PURE MATHS 4 (A) TEST PAPER 9 : ANSWERS AND MARK SCHEME

1. $\frac{-3+i}{1-2i}\cdot\frac{1+2i}{1+2i}=\frac{-5-5i}{5}=-1-i$ Mod = $\sqrt{2}$, arg = $-3\pi/4$ M1 A1 A1 A1 4

2. $\frac{1}{4}n^2(n+1)^2+\frac{1}{2}n(n+1)-(n+1)=\frac{1}{4}(n+1)(n^3+n^2+2n-4)$ M1 A1 A1
$=\frac{1}{4}(n+1)(n-1)(n^2+2n+4)=\frac{1}{4}(n^2-1)((n+1)^2+3)$ M1 A1 5

3. (a) Curve sketched, with loop B4
(b) Circle sketched; centre = $(a, \pi/2)$ B1 B1 6

4. (a) f(–2) = 2·86, f(–1) = –0·37, so root between –2 and –1 M1 A1
(b) Successive estimates : –1·5, –1·25, –1·125 M1 A1 A1 A1
(c) $x^2>2$ $x<-\sqrt{2}$ or $x>\sqrt{2}$ M1 A1 A1 9

5. (a) Integrating factor = $e^{\int -2\,dt}=e^{-2t}$ $e^{-2t}\frac{dx}{dt}-2xe^{-2t}=3te^{-2t}$ B1 M1 A1
$\frac{d}{dt}(xe^{-2t})=3te^{-2t}$ $xe^{-2t}=3(-\frac{1}{2}te^{-2t}-\frac{1}{4}e^{-2t})+c$ A1 M1 A1
$x=ce^{2t}-\frac{3}{2}t-\frac{3}{4}$ A1
(b) $c-0{\cdot}75=1{\cdot}25$ $c=2$ $t=1{\cdot}5$: $x=2e^3-3\approx 37{\cdot}2$ M1 A1 M1 A1 11

6. (a) Points at (3, 4), $(1/2, -(\sqrt{3})/2)$ B1 B1
(b) $ww^*=(3+4i)(3-4i)=25$, real M1 A1
(c) $|w|=5$, arg $w=\arctan(4/3)\approx 0{\cdot}93°$ B1 B1
$|z|=1$, arg $z=\arctan(-\sqrt{3})=-\pi/3$ B1 B1
(d) $r=|wz|=5$ $\theta=\arg(wz)=-0{\cdot}12$ M1 A1 M1 A1 12

7. (a) Area $=\frac{1}{2}\cdot 2\int_0^{\pi/6}16\cos^2 3\theta\,d\theta=\int_0^{\pi/6}8(\cos 6\theta+1)\,d\theta$ B1 M1 A1
$=[\frac{4}{3}\sin 6\theta+8\theta]_0^{\pi/6}=\frac{4\pi}{3}$ M1 A1 A1
(b) $y=r\sin\theta=4\cos 3\theta\sin\theta$ M1 A1
$dy/d\theta=4(\cos 3\theta\cos\theta-3\sin 3\theta\sin\theta)=0$ at required points M1 A1
$\cos 3\theta\cos\theta=3\sin 3\theta\sin\theta$ $3\tan\theta\tan 3\theta=1$ M1 A1 12

8. (a) (i) Aux. eqn. is $2u^2-6u+17=0$ $u=(3\pm 5i)/2$ B1 M1 A1
$y=e^{3x/2}(a\sin 5x/2+b\cos 5x/2)$ M1 A1
(ii) Let P.I. be $px+q$ $-6p+17px+17q=17x+1$ M1 M1
$p=1, q=7/17$ A1 A1
$y=e^{3x/2}(a\sin 5x/2+b\cos 5x/2)+x+7/17$ A1
(b) $y'=e^{3x/2}[\frac{3}{2}(a\sin 5x/2+b\cos 5x/2)+\frac{5}{2}(a\cos 5x/2-b\sin 5x/2)]+1$ M1 A1
$y(0)=1: b+7/17=1$ $b=10/17$ B1
$y'(0)=-1: 3b/2+5a/2=-2$ $5a/2=-49/17$ $a=-98/85$ M1 A1
$y=e^{\frac{3x}{2}}\left(\frac{10}{17}\cos\frac{5x}{2}-\frac{98}{85}\sin\frac{5x}{2}\right)+x+\frac{7}{17}$ A1 16