + 14i$.
   (a) Show on an Argand diagram the points P and Q which represent the complex numbers z_1 and $z_1 z_2$ respectively. **(2 marks)**
   (b) Find the values of a and b. **(4 marks)**
   (c) Verify that $|z_1 z_2| = |z_1| \times |z_2|$. **(3 marks)**
   (d) State the value of $z_2 z_2^*$. **(2 marks)**

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7. (a) If $x = e^t$ and y is a function of x,

   (i) show that $\frac{dy}{dx} = e^{-t}\frac{dy}{dt}$, **(2 marks)**

   (ii) obtain an expression for $\frac{d^2y}{dx^2}$ in terms of t, $\frac{dy}{dt}$ and $\frac{d^2y}{dt^2}$. **(5 marks)**

   (b) Use the substitution $x = e^t$ to find the general solution of the differential equation

$$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0.$$

(6 marks)

8. (a) Sketch on a single diagram the curves with polar equations $r = a\cos 2\theta$ and $r = 2a\sin^2\theta$, for $0 \le \theta < \frac{\pi}{4}$, where $a > 0$. **(4 marks)**

   Calculate, in terms of a and π,

   (b) the polar co-ordinates of the point, other than the pole, at which the two curves intersect, **(5 marks)**

   (c) the finite area enclosed between the curve $r = a\cos 2\theta$, $0 \le \theta \le \frac{\pi}{4}$, and the initial line. **(7 marks)**

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