PURE MATHS 4 (A) TEST PAPER 8 : ANSWERS AND MARK SCHEME

1.	$a^2 + (1-a)^2 = 5/8$ $\quad$ $16a^2 - 16a + 3 = 0$	M1 A1	
	$(4a-1)(4a-3) = 0$ $\quad$ $a = 1/4$ or $a = 3/4$	A1 A1	4
2.	(a) Half-line through pole ($y = -x\sqrt{3}$) and vertical line ($x = -1$)	B1 M1 A1	
	(b) $r = \sec \pi/3 = 2$ $\quad$ Point has polar co-ords $(2, 2\pi/3)$	M1 A1 A1	6
3.	$\Sigma\,(9r^2 - 3r - 2) = 9 \times \frac{1}{6}n(n+1)(2n+1) - 3 \times \frac{1}{2}n(n+1) - 2n$	M1 A1 A1 A1	
	$= 3n^3 + 9n^2/2 + 3n/2 - 3n^2/2 - 3n/2 - 2n = 3n^3 + 3n^2 - 2n$	M1 A1 A1	7
4.	(a) Graphs sketched : (i) asymptotes $x = -1$, $y = 0$, through $(0, 1)$	B2	
	(ii) asymptotes $x = 1$, $y = 1$, through $(0, 0)$	B2	
	(b) From graphs or otherwise, solution set is $-1 < x < 1$	M1 A1 A1	7
5.	$u^2 + 9u = 0$ has roots $u = \pm 3\text{i}$, so C.F. is $y = a \sin 3x + b \cos 3x$	M1 A1 A1	
	For P.I., let $y = p \sin x + q \cos x$ $\quad$ Then $y'' = -y$	M1 A1	
	$8y = \sin x + \cos x$ $\quad$ $p = q = 1/8$	M1 A1	
	$y = a \sin 3x + b \cos 3x + (\sin x + \cos x)/8$	A1	8
6.	(a) Graphs sketched : $y = \ln(x+1)$ through $(0, 0)$,	B2	
	$y = 2 \cos 3x$ through $(0, 1)$, $(\pi/6, 0)$, $(\pi/2, 0)$, $(5\pi/6, 0)$	B2	
	(b) Let $f(x) = 2 \cos 3x - \ln(x+1)$ $\quad$ $f(0) = 2$, $f(1) = -2{\cdot}67$	M1 A1	
	(c) $f'(x) = -6 \sin 3x - 1/(x+1)$ $\quad$ $\pi/6 - f(\pi/6) / f'(\pi/6) =$	B1 B1 M1	
	$0{\cdot}4603$ $\quad$ Repeating the process, get $0{\cdot}4601$	A1 M1 A1	12
7.	(a) $y = \dfrac{1}{z}$, so $\dfrac{dy}{dx} = -\dfrac{1}{z^2}\dfrac{dz}{dx}$ $\quad$ $-\dfrac{1}{z^2}\dfrac{dz}{dx} + \dfrac{1}{xz} = \dfrac{x^3}{z^2}$ $\quad$ $\dfrac{dz}{dx} - \dfrac{z}{x} = -x^3$	M1 A1 M1 A1 A1	
	(b) Int. factor is $e^{\int -1/x\,dx} = 1/x$ $\quad$ $\dfrac{1}{x}\dfrac{dz}{dx} - \dfrac{z}{x^2} = -x^2$	M1 A1 A1	
	$\dfrac{d}{dx}\left(\dfrac{z}{x}\right) = -x^2$ $\quad$ $\dfrac{z}{x} = -\dfrac{x^3}{3} + c$ $\quad$ $z = cx - \dfrac{x^4}{3}$	M1 A1	
	$y = \dfrac{3}{kx - x^4}$ $\quad$ $y(1) = 1 : k = 4$ $\quad$ $y = \dfrac{3}{x(4 - x^3)}$	A1 M1 A1	13
8.	(a) Conjugate of $2 - \text{i}$, so $m = 2$, $n = 1$	M1 A1 (both)	
	(b) Equation is $(x-1)(x-2+\text{i})(x-2-\text{i}) = 0$	B1	
	i.e. $(x-1)(x^2 - 4x + 5) = 0$ $\quad$ $b = -5$, $c = 9$, $d = -5$	M1 A1 A1 A1	
	(c) Points at $(1, 0)$, $(2, -1)$, $(2, 1)$	B1 B1 B1	
	(d) 1 has modulus 1, argument 0	B1	
	$2 + \text{i}$ has modulus $\sqrt{5}$, argument $\arctan(1/2) = 0{\cdot}463$	B1 B1	
	$2 - \text{i}$ has modulus $\sqrt{5}$, argument $-0{\cdot}463$	B1 B1	
	(e) $\dfrac{2-\text{i}}{2+\text{i}} = \dfrac{(2-\text{i})(2-\text{i})}{(2+\text{i})(2-\text{i})} = \dfrac{3-4\text{i}}{5} = \dfrac{3}{5} - \dfrac{4}{5}\text{i}$	M1 A1 A1	18